PROCEEDINGS OF SCIENCE

The effects of the birth of particles in the field of waves of high intensity

Ekaterina Dmitrieva[a,b,*] **and Petr Satunin**[a,b]

[a] *Institute for Nuclear Research of the Russian Academy of Sciences, 60th October Anniversary Prospect, 7a, 117312 Moscow, Russia*

[b] *Moscow State University, Leninskiye Gory, 119991 Moscow, Russia*

E-mail: edmitrieva@inr.ru, satunin@ms2.inr.ac.ru

We study the process of massive particle production in a plane wave of massless field of high intensity, in a toy model of two interacting scalar fields. We solve the Heisenberg equation for quantum amplitudes, and find resonantly growing solution similar to parametric resonance. We complement the study [1] in which only the case of small masses has been considered, extending to the case of arbitrary masses. We present the instability threshold for the plane wave amplitude depending on the mass. We have shown that the particle production effect is also observed outside the low-mass approximation.

International Conference on Particle Physics and Cosmology (ICPPCRubakov2023)
02-07, October 2023
Yerevan, Armenia

[*] Speaker

PoS(ICPPCRubakov2023)047

1. Introdoction

The process of particle formation in intense external fields contains interesting phenomena [2–7], such as the creation of particles in strong electromagnetic fields [7, 8]. These processes cannot always be described by the standard perturbation theory [9–14]. For example, phenomena similar to parametric resonance are described in [15–19]. In order for parametric resonance to occur, the time-fluctuating field must interact with the field associated with the forming particles. Such conditions may be possible at the reheating stage after inflation [6, 9, 18, 20–22].

A toy model of a similar process, for which the oscillating field depends on both time and spatial coordinate, was studied by A.Arza[1]. This model includes two scalar fields. In the work mentioned above, it was demonstrated that the formation of massive particles is possible with a sufficiently large amplitude of a plane wave of a massless field, so large that it is unstable. The author considered the case of an approximation of low mass compared to the frequency of a plane wave. In this article, we study how the result will change if we do not use this approximation and consider the more general case of any mass.

2. Instability in two-scalar model

Consider two interacting scalar fields ϕ and χ,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 - g\phi\chi^2, \tag{1}$$

where m_χ is a mass of χ while ϕ is massless, g is a coupling constant,

$$\phi(\vec{x}, t) = \frac{\sqrt{2\rho_\phi}}{\omega_p}\cos(\vec{p}\cdot\vec{x} - \omega_{\vec{p}}t), \tag{2}$$

Here $\vec{p}$ and $\omega_{\vec{p}} = \sqrt{\vec{p}^2}$ are spatial momentum and frequency of the plane wave, ρ_ϕ is its averaged energy density.

The corresponding equations of motion (EoM) of the model (1) will be

$$\Box\phi = -g\chi^2, \tag{3}$$

$$(\Box + m_\chi^2)\chi = -2g\phi\chi. \tag{4}$$

We are interested in the case when ϕ is a beam of particles with momentum $\vec{p}$. In this case, we can consider ϕ as a classical field. We consider time scales in which ϕ is not significantly depleted. This allows us to neglect the back reaction. Then we can represent ϕ as a monochromatic plane wave (2). And we will write the quantum field χ using the creation and annihilation operators in Fourier as

$$\chi = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\Omega_{\vec{k}}}}\left(\chi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}} + \chi_{\vec{k}}(t)^\dagger e^{-i\vec{k}\cdot\vec{x}}\right), \tag{5}$$

where $\Omega_{\vec{k}} = \sqrt{k^2 + m_\chi^2}$ and the operators $\chi_{\vec{k}}$ and $\chi_{\vec{k}}^\dagger$ simultaneously satisfy the commutation relations $[\chi_{\vec{k}}, \chi_{\vec{k}`}] = 0$, $[\chi_{\vec{k}}, \chi\dagger_{\vec{k}`}] = (2\pi)^3\delta^3(\vec{k} - \vec{k}`)$. Substituting the decomposition (5) into

EoM (4) with (2) and denoting $A_{\vec{k}} = \chi_{\vec{k}} + \chi^\dagger_{-\vec{k}}$, as a result, we get,

$$(\partial_t^2 + \Omega_{\vec{k}}^2) A_{\vec{k}} = -\omega_{\vec{p}}^2 \alpha \left(\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}} A_{\vec{k}-\vec{p}} e^{-i\omega_{\vec{p}} t} + \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}} A_{\vec{k}+\vec{p}} e^{i\omega_{\vec{p}} t} \right), \tag{6}$$

where $\alpha \equiv \frac{g\sqrt{2\rho_\phi}}{\omega_{\vec{p}}^3}$. Let's pick out the standard oscillation part of $\chi_{\vec{k}}$ as $\chi_{\vec{k}} = a_{\vec{k}}(t) e^{-i\Omega_{\vec{k}} t}$ and $\chi^\dagger_{\vec{k}} = a^\dagger_{\vec{k}}(t) e^{i\Omega_{\vec{k}} t}$ and substitute into the equation (6). The time evolution for the amplitudes $a_{\vec{k}}$, $a^\dagger_{\vec{k}}$ is governed by the equation,

$$\begin{aligned} e^{-i\Omega_{\vec{k}} t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}} \dot{a}_{\vec{k}}) + e^{i\Omega_{-\vec{k}} t}(\ddot{a}^\dagger_{-\vec{k}} + 2i\Omega_{-\vec{k}} \dot{a}^\dagger_{-\vec{k}}) = \\ = -\omega_{\vec{p}}^2 \alpha \left(\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}} \left(a^\dagger_{-\vec{k}-\vec{p}} e^{i(\Omega_{-\vec{k}-\vec{p}} + \omega_{\vec{p}})t} + a_{\vec{k}+\vec{p}} e^{-i(\Omega_{\vec{k}+\vec{p}} - \omega_{\vec{p}})t} \right) + \right. \\ \left. + \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}} \left(a^\dagger_{-\vec{k}+\vec{p}} e^{i(\Omega_{-\vec{k}+\vec{p}} - \omega_{\vec{p}})t} + a_{\vec{k}-\vec{p}} e^{-i(\Omega_{\vec{k}-\vec{p}} + \omega_{\vec{p}})t} \right) \right). \end{aligned} \tag{7}$$

The right column of equation (7) can be obtained from the left column using Hermitian conjugation and $k \to -k$ substitution. Since we are considering the moment when $t = 0$ and there are no particles, then the main process that we will take into account is $\phi \to 2\chi$. Denoting

$$\sigma_{\vec{p}-\vec{k}} = -\omega_{\vec{p}}^2 \alpha \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{p}-\vec{k}}}}, \quad \text{and} \quad \sigma_{\vec{p}+\vec{k}} = -\omega_{\vec{p}}^2 \alpha \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{p}+\vec{k}}}}, \tag{8}$$

we will consider only those terms of the equation (7) that correspond to the creation of particles, let us rewrite eq. (7) for brevity as

$$e^{-i\Omega_{\vec{k}} t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}} \dot{a}_{\vec{k}}) = \sigma_{\vec{p}-\vec{k}} a^\dagger_{\vec{p}-\vec{k}} e^{i(\Omega_{\vec{p}-\vec{k}} - \omega_{\vec{p}})t} + \sigma_{\vec{p}+\vec{k}} a^\dagger_{-\vec{p}-\vec{k}} e^{i(\Omega_{-\vec{p}-\vec{k}} + \omega_{\vec{p}})t}. \tag{9}$$

The term with $a^\dagger_{\vec{p}-\vec{k}}$ is related to the production of χ particles with momentum $\vec{p} - \vec{k}$, $a^\dagger_{-\vec{p}-\vec{k}}$ — with momentum $(-\vec{p} - \vec{k})$. At the same time, we take into account that $\Omega_{-\vec{p}-\vec{k}} = \Omega_{\vec{p}+\vec{k}}$ by definition. It turns out that the resonance related to the last term, is weaker than the first one (details will be given in [23]). Thus, we obtain the equation

$$e^{-i\Omega_{\vec{k}} t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}} \dot{a}_{\vec{k}}) = \sigma_{\vec{p}-\vec{k}} a^\dagger_{\vec{p}-\vec{k}} e^{i(\Omega_{\vec{p}-\vec{k}} - \omega_{\vec{p}})t}. \tag{10}$$

This equation can be simplified if $a_{\vec{k}}(t)$ vary slower with time than $\chi_{\vec{k}}(t)$ in a way that we can neglect the second derivative in (10),

$$|\ddot{a}_{\vec{k}}| \ll |\Omega_{\vec{k}} \dot{a}_{\vec{k}}|, \tag{11}$$

which limit is referred as rotating wave approximation (RWA) in [1]:

$$\dot{a}_{\vec{k}} = i \frac{\sigma_{\vec{p}-\vec{k}}}{2\Omega_{\vec{k}}} a^\dagger_{\vec{p}-\vec{k}} e^{i\epsilon_{\vec{k}} t}, \tag{12}$$

where $\epsilon_k = \Omega_{\vec{k}} + \Omega_{\vec{p}-\vec{k}} - \omega_p$. We can obtain the same equation as (12) for the Hermitian conjugate case.

2.1 Solution

The approximated solution [1] of equation (12) reads (in our notations),

$$a_{\vec{k}}(t) = e^{i\epsilon_{\vec{k}}t/2} a_{\vec{k}}(0) \left(\cosh(s^0_{\vec{k}}t) - i\frac{\epsilon_{\vec{k}}}{2s^0_{\vec{k}}} \sinh(s^0_{\vec{k}}t)) + i\frac{\sigma_{\vec{p}-\vec{k}}}{2s_{\vec{k}}\Omega_{\vec{k}}} a^\dagger_{\vec{p}-\vec{k}}(0) \sinh(s^0_{\vec{k}}t) \right), \tag{13}$$

where

$$s^0_{\vec{k}} = \frac{1}{2}\sqrt{\frac{\sigma^2_{\vec{p}-\vec{k}}}{\Omega^2_{\vec{k}}} - \epsilon^2_{\vec{k}}}. \tag{14}$$

Substituting the solution (13),(14) into the approximation (11), we obtain:

$$\epsilon_{\vec{k}} \ll \Omega_{\vec{k}}, \qquad s^0_{\vec{k}} \ll \Omega_{\vec{k}}. \tag{15}$$

The first condition reads, $\Omega_{\vec{p}-\vec{k}} \ll \omega_{\vec{p}}$, which restricts ourselves to the cases of small $m_\chi \ll \omega_p$. The second condition reduces to $\alpha \ll 1$. We must pay special attention to the case when either $\Omega_{\vec{k}}$ or $\Omega_{\vec{p}-\vec{k}}$ approaches to zero, because $s^0_{\vec{k}}$ could blow up. For example, when $m_\chi = 0, \Omega_{\vec{p}-\vec{k}} = 0$ if $\vec{k} = \vec{p}$. The opposite limit $m_\chi \gg \omega_p$ is of great interest as well. For this reason it is worth solving the equation (10) without RWA approximation. The solution of eq. (10) can be obtained by the Bogolyubov transformations [6, 21], details will be given in [23].

$$\begin{aligned} a_{\vec{k}}(t) = e^{i\epsilon_{\vec{k}}t/2} \Bigg[a_{\vec{k}}(0) \left(\cosh(s_{\vec{k}}t) \; - \; i\frac{\epsilon^2_{\vec{k}}/4 - s^2_{\vec{k}} - \Omega_{\vec{k}}\epsilon_{\vec{k}}}{s_{\vec{k}}(\epsilon_{\vec{k}} - 2\Omega_{\vec{k}})} \sinh(s_{\vec{k}}t) \right) - \\ - a^\dagger_{\vec{p}-\vec{k}}(0) \cdot i\frac{\sigma_{\vec{p}-\vec{k}}}{s_{\vec{k}}(\epsilon_{\vec{k}} - 2\Omega_{\vec{k}})} \sinh(s_{\vec{k}}t) \Bigg], \end{aligned} \tag{16}$$

where

$$s_{\vec{k}} = \sqrt{-\frac{\epsilon^2_{\vec{k}}}{4} - 2\Omega^2_{\vec{k}} + \epsilon_{\vec{k}}\Omega_{\vec{k}} + \sqrt{\Omega^2_{\vec{k}}\epsilon^2_{\vec{k}} + 4\Omega^4_{\vec{k}} + \sigma^2_{\vec{p}-\vec{k}} - 4\epsilon_{\vec{k}}\Omega^3_{\vec{k}}}}. \tag{17}$$

2.2 Conditions of instability

It can be seen from solution (16) that s must be real, otherwise the solutions will disappear. This means that modes that demonstrate parametric resonance must satisfy the condition

$$s^2_{\vec{k}} > 0. \tag{18}$$

Now we will move on to dimensionless expressions: $\vec{\kappa} = \vec{k}/\omega_{\vec{p}}, \mu = m_\chi/\omega_{\vec{p}}, \vec{\nu} = \vec{p}/\omega_{\vec{p}}$ and $\beta_{\vec{\kappa}} = \sqrt{\kappa^2 + \mu^2}$. Also $\eta_{\vec{\kappa}} = s_{\vec{k}}/\omega_{\vec{p}}$, which in terms of dimensionless parameters will have the form:

$$\begin{aligned} \eta_{\vec{\kappa}} = \Bigg(-\frac{1}{4}(\beta_{\vec{\kappa}} + \beta_{\vec{\nu}-\vec{k}} - 1)^2 - 2\beta^2_{\vec{\kappa}} + (\beta_{\vec{\kappa}} + \beta_{\vec{\nu}-\vec{\kappa}} - 1)\beta_{\vec{k}} + \\ + \sqrt{\beta^2_{\vec{\kappa}}(\beta_{\vec{\kappa}} + \beta_{\vec{\nu}-\vec{\kappa}} - 1)^2 + 4\beta^4_{\vec{\kappa}} - 4(\beta_{\vec{\kappa}} + \beta_{\vec{\nu}-\vec{\kappa}} - 1)\beta^3_{\vec{\kappa}} + \frac{\beta_{\vec{\kappa}}\alpha^2}{\beta_{\vec{\nu}-\vec{\kappa}}}} \Bigg)^{\frac{1}{2}}. \end{aligned} \tag{19}$$

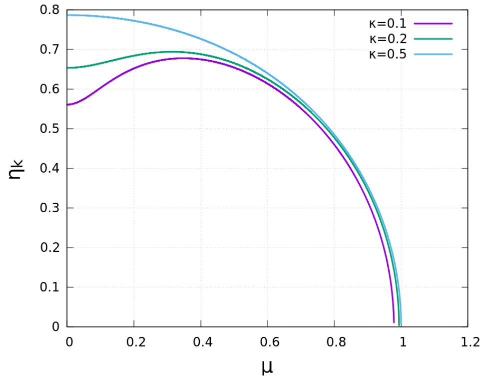

Figure 1: $\eta_{\vec{\kappa}}$ as a μ function for $\alpha = 1$ and $\kappa = 0.1, 0.2, 0.5$.

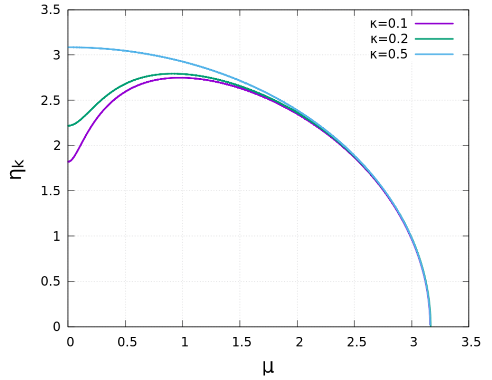

Figure 2: $\eta_{\vec{\kappa}}$ as a μ function for $\alpha = 10$ and $\kappa = 0.1, 0.2, 0.5$.

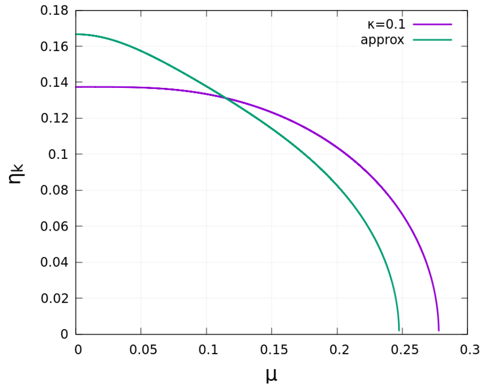

Figure 3: Dependence $\eta_{\vec{\kappa}}(\mu)$ for $\alpha = 0.1$ and $\kappa = 0.1$ for approximation and without it.

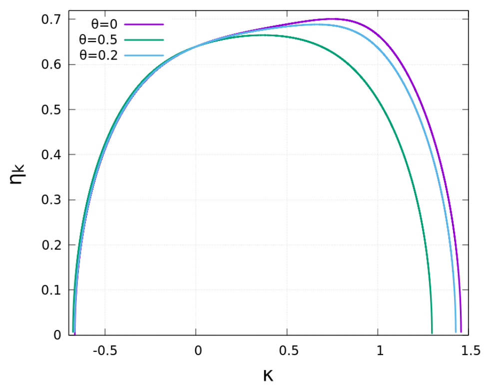

Figure 4: Dependence $\eta_{\vec{\kappa}}(\kappa)$ for $\alpha = 1$, $\mu = \sqrt{\alpha}/2$, $\theta = 0, 0.2, 0.5$.

Here we have defined θ as is the angle between $\vec{k}$ and $\vec{p}$. Consider the case $\nu = 1$, then the plots of $\eta_{\vec{\kappa}}$ for $\theta = 0$ and various values of κ and α are shown in figures 1, 2.

The instability boundary corresponds to the equation $\eta^2_{\vec{\kappa}} = 0$, therefore, we solve it and find the instability limits for κ using numerical calculations. For fixed values of α and $\mu = \sqrt{\alpha}/2$, a solution can be found analytically. The result of such calculations is shown in the figure 4.

With a fixed κ, we can express the dependence of α on μ from equation $\eta^2_{\vec{\kappa}} = 0$:

$$\alpha(\mu) = \frac{1}{4}\sqrt{\frac{\beta_{\vec{\nu}-\vec{\kappa}}}{\beta_{\vec{\kappa}}}(\beta_{\vec{\kappa}} + \beta_{\vec{\nu}-\vec{\kappa}} - 1)^2(-3\beta_{\vec{\kappa}} + \beta_{\vec{\nu}-\vec{\kappa}} - 1)^2}. \tag{20}$$

This dependence for various κ and $\theta = 0$ is shown in the figure 5. This graph can be considered as a stability boundary, and we also see that the greater μ we are interested in, the greater α we need to take. The dependence $\eta_{\vec{\kappa}}(\kappa)$ for $\mu = \sqrt{\alpha}/2$ and fixed θ is shown in the figures 9, 8, 7. From these graphs, we can see that for $\alpha \ll 1$, the result completely coincides with the approximated result [1].

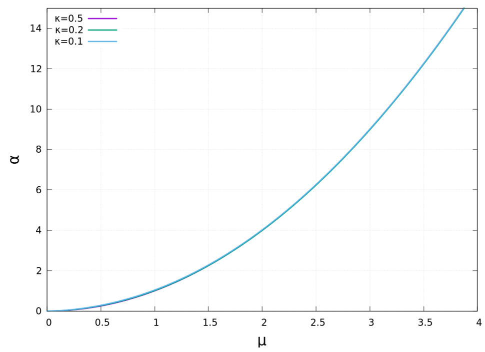

Figure 5: Dependence $\alpha(\mu)$ at fixed $\kappa = 0.1, 0.2, 0.5$ at $\theta = 0$. The larger μ, the larger α is required to start the process. $\alpha \sim \rho_\phi$ for large masses, a large ρ_ϕ is required.

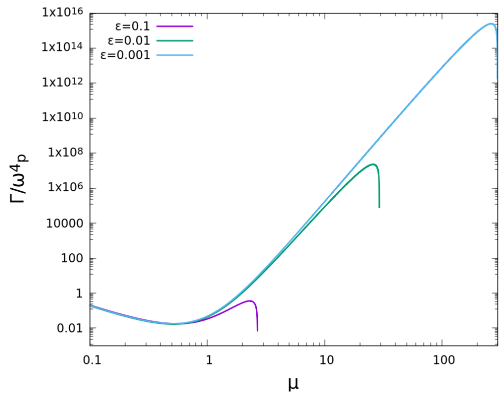

Figure 6: Dependence of $\frac{\Gamma}{\omega_{\vec{p}}^4}$ on μ for the values $\alpha + \epsilon, \mu - \epsilon$ shown in figure 5.

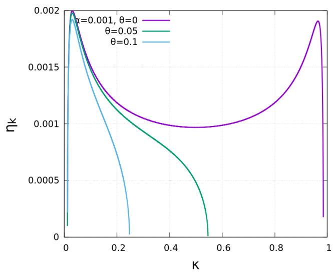

Figure 7: Dependence $\eta_{\vec{\kappa}}(\kappa)$ for $\alpha = 0.001$, $\mu = \sqrt{\alpha}/2$ and fixed $\theta = 0.1, 0.05, 0.1$..

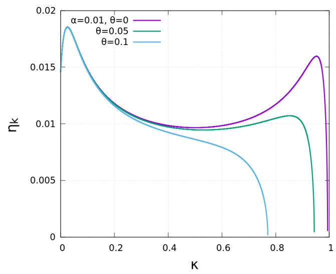

Figure 8: Dependence $\eta_{\vec{\kappa}}(\kappa)$ for $\alpha = 0.01$, $\mu = \sqrt{\alpha}/2$ and fixed $\theta = 0.1, 0.05, 0.1$.

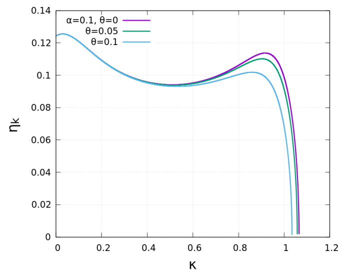

Figure 9: Dependence $\eta_{\vec{\kappa}}(\kappa)$ for $\alpha = 0.1$, $\mu = \sqrt{\alpha}/2$ and fixed $\theta = 0.1, 0.05, 0.1$.

3. Decay rate

In general, to use the assumption of asymptotic states, it is required that the mass of the particles formed be below than the mass of the decaying particles. But in our case, the beam has a very high energy density, and the final particles are not in asymptotic states, since they continue to interact with the decaying field. Therefore, to find decay rate Γ, we use theory that is used in post-inflationary cosmology [24],[25, 26]. Thus, we define the decay rate for process $\phi_i \longrightarrow \phi_j + \phi_k$ as [1],[25]:

$$\Gamma = \int \frac{d^3 p_j}{(2\pi)^3} \int \frac{d^3 p_k}{(2\pi)^3} \frac{d|S_{fi}|^2}{dt}. \tag{21}$$

Scattering matrix S_{fi} of the transition between the states $|i\rangle$ and $|f\rangle$ [1]:

$$S_{fi} = \int_0^T dt \int d^3x \langle f|H_I|i\rangle, \tag{22}$$

where $H_I = g\phi\chi^2$, T - transition time from the initial state $|i\rangle$ which contains one ϕ particle with momentum $\vec{p}$ to the final state $|f\rangle$ consisting of two χ particles with momenta $\vec{k}$ and $\vec{q}$ respectively.

We consider the case of the absence of χ particles at the initial moment of time and take into account that the back reaction has not yet been observed at the final moment of time. Therefore, we take the time interval equal to $T = \frac{1}{2s_{\vec{k}}}$, since it is quite short. During this time, the ϕ will not be significantly depleted, and other processes have not yet begun. We will move on to dimensionless parameters and as a result we will get

$$\Gamma = \frac{\omega_{\vec{p}}^4 \alpha}{16(2\pi)^5} \int_0^{\pi} \int_{\kappa_-}^{\kappa_+} \frac{\sinh(\theta)\kappa^2 d\kappa d\theta}{\beta_{\vec{k}}\beta_{\vec{v}-\vec{\kappa}}} \Big(16C_1^2\eta^2(\sinh(2) - 4\eta_{\vec{v}-\vec{\kappa}}\sinh(1)) + \frac{(1 - C_2^2 + C_1^2)^2}{4\eta_{\vec{v}-\vec{\kappa}}} + \tag{23}$$
$$+\eta_{\vec{v}-\vec{\kappa}}(1 - (C_2^2 - C_1^2)^2(\cosh(1) + (1))) + 2\eta^3_{\vec{v}-\vec{\kappa}}\sinh(2)(1 + C_2^2 - C_1^2)\Big)$$

where $C_1 = \frac{\frac{\epsilon_{\vec{k}}^2}{4} - \eta_{\vec{\kappa}}^2 - \beta_{\vec{\kappa}}\epsilon_{\vec{k}}}{\eta_{\vec{\kappa}}(\epsilon_{\vec{\kappa}} - 2\beta_{\vec{\kappa}})}$ and $C_2 = \frac{\sigma_{\vec{v}-\vec{\kappa}}}{\eta_{\vec{\kappa}}(\epsilon_{\vec{\kappa}} - 2\beta_{\vec{\kappa}})}$.

The result of integrating formula (23) is shown in Figure 6. The boundaries of integration are $\kappa_- = 0.1$ and $\kappa_+ = 0.9$. When drawing this graph, we take the values of μ and α at the boundary of instability (20), which is shown in Figure 5 and deviate from them by the value of ϵ as follows $\alpha + \epsilon, \mu - \epsilon$. As a result, we can conclude that the Γ value becomes greater the closer we get to the boundary of instability.

4. Standing wave

Let's replace the running wave with a standing wave. If eight running waves overlap each other in a rectangular area, then a standing wave is formed under condition (25) [27]:

$$\phi^s(\xi, t) = 8\frac{\sqrt{2\rho_\phi}}{\omega}\cos(p_x\xi_x)\cos(p_y\xi_y)\cos(p_z\xi_z)\cos(\omega t), \tag{24}$$

where ω is the frequency of the waves that form a standing wave. The condition for the formation of a standing wave

$$p_x = \frac{\pi n_1}{a}, p_y = \frac{\pi n_2}{b}, p_z = \frac{\pi n_3}{c}; n_1, n_2, n_3 = 1, 2, 3, ..., \tag{25}$$

where a, b and c are the sizes of the area in which the standing wave is formed. Similarly to the case of a running wave, for a standing wave we write χ as

$$\chi = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\Omega_{\vec{k}}}}\Big(\chi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{\xi}} + \chi_{\vec{k}}(t)^\dagger e^{-i\vec{k}\cdot\vec{\xi}}\Big). \tag{26}$$

Consider the right-hand side of of the equation (4), where we replaced ϕ with ϕ^s and after doing the same steps as for the running wave after the transformation we get

$$-2g\phi^s\chi = -2g\int\frac{d^3k}{(2\pi)^3}\frac{\sqrt{2\rho_\phi}}{2\omega\sqrt{2\Omega_{\vec{k}}}}\Big(\chi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{\xi}} + \chi_{\vec{k}}(t)^\dagger e^{-i\vec{k}\cdot\vec{\xi}}\Big)(e^{i\omega t} + e^{-i\omega t})$$
$$\times\Big(e^{i(p_x\xi_x + p_y\xi_y + p_z\xi_z)} + e^{i(p_x\xi_x - p_y\xi_y + p_z\xi_z)} + e^{i(-p_x\xi_x + p_y\xi_y + p_z\xi_z)} + e^{i(-p_x\xi_x - p_y\xi_y + p_z\xi_z)}$$
$$+e^{i(p_x\xi_x + p_y\xi_y - p_z\xi_z)} + e^{i(p_x\xi_x - p_y\xi_y - p_z\xi_z)} + e^{i(-p_x\xi_x + p_y\xi_y - p_z\xi_z)} + e^{i(-p_x\xi_x - p_y\xi_y - p_z\xi_z)}\Big)$$

Here we will introduce the designation $\vec{\xi} = (\xi_x, \xi_y, \xi_z)$, $\vec{p}_1 = (p_x, p_y, p_z)$, $\vec{p}_2 = (-p_x, p_y, p_z)$, $\vec{p}_3 = (p_x, -p_y, p_z)$, $\vec{p}_4 = (-p_x, -p_y, p_z)$ and we will denote $A_k = \chi_k + \chi^\dagger_{-k}$ and take into account that the left part is the same as for the running wave, we get:

$$(\partial_t^2 + \Omega^2_{\vec{k}})A_{\vec{k}} = -\alpha\omega^2 \sum_{j=1}^{4} \int \frac{d^3k}{(2\pi)^3}\Big[e^{-i\omega t}\Big(\frac{A_{\vec{k}-\vec{p}_j}}{\sqrt{2\Omega_{\vec{k}-\vec{p}_j}}} + \frac{A_{\vec{k}+\vec{p}_j}}{\sqrt{2\Omega_{\vec{k}+\vec{p}_j}}}\Big) + e^{i\omega t}\Big(\frac{A_{\vec{k}-\vec{p}_j}}{\sqrt{2\Omega_{\vec{k}-\vec{p}_j}}} + \frac{A_{\vec{k}+\vec{p}_j}}{\sqrt{2\Omega_{\vec{k}+\vec{p}_j}}}\Big)\Big] \tag{27}$$

Based on the result obtained, it can be concluded that when considering a standing wave, the mechanism and amplitude of the process remain the same as for a running wave, but additional terms appear in the equations. Denote $\chi_k = a_k e^{-i\Omega_k t}$, and as a result, we get group of equations for a_k:

$$\Big[\ddot{a}_k + i\dot{a}_k(\epsilon_{a_j-k} - 2\Omega_k) + a_k(-\frac{\epsilon^2_{p_j-k}}{4} + \epsilon_{p_j-k}\Omega_k)\Big] = \sigma_{p_j-k}a^\dagger_{p_j-k}, \tag{28}$$

where $\sigma_{\vec{p}_j-\vec{k}} = -\omega^2\alpha\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}_j}}}$, $\epsilon_{\vec{p}_j-\vec{k}} = \Omega_{\vec{k}} + \Omega_{\vec{p}_j-\vec{k}} - \omega$.

As a consequence, we get 4 equations for different momenta, but the condition (25) is imposed on them, and also all momenta have the same modulus value. For each of these four equations, we can find the appropriate solution:

$$a^j_{\vec{k}}(t) = e^{i\epsilon_{\vec{p}_j-\vec{k}}t/2}\Big[a_{\vec{k}}(0)(\cosh(s_{\vec{p}_j-\vec{k}}t) - i\frac{\frac{\epsilon^2_{\vec{p}_j-\vec{k}}}{4} - s^2_{\vec{p}_j-\vec{k}} - \Omega_{\vec{k}}\epsilon_{\vec{p}_j-\vec{k}}}{s_{\vec{p}_j-\vec{k}}(\epsilon_{\vec{p}_j-\vec{k}} - 2\Omega_{\vec{k}})}\sinh(s_{\vec{p}_j-\vec{k}}t)) - i\frac{\sigma_{\vec{p}_j-\vec{k}}}{s_{\vec{p}_j-\vec{k}}(\epsilon_{\vec{p}_j-\vec{k}} - 2\Omega_{\vec{k}})}a^\dagger_{\vec{p}_j-\vec{k}}(0)\sinh(s_{\vec{p}_j-\vec{k}}t)\Big],$$

$j = 1, .., 4$.

As a result, we conclude that particles can be generated by four traveling waves and momenta p_j of different directions, which were formed from the initial standing wave. These waves can be studied independently, and the process of particle generation is described in the same way as for a traveling wave.

5. Conclusion

In this article, we examined a toy model with the interaction of $g\phi\chi^2$ and studied the decay of a plane wave of a massless field into massive particles $\phi \to 2\chi$ in this model. We have found under what conditions parametric resonance is possible (see Fig. 5). We have indicated that the decay occurs at a field amplitude above the threshold value not only in the case of low mass ($m_\chi \ll \omega_p$) considered in [1], but for arbitrary masses. In the case of large masses ($m_\chi \gg \omega_p$), the required threshold amplitude of the field ϕ is significantly larger compared to the case of small masses. We also extended the obtained result to the case of a standing wave of initial massless scalar field, which can be further generalized to the case of scalar millicharges production standing electromagnetic wave. The obtained result can be used to study nonperturbative effects that can be included in collider physics, astrophysics and cosmology [12, 13].

Acknowledgements. The authors thank Maxim Fitkevich and Dmitry Kirpichnikov for valuable discussions. The work is supported by RSF grant 21-72-10151

References

[1] A. Arza, *Production of massive bosons from the decay of a massless particle beam*, *Phys. Rev. D* **105** (2022) 036004 [2009.03870].

[2] A. Nikishov, *S matrix in quantum electrodynamics with external field*, *Theoretical and Mathematical Physics* **20** (1974) 653.

[3] A. Arza, *Photon enhancement in a homogeneous axion dark matter background*, *Eur. Phys. J. C* **79** (2019) 250 [1810.03722].

[4] G. Fiore and G. Modanese, *General properties of the decay amplitudes for massless particles*, *Nucl. Phys. B* **477** (1996) 623 [hep-th/9508018].

[5] S.P. Gavrilov and D.M. Gitman, *Vacuum instability in external fields*, *Phys. Rev. D* **53** (1996) 7162 [hep-th/9603152].

[6] E.A. Calzetta and B.-L.B. Hu, *Nonequilibrium Quantum Field Theory*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2023), 10.1017/9781009290036.

[7] E.T. Akhmedov, A.V. Anokhin and D.I. Sadekov, *Currents of created pairs in strong electric fields*, *Int. J. Mod. Phys. A* **36** (2021) 2150134 [2012.00399].

[8] Z. Lei, B. Hu and X. Zhao, *Pair production in strong electric fields*, 2201.01746.

[9] L. Kofman, A.D. Linde and A.A. Starobinsky, *Towards the theory of reheating after inflation*, *Phys. Rev. D* **56** (1997) 3258 [hep-ph/9704452].

[10] A. Arza, T. Schwetz and E. Todarello, *How to suppress exponential growth—on the parametric resonance of photons in an axion background*, *JCAP* **10** (2020) 013 [2004.01669].

[11] P.B. Greene, L. Kofman and A.A. Starobinsky, *Sine-Gordon parametric resonance*, *Nucl. Phys. B* **543** (1999) 423 [hep-ph/9808477].

[12] D.G. Figueroa and F. Torrenti, *Parametric resonance in the early Universe—a fitting analysis*, *JCAP* **02** (2017) 001 [1609.05197].

[13] M. Desroche, G.N. Felder, J.M. Kratochvil and A.D. Linde, *Preheating in new inflation*, *Phys. Rev. D* **71** (2005) 103516 [hep-th/0501080].

[14] D. Boyanovsky and L. Lello, *Time evolution of cascade decay*, *New Journal of Physics* **16** (2014) 063050.

[15] K.A. Beyer, G. Marocco, C. Danson, R. Bingham and G. Gregori, *Parametric co-linear axion photon instability*, *Phys. Lett. B* **839** (2023) 137759 [2108.01489].

[16] J.F. Dufaux, G.N. Felder, L. Kofman, M. Peloso and D. Podolsky, *Preheating with trilinear interactions: Tachyonic resonance*, *JCAP* **07** (2006) 006 [hep-ph/0602144].

[17] J.A. Dror, K. Harigaya and V. Narayan, *Parametric resonance production of ultralight vector dark matter*, *Phys. Rev. D* **99** (2019) 035036.

[18] R. Allahverdi, R.H.A.D. Shaw and B.A. Campbell, *Parametric resonance for complex fields*, *Phys. Lett. B* **473** (2000) 246 [hep-ph/9909256].

[19] S. Kasuya and M. Kawasaki, *Restriction to parametric resonant decay after inflation*, *Phys. Lett. B* **388** (1996) 686 [hep-ph/9603317].

[20] M.A. Amin, M.P. Hertzberg, D.I. Kaiser and J. Karouby, *Nonperturbative Dynamics Of Reheating After Inflation: A Review*, *Int. J. Mod. Phys. D* **24** (2014) 1530003 [1410.3808].

[21] D.S. Gorbunov and V.A. Rubakov, *Introduction to the theory of the early universe: Cosmological perturbations and inflationary theory* (2011), 10.1142/7873.

[22] Y. Shtanov, J.H. Traschen and R.H. Brandenberger, *Universe reheating after inflation*, *Phys. Rev. D* **51** (1995) 5438 [hep-ph/9407247].

[23] E.A. Dmitrieva and P.S. Satunin in preparation.

[24] M. Scully and M. Zubairy, *Quantum Optics*, Quantum Optics, Cambridge University Press (1997).

[25] N. Herring, B. Pardo, D. Boyanovsky and A.R. Zentner, *Particle decay in post inflationary cosmology*, *Phys. Rev. D* **98** (2018) 083503.

[26] D. Boyanovsky, *Condensates and quasiparticles in inflationary cosmology: Mass generation and decay widths*, *Phys. Rev. D* **85** (2012) 123525.

[27] I.V. Savelyev, *Course of general physics: Textbook. In 3 vols. 3. Quantum new optics. Atomic physics. Solid state physics. Physics of the Atomic nucleus and Elementary Particles.*, M.; Nauka, Gl. ed. phys.—mat. lit. (1987).