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Top quark polarization and gluon spin and transversity in the nucleon

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Top pair production at LHC is a prime example of production that proceeds primarily via gluon fusion. Decays of polarized top pairs through various channels produce a variety of correlations among the decay products - particles and jets. Combinations of the gluon distributions, either polarized or unpolarized, can be accessed experimentally through angular dependences of decay products, as will be shown, along with predictions from a spectator model of gluon distributions.
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1. Introduction

The structure of the hadrons, as determined by QCD, has been studied for many years, especially focused on the nucleon. Theoretical efforts to model that structure via interacting quarks and gluons are constrained by non-perturbative physics at nucleon size. Experimental probes at lepton accelerators have informed the field since the early measurements of the electromagnetic form factors and later, the parton distributions. Complicating and enriching the understanding of the constituent structure have been measurements of spin dependence. The picture of the nucleon as a three quark system was disproven by longitudinal spin dependence. The "spin crisis" of 1987 arose from the EMC experiment [1] A flowering of different interpretations followed, with attention to gluon contributions and orbital angular momenta. Various spin dependent distributions of quarks and gluons were defined and connected with different kinds of measurements.

There are several kinds of gluon and quark distributions within the nucleon and other hadrons. The most general structures are the Generalized Transverse Momentum Distribution Functions (GTMD's), that depend on the virtual photon+nucleon scattering variables $(x, \xi, \Delta, k_t^2, Q^2)$. Integrations over k_t^2 or Δ reduce these to Generalized Parton Distributions (GPDs) or Transverse Momentum Distributions (TMDs), respectively. Unintegrated models for either of these distributions default back to GTMDs, and through Fourier transforms, to Wigner distributions [2]. The model that will be in the background here, providing a direction for coupling the top pairs to the gluon distributions,is a particular extension of a model for GPD's - the Reggeized spectator model referred to as the "flexible parameterization" scheme [3], which has a natural generalization to the gluon and sea quark GPDs [4]. The model for gluon GPDs, with parameterization fixed by various constraints, is directly related to the gluon transversity pdf, $h_1^g(x, Q^2)$ and for non-exclusive or semi-inclusive processes (SIDIS), is the TMD $h_1^g(x, \vec{k}_T^2)$. We carry over a spectator picture for quark color triplet and spectator diquark color anti-triplet to the gluon plus spectator picture. This makes the struck nucleon to have a color octet gluon plus a color octet with baryon number 1. The struck gluon interacts with a probing virtual photon by charged pair of produced quarks in a color octet symmetric state [4]. These details will be described in work in progress.

The gluon TMDs are the distributions that can be measured in the process being advocated here, the production of single top-antitop pairs in proton+proton interactions at the LHC. To adumbrate the spin correlations for the gluons and the top pairs, I will use the GPDs that can be formally connected to the amplitudes for the process. Thereafter, the substitution of TMDs for the GPDs is straightforward.

The emphasis in this paper, will be on the top-antitop spin correlations, since these are seldom discussed. Relevant features of the gluon distributions have been discussed in Ref. [4] and details will be presented in a forthcoming paper.

2. Top-Antitop Spin Correlations

Before the discovery of the top quark at the Fermilab Tevatron, one proposed method to disentangle the signal for top quark production from the daunting background of multiple hadron events was to concentrate on the spin correlations of the top and antitop decay products. The "golden events" were expected to be the dilepton events in which two very energetic opposite sign leptons would signal the weak decays of each top into b-quarks and W's, the latter decaying leptonically. The actual observations of top quarks by the D0 [5] and CDF [6] groups did not use the spin correlations. Nevertheless, these correlations provide a test of the QCD mechanism [7]. The LHC now produces many more top quarks. The higher energy makes quark-antiquark annihilation less important than gluon fusion. Gluon fusion, involving the merging of two vector particles, gives rise to quite distinct spin correlations among the top decay products. I will present the corresponding spin density matrices and angular correlations.

What is known about single polarization of the top only or antitop only in the pair production? Recent determination at the LHC of top single spin asymmetry (SSA) are small - from ATLAS $A_p = -0.035 \pm 0.040$ and from CMS $A_p = 0.005 \pm 0.01$ [8]. An explanation based on one loop QCD calculations with an ansatz for "recombination" [9], predicts peaks close to -0.05 vs. p_T, over a range of x_F as shown in Fig. 1. This prediction is within the small values and their uncertainties determined by both CMS and ATLAS. The measurement of such single spin asymmetries will test an important mechanism whereby QCD can perturbatively generate appropriate interferences to polarize heavy quarks [9] or non-perturbatively through sets of light quark pair production [10] work in progress.

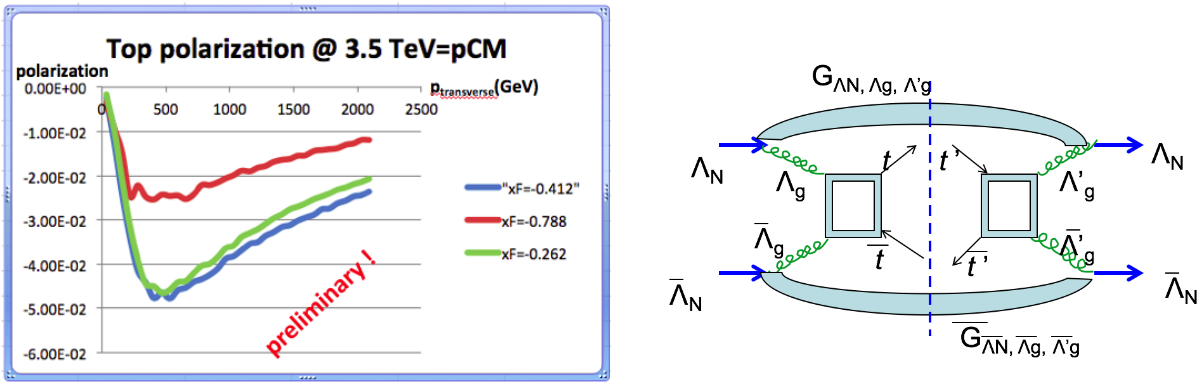

Figure 1: Predicted top polarization from p+p collisions at 3.5 TeV vs. p_T at 3 values of x_F, based on perturbative QCD model of Dharmaratna and Goldstein Ref. [9]. (left) Illustration of cross section for definite helicities in p+p $\rightarrow$ t+$\bar{\text{t}}$ +X. (right)

The spin correlations for the top-antitop pairs produced by unpolarized p+$\bar{\text{p}}$ or p+p collisions can be calculated precisely at tree level QCD for the quarks or gluons and folded into the relevant parton distribution functions. At this point there are enough top pairs in the data from ATLAS and CMS to begin to use the spin correlations as probes of the production mechanism. It has become clear that heavy hadron production [11] or even Higgs boson production at very high energies could provide a measure of the *polarized gluon* distributions in the protons. It is now possible, and especially interesting to use the top pair spin correlation as a lever to disentangle the gluon polarization distributions.

In the following we present the tree level production mechanisms for $\text{g}_{X\,\text{or}\,Y} + \text{g}_{X\,\text{or}\,Y} \rightarrow \text{t}_\pm + \bar{\text{t}}_\pm + \text{X}$, where the gluon subscripts are linear polarization directions and the t-quark subscripts are helicities. The existence of top spin correlations at the LHC has been established. A standard

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parametrization is to represent the top-antitop cross section asymmetries as

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4}(1 + B_1\cos\theta_1 + B_2\cos\theta_2 - C_{helicity}\cos\theta_1\cos\theta_2) \tag{2.1}$$

where the polar angles θ_1, θ_2, for the decay product leptons from the top and antitop, are measured relative to the t-direction in the t+$\bar{t}$ center of momentum. The measurements of $C_{helicity}$ by ATLAS and CMS agree with the QCD calculations of Ref. [12]. In general, however, it is the azimuthal dependences that are important for different polarized initial states. The full angular dependences of the top-antitop spin correlations as they depend on gluon distributions are shown in the following.

3. Gluon-top pair observables

The QCD amplitudes for g+g→ t+ $\bar{t}$ are well known at tree level [9, 13]. Let the amplitudes be $A_{\Lambda_{g1},\Lambda_{g2};t,\bar{t}}(\hat{s},\hat{t})$ with $\Lambda_{g1},\Lambda_{g2}$ the gluon helicities, t ,$\bar{t}$ the top and antitop helicities, and $\hat{s},\hat{t}$ the kinematic invariants in an arbitrary frame. Let $g^{(1)}_{\Lambda_{N1},\Lambda_{X1},\Lambda_{g1}}(x_1,k_T,M^2_{X1})$ be the amplitude for proton number 1 to emit a gluon no.1 with longitudinal momentum fraction x_1 and transverse momentum k_T with respect to the proton 3-momentum in the collider frame (p + p CM), along with an unspecified residual $X1$ of mass M_{X1}, with corresponding $g^{(2)}$ for the other proton. The overall amplitude for N_1 + N_2 → t+ X_1 +$\bar{t}$ + X_2 is then

$$g^{(1)}_{\Lambda_{N1},\Lambda_{X1},\Lambda_{g1}} g^{(2)}_{\Lambda_{N2},\Lambda_{X2},\Lambda_{g2}} A_{\Lambda_{g1},\Lambda_{g2};t,\bar{t}}\,. \tag{3.1}$$

To construct differential cross sections for unpolarized colliding protons, this amplitude must be combined with its conjugate and summed and integrated over unobserved quantities as in Fig. 1. The terms can be rearranged to correspond to gluon distributions and hard scattering amplitude products

$$G^{(1)}_{\Lambda_{N1},\Lambda_{g1},\Lambda'_{g1}} = \sum_{\Lambda_{X1}}\int_{X_1} g^{(1)*}_{\Lambda_{N1},\Lambda_{X1},\Lambda'_{g1}} g^{(1)}_{\Lambda_{N1},\Lambda_{X1},\Lambda_{g1}}\,. \tag{3.2}$$

Then the multiple sum can be written more compactly as a double density matrix,

$$\rho_{t',\bar{t}';t,\bar{t}} = \sum_{\Lambda_{g1},\Lambda_{g2},\Lambda'_{g1},\Lambda'_{g2}} \sum_{\Lambda_{N2},\Lambda_{N1}} G^{(2)}_{\Lambda_{N2},\Lambda_{g2},\Lambda'_{g2}} G^{(1)}_{\Lambda_{N1},\Lambda_{g1},\Lambda'_{g1}} A^*_{\Lambda'_{g1},\Lambda'_{g2};t',\bar{t}'} A_{\Lambda_{g1},\Lambda_{g2};t,\bar{t}}\,. \tag{3.3}$$

When the summations over the various unmeasured helicities are carried out and parity relations are used, four distinct terms arise, each a variation of the form

$$\rho^{LP,LP}_{t',\bar{t}';t,\bar{t}} = \left[A^*_{LL,t',\bar{t}}A_{RR,t,\bar{t}} + A^*_{LR,t',\bar{t}}A_{RL,t,\bar{t}} + A^*_{RL,t',\bar{t}}A_{LR,t,\bar{t}} + A^*_{RR,t',\bar{t}}A_{LL,t,\bar{t}}\right], \tag{3.4}$$

so that

$$\rho_{t',\bar{t}';t,\bar{t}} = \sum_{\Lambda_{N1},\Lambda_{N2}} \{G^{(2)}_{\Lambda_{N2},UP}\rho^{UP,UP}_{t',\bar{t}';t,\bar{t}} G^{(1)}_{\Lambda_{N1},UP} + G^{(2)}_{\Lambda_{N2},UP}\rho^{UP,LP}_{t',\bar{t}';t,\bar{t}} G^{(1)}_{\Lambda_{N1},LP} + \\ + G^{(2)}_{\Lambda_{N2},LP}\rho^{LP,UP}_{t',\bar{t}';t,\bar{t}} G^{(1)}_{\Lambda_{N1},UP} + G^{(2)}_{\Lambda_{N2},LP}\rho^{LP,LP}_{t',\bar{t}';t,\bar{t}} G^{(1)}_{\Lambda_{N1},LP}\}\,. \tag{3.5}$$

The subscripts R, L correspond to gluon helicities ± 1. Because of parity relations the combination of gluon distributions that appear in the summation is limited. The two independent combinations correspond to linear polarization states:

$$G^{(1)}_{\Lambda_{N1},R,R} + G^{(1)}_{\Lambda_{N1},L,L} = G^{(1)}_{\Lambda_{N1},XX} + G^{(1)}_{\Lambda_{N1},YY} = G^{(1)}_{\Lambda_{N1},UP}, \tag{3.6}$$

$$G^{(1)}_{\Lambda_{N1},R,L} + G^{(1)}_{\Lambda_{N1},L,R} = G^{(1)}_{\Lambda_{N1},YY} - G^{(1)}_{\Lambda_{N1},XX} = G^{(1)}_{\Lambda_{N1},LP}. \tag{3.7}$$

The UP and LP subscripts on the right are for unpolarized and linearly polarized gluons. The $\hat{X}$ & $\hat{Y}$ directions are transverse to the gluon 3-momentum $\vec{k}_1$, with $\hat{X}$ in the g_1+$g_2 \to$ t+$\bar{t}$ scattering plane. For gluon number 2, the 3-momentum $\vec{k}_2$ is neither parallel nor anti-parallel to $\vec{k}_1$, in general, but the $X-Z$-planes coincide. So the $\hat{X}$ direction for g_2 differs from g_1, but the $\hat{Y}$ directions coincide.

The tree level hard scattering amplitudes $A_{\Lambda_{g1},\Lambda_{g2};t,\bar{t}}$ can be evaluated in the CM frame in terms of the variables $\hat{s}$, θ, β and the color factors for the $(8)\otimes(8)\to(3)\otimes(\bar{3})$. The azimuthal dependence enters these density matrix elements. The double density matrix elements will involve azimuthal dependences $I, e^{\pm 2i\phi}, e^{\pm 4i\phi}$. With these amplitudes, and their relations to the gluon he-

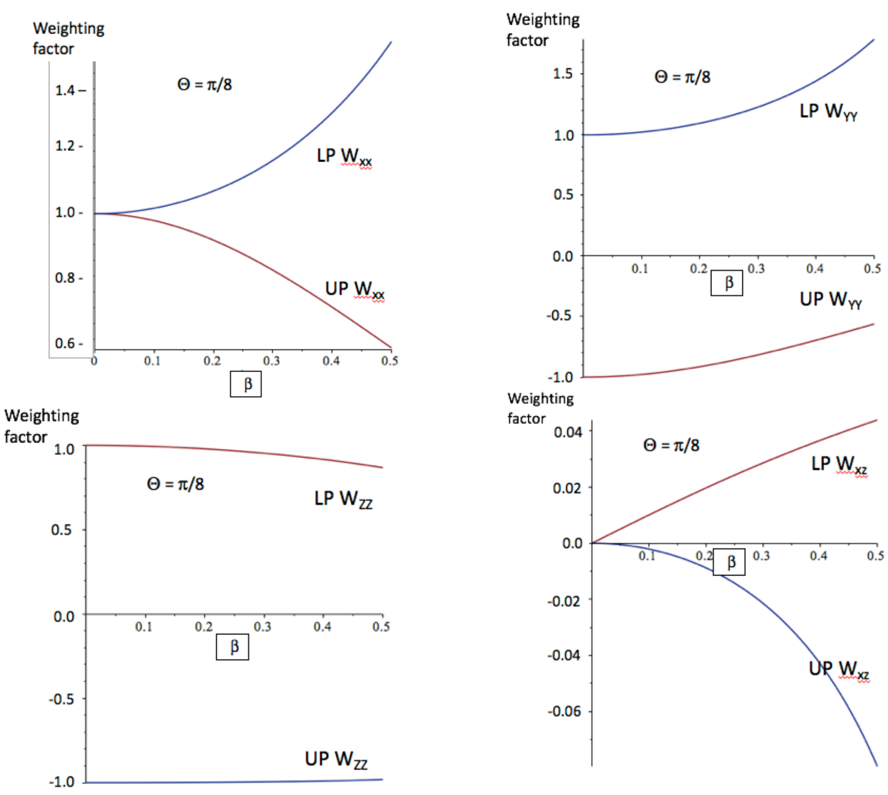

Figure 2: Weighting for Cartesian components of $\hat{p}(\mu^+)$, $\hat{p}(\mu^-)$, plotted for varying β, the magnitude of relativistic velocity of the top in the t+$\bar{t}$ Center-of-Mass frame. Each is plotted for unpolarized and transverse-linear polarized gluon distributions. Each lepton momentum is evaluated in the corresponding top or anti-top rest frame, with directions defined by CM.

licities or transversities, the top-antitop decays provide markers of polarizations.

4. Top decay distributions

The top quark decays through the weak interaction, primarily into a W and b-jet. This happens rapidly, since the top mass is so large, making the weak coupling sufficient to produce the decay probability competitive with the strong interaction. Because of this rapid quark decay, there is not enough time for top quarks to form top hadronic states. There are no top mesons or baryons.The W in the decay, in turn, can decay into a charged quark pair or lepton + neutrino pair. The semi-leptonic decays of the top quark afford the best opportunity for polarization analysis [14]. The opposite-sign leptons usually have very high transverse momenta and are accompanied by b-quark jets. So the double correlation of top spins is manifested in the joint decay distributions into leptons and b-jets. Other decay channels involved are presented elsewhere [14].

As shown in Ref. [14], the amplitude for a polarized top quark at rest to decay into a measured b-quark and antilepton (or u-quark) along with an unobserved neutrino (or ignoring the $\bar{d}$ jet) is completely determined in the Standard Model. Since the neutrino is not observed, its 3-momentum is fixed to lie on an ellipse in a lepton-b-quark coordinate system. Using an associated ellipse construction for the unmeasured neutrino momentum allows for the specification of the top-antitop center of mass and the related polarization. This process is described in detail in ref.[?]. An alternative approach to extracting missing momenta and polarizations uses cubic equations from momentum conservation.

The quark or gluon spin correlations are transmitted to the decay products. The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions, which has been expressed as a weighting factor [7] in the light quark-antiquark annihilation mechanism.

The gluon fusion mechanism for the weighting factor for unpolarized gluons, is summed over gluon helicities. This gives rise to a fourth order angular distribution:

$$\begin{aligned} W(\theta, p, p_{\bar{l}}, p_l) = \frac{1}{4} - \frac{1}{4}\{ & [(1-\beta^2)^2 + \sin^4\theta)] (\hat{p}_{\bar{l}})_x (\hat{p}_l)_{\bar{x}} + \\ & +[-(1-\beta^2)^2 - (1-2\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_y (\hat{p}_l)_{\bar{y}} + \\ & +[(1-\beta^4) - 2\beta^2\sin^2\theta + \sin^4\theta](\hat{p}_{\bar{l}})_z (\hat{p}_l)_{\bar{z}} + \\ & +2(\beta/\gamma)\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x (\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z (\hat{p}_l)_{\bar{x}}]\} \\ & /4\left[(1-\beta^4) + 2\beta^2\sin^2\theta + (1-2\beta^2)\sin^4\theta\right]. \end{aligned} \tag{4.1}$$

where m is the top quark mass, θ is the top quark production angle in the quark-antiquark or $\bar{t}t$ CM frame, p is the light quark or gluon CM momentum, β is the magnitude of the relativistic velocity of the top or antitop quark in the CM, $\hat{p}_{\bar{l}}$ is the l^+ momentum direction in the top rest frame and $\hat{p}_l$ is the corresponding l^- direction in the antitop rest frame.

We can now separate the dilepton angular distributions into different components for the four different combinations of gluon distributions. For the (LP, LP) case, which measures the linearly polarized gluon pair.

$$\begin{aligned} W^{(LP,LP)}(\theta, p, p_{\bar{l}}, p_l) = -\frac{1}{4} + \frac{1}{4}\{ & [(1-\beta^4) + \beta^2\sin^2\theta(-2 + (2-\beta^2)\sin^2\theta)](\hat{p}_{\bar{l}})_x (\hat{p}_l)_{\bar{x}} + \\ & +[(1-\beta^4) + \beta^2\sin^2\theta(2-\beta^2\sin^2\theta)](\hat{p}_{\bar{l}})_y (\hat{p}_l)_{\bar{y}} + \end{aligned}$$

$$+[-(1-\beta^2)^2+\beta^2(2-\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}+ \\ -4(\beta^2/\gamma)\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}}-(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\} \\ /\left[(1-\beta^2)^2+\beta^4\sin^4\theta\right]. \tag{4.2}$$

In Figure 2 we show the directional correlation distributions for an unpolarized gluon distribution and a linear-transverse polarized gluon distribution. We have not included any particular values of gluon distribution functions. For that, we would convolute our spectator model distributions with the weights. The distributions are rich in dependences on the energies and angles for the t + $\bar{t}$ pair and the dilepton momenta. The W_{ij} is the θ and β dependent factor multiplying the Cartesian components of $\hat{p}(\mu^+)_i\,\hat{p}(\mu^-)_j$, plotted for varying β, the magnitude of relativistic velocity of the top in the t + $\bar{t}$ Center-of-Mass frame. The lepton momenta are determined in their top or antitop rest frames. Coordinates in the t + $\bar{t}$ CM are determined as follows. The t + $\bar{t}$ pair have momentum $\vec{p}_{t+\bar{t}}$ in the p+p collider CM. In the t + $\bar{t}$ CM the pair of gluons (or quark-antiquark) has zero 3-momentum, so the orientation of one gluon relative to the top in the t + $\bar{t}$ CM is the angle θ boosted from the p+p CM.

For illustration we chose the polar angle of the t + $\bar{t}$ CM to be $\theta=\pi/8$ and varied β. The resulting weighting factors are remarkable for the clear distinction between unpolarized and polarized gluons.

5. Gluon TMDs

Gluon TMDs and GPDs are more complicated (at leading twist and beyond) than their valence quark counterparts. For one thing, several of the gluon analogs of the quark TMDs at small x, like the Sivers function, allow two kinds of gluon distributions. Which of the two is probed is process dependent. These interesting distinctions have been studied over time, beginning with the realization that there are two ways to insert gauge links into the hadronic matrix elements of the gluon-gluon correlator [15]:

$$\Gamma^{\mu\nu[\mathscr{U},\mathscr{U}']}(x,\mathbf{k}_T)\equiv\int\frac{d(\xi\cdot P)d^2\xi_T}{(P\cdot n)^2(2\pi)^3}e^{i(xP+k_T)\cdot\xi}\langle P\mid \mathrm{Tr}_c[F^{+\nu}(0)\mathscr{U}_{[0,\xi]}F^{+\mu}(\xi)\mathscr{U}'_{[\xi,0]}]\mid P\rangle\,|_{\xi\cdot n=0} \tag{5.1}$$

$$\text{with}\quad \mathscr{U}_{[0,\xi]}=\mathscr{P}\exp\left(-\mathrm{ig}\int_{\mathscr{C}[0,\xi]}\mathrm{ds}_\mu \mathrm{A}^\mu(\mathrm{s})\right), \tag{5.2}$$

for a contour $\mathscr{C}[0,\xi]$ (wherein the 4-vector notation $\xi=[0^+,\xi^-,\xi_T]$). The path determined gauge links can point to $\xi^-\to\pm\infty^-$.

At small x, pairing these two path definitions corresponds to Weizäcker-Williams $[++]$ and dipole $[+-]$ distributions, respectively. For Weizäcker-Williams (WW), γ^* + g $\to\gamma'$ + g$'$, the incoming γ^* can be replaced by equivalent on-shell gammas for which there will be no initial state interactions. The dipole approximation (DP) has the $\gamma^*\to$ q +$\bar{q}$ Fock states and the quark or antiquark interacts with the gluon, hence an initial state interaction. These two are distinguished by the color structure, the f-type or d-type coupling, at the tree level. Because the top pair production has to occur at moderate values of x at the LHC, only the WW case is of relevance, simplifying the analysis of future data on top pair production cross sections and spin asymmetries. The initial state interactions of quark-antiquark pairs that would arise from energetic off-shell gammas in the dipole picture do not complicate the relation to the gluon fusion picture.

The double helicity flip for GPDs **does not mix** with quark distributions, which makes gluon transversity unique and useful. In the definition of transversity [16] for on-shell gluons or photons, wherein there are no helicity 0 states, the transversity states written as linear combinations of helicity states are:

$$
\begin{aligned}
|+1)_{trans} &= \{|+1\rangle + |-1\rangle\}/2 = |-1)_{trans}\,, \\
|0)_{trans} &= \{|+1\rangle - |-1\rangle\}/\sqrt{2}\,, \\
\text{helicity} \quad |\pm 1\rangle &= \{\mp\hat{x} - i\hat{y}\}/\sqrt{2}\,.
\end{aligned}
$$

where the two-body scattering plane is the X-Z plane, with $\hat{y}$ along the normal to the scattering plane. That this non-mixture - of gluon double flip distributions with quark distributions - carries over to gluon TMDs should follow from integration of the more general GTMDs for double helicity flip gluons.

In a forthcoming publication [4] we will present our explicit model for the gluon GTMDs, generalizing from the Regge-diquark spectator model, the "flexible model". We will address some questions that are unique to gluon distributions: how are the t and skewness ξ dependences normalized? How is the small x behavior accounted for? What is the connection to the Pomeron?

In hadronic collision processes, the gluon distributions are folded into the more probable initial and final state interactions. Nevertheless, we will see that at the LHC, the production of top pairs can enhance the ability to separate out a form of polarized gluon contributions.

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