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Stable solutions in Horndeski theory

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The construction of completely stable solutions within the framework of Horndeski theory is significantly limited by the "no-go" theorem. However, in this article, we reveal a novel approach to constructing stable solutions within the general Horndeski framework. Our research focuses on scenarios in which the previously studied unitary gauge has a singularity. We construct a spatially flat, stable solution described by general relativity in combination with non-canonical scalar fields. Following this, we analyze the effect of perturbations from an anisotropic background on these solutions' stability. This investigation provides valuable information about the dynamics of cosmological models using non-canonical scalar fields.

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1. Introduction

In its classical version, the inflation scenario assumes that under typical initial conditions and with a simple inflationary potential requiring minimal fine-tuning, inflation has the ability to generate exponentially large spaces. These regions are typically characterized by uniformity, isotropy, and flatness, along with an almost scale-independent spectrum of density fluctuations and gravitational waves. These fluctuations are adiabatic, Gaussian and have predictable properties. Classical inflation implies the assumption that volume serves as a natural measure: despite the low probability of obtaining a piece of space with the right initial conditions, the inflated areas cover an extremely large volume, thus their properties form the basis for further forecasts.

Initially, achieving the observed amplitude of the primordial density fluctuations demands meticulous parameter fine-tuning across various inflationary potentials. Moreover, the likelihood of a spatial region possessing precise initial conditions to initiate inflation is exponentially small[1, 2]. Conventional statistical mechanics reasoning suggests that even with simple inflaton potentials, there are more cosmological solutions that are homogeneous and flat without undergoing prolonged inflation[2].

The primary conceptual challenge arises from the multiverse problem, also known as the measure problem, which arises from eternal inflation.[3] Despite assuming smooth and classical evolution of the inflaton, inflation eventually ends when the inflaton reaches the bottom of its potential. However, in the general case, classical evolution is sometimes interrupted by large quantum fluctuations, including fluctuations that push the inflaton field uphill, away from its expected path. As a result, inflation amplifies these rare quantum fluctuations, leading to eternal inflation.

Continuing this line of thought, multiple quantum leaps occur as the inflaton evolves, leading to different cosmological properties in different volumes of space. This eternal multiverse model provides a variety of possibilities, with different outcomes repeating infinitely.

In the context of classical inflation, where volume is the natural measure, most of today's volumes remains in an inflating state, while non-inflating volumes (bubbles) are expected to be exponentially younger than the observable universe[4, 5].

According to this generally accepted point of view, the cause of the beginning of the expansion of the Universe lies in the effects of quantum gravity. That is, the resolution of this issue is inseparable from the issue of the ultraviolet completion of General Relativity. However, there are alternative scenarios that do not require the construction of such a completion. We are talking about the Genesis and Bounce Universe scenarios. [6–8] The bounce Universe solution suggests that the current expansion may be caused by a smooth "bounce" from an earlier phase of Minkowski space. Over time, it begins to expand smoothly, connecting with the familiar FLRW cosmology, thus resolving the Big Bang singularity. Another option is the Genesis scenario.This unperturbed solution describes a universe that is asymptotically Minkowski in the past, expanding with increasing energy density until it leaves the regime of validity of the effective field theory and reheats. This solution is a dynamical attractor, and the universe is driven towards it, even if it was initially contracting. Both of these scenarios require violating the null energy condition (NEC), which is impossible in General Relativity.

Horndeski theory [9–12](See [13] for a review) is the most general scalar-tensor theory of gravity with one additional scalar field (we denote it by π) whose equations of motion do not exceed the second order despite the presence of higher derivatives in the Lagrangian.

However, the construction of cosmological solutions that are stable over the entire time axis faces a significant limitation due to the existence of the so-called no-go theorem [14–16]. This theorem was originally formulated in terms of the unitary gauge, where perturbations of the scalar field π are set to zero. The statement of this theorem asserts that, in General Horndeski Theory, without encountering strong coupling, gradient instabilities, or ghosts in the quadratic action, it is impossible to construct a nonsingular solution with complete evolution over the interval $t = (-\infty, \infty)$.

Nevertheless, there is a well-known example of a stable solution in Horndeski theory: empty Minkowski space. However, in this case, choosing the unitary gauge is not possible due to the singularity of the coefficients in the quadratic action. Consequently, the no-go theorem does not apply in this scenario.

First, we consider the quadratic action for scalar perturbations on the FLRW background. We then integrate out the constraints in terms of the Bardeen variables and consider all possible variants of singularities that may arise during this process. It is noteworthy to note that we find that, when the unitary gauge becomes singular and the background scalar field is non-trivial (i.e., $\dot{\pi} \neq 0$), the scalar modes do not exhibit any dynamics. Conversely, if the background scalar field is static, for example in Minkowski space, the scalar modes manifest as ordinary Lorentz-invariant waves with a speed of sound equal to $c_S^2 = 1$ at high momenta. Finally, in section 4, we present a specific action choice for a stable theory with a bounce, and we further evaluate how deviations from an isotropic background affect the stability of this solution.

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2. Scalar perturbations in terms of Bardeen variables

We consider the General Horndeski theory with the following Lagrangian:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right), \tag{1a}$$

$$\mathcal{L}_2 = F(\pi, X), \tag{1b}$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi, \tag{1c}$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right], \tag{1d}$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}^{\ \ \nu}\right], \tag{1e}$$

where π is the scalar field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X} = \partial G_4/\partial X$, etc.

In this paper we consider spatially flat FLRW background:

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right). \tag{2}$$

The decomposition of metric perturbations $h_{\mu\nu}$ into helicity components in the general case has the form

$$h_{00} = 2\Phi \tag{3a}$$

$$h_{0i} = -\partial_i\beta + Z_i^T, \tag{3b}$$

$$h_{ij} = -2\Psi\delta_{ij} - 2\partial_i\partial_j E - \left(\partial_i W_j^T + \partial_j W_i^T\right) + h_{ij}, \tag{3c}$$

and perturbation of scalar field π we denote as $\delta\pi = \chi$.

In our work we are interested in the scalar sector of perturbations. After partial fixation of the gauge $E = 0$, the quadratic action for this sector has the form:

$$S^{(2)} = \int \mathrm{d}t\,\mathrm{d}^3x\,a^3 \Bigg(A_1\,(\dot\Psi)^2 + A_2\,\frac{(\overrightarrow{\nabla}\Psi)^2}{a^2} + A_3\,\Phi^2 + A_4\,\Phi\frac{\overrightarrow{\nabla}^2\beta}{a^2} + A_5\,\dot\Psi\frac{\overrightarrow{\nabla}^2\beta}{a^2} + A_6\,\Phi\dot\Psi$$
$$+A_7\,\Phi\frac{\overrightarrow{\nabla}^2\Psi}{a^2} + A_8\,\Phi\frac{\overrightarrow{\nabla}^2\chi}{a^2} + A_9\,\frac{\overrightarrow{\nabla}^2\beta}{a^2}\dot\chi + A_{10}\,\chi\ddot\Psi + A_{11}\,\Phi\dot\chi + A_{12}\,\chi\frac{\overrightarrow{\nabla}^2\beta}{a^2} + A_{13}\,\chi\frac{\overrightarrow{\nabla}^2\Psi}{a^2}$$
$$+A_{14}\,(\dot\chi)^2 + A_{15}\,\frac{(\overrightarrow{\nabla}\chi)^2}{a^2} + A_{16}\,\dot\chi\frac{\overrightarrow{\nabla}^2\Psi}{a^2} + A_{17}\,\Phi\chi + A_{18}\,\chi\dot\Psi + A_{19}\,\Psi\chi + A_{20}\,\chi^2\Bigg). \tag{4}$$

Here dot denotes the derivative with respect to the cosmic time t, coefficients A_i are the combinations of the Lagrangian functions, their derivatives and background.

This action is invariant with respect to the residual gauge transformations:

$$\Phi \to \Phi + \dot\xi_0, \quad \beta \to \beta - \xi_0 + a^2\dot\xi_S, \quad \chi \to \chi + \xi_0\dot\pi, \quad \Psi \to \Psi + \xi_0 H,$$

where H is the Hubble parameter and ξ_0 is the gauge function.

So the action can be rewritten in explicitly gauge-invariant form by introducing new variables (Bardeen variables):

$$\mathcal{X} = \chi + \dot\pi\frac{\beta}{a^2}, \tag{5a}$$

$$\mathcal{Y} = \Psi + H\frac{\beta}{a^2}, \tag{5b}$$

$$\mathcal{Z} = \Phi + \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\beta}{a^2}\right]. \tag{5c}$$

In terms of this variables the action (4) takes the form:

$$S^{(2)} = \int \mathrm{d}t\,\mathrm{d}^3x\,a^3 \Bigg(A_1\left(\dot{\mathcal{Y}}\right)^2 + A_2\,\frac{(\overrightarrow{\nabla}\mathcal{Y})^2}{a^2} + A_3\,\mathcal{Z}^2 + A_6\,\mathcal{Z}\dot{\mathcal{Y}} + A_7\,\mathcal{Z}\frac{\overrightarrow{\nabla}^2\mathcal{Y}}{a^2} + A_8\,\mathcal{Z}\frac{\overrightarrow{\nabla}^2\mathcal{X}}{a^2}$$
$$+A_{10}\,\mathcal{X}\ddot{\mathcal{Y}} + A_{11}\,\mathcal{Z}\dot{\mathcal{X}} + A_{13}\,\mathcal{X}\frac{\overrightarrow{\nabla}^2\mathcal{Y}}{a^2} + A_{14}\left(\dot{\mathcal{X}}\right)^2 + A_{15}\,\frac{(\overrightarrow{\nabla}\mathcal{X})^2}{a^2} + A_{16}\,\dot{\mathcal{X}}\frac{\overrightarrow{\nabla}^2\mathcal{Y}}{a^2}$$
$$+A_{17}\,\mathcal{Z}\mathcal{X} + A_{18}\,\mathcal{X}\dot{\mathcal{Y}} + A_{20}\mathcal{X}^2\Bigg). \tag{6}$$

After we have integrated all the constraints and done one more variable substitution

$$\zeta = \mathcal{Y} + \eta\mathcal{X}, \quad \eta = \frac{3A_{11}A_4 - 2A_{10}A_3}{4A_1A_3 - 9{A_4}^2}, \tag{7}$$

we get the following action:

$$S^{(2)} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left(\mathcal{G}_S \left(\dot{\zeta}\right)^2 - \mathcal{F}_S \frac{\left(\overrightarrow{\nabla}\zeta\right)^2}{a^2} \right), \tag{8}$$

where

$$\mathcal{G}_S = \frac{4}{9}\frac{A_3 {A_1}^2}{{A_4}^2} - A_1, \tag{9a}$$

$$\mathcal{F}_S = -\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{aA_1A_7}{3A_4}\right] - A_2 = \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{aA_5 \cdot A_7}{2A_4}\right] - A_2. \tag{9b}$$

3. $A_4 = 0$ case

The key point is that the action (8) has a singularity at $A_4 = 0$. If we consider this variant separately and do all the previous procedures, we obtain the following action:

$$S^{(2)} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left(A_2 \frac{\left(\overrightarrow{\nabla}\zeta\right)^2}{a^2} - \frac{1}{9}\frac{{A_1}^2}{A_3}\frac{\left(\overrightarrow{\nabla}^2\zeta\right)^2}{a^4} \right) \tag{10}$$

which means the absence of dynamics of the field ζ.

From the view of the $\mathcal{Z}$–constraint from action (6), we can distinguish two other special cases: $A_4 = 0, A_{11} = 0$ and $A_4 = 0, \dot{\pi} = 0$ (we know that $A_3 = \frac{3}{2}A_4H - \frac{1}{2}A_{11}\dot{\pi}$).

$A_{11} = 0$:

In this case, the action (6) takes the following form:

$$S^{(2)} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3\, m\mathcal{Y}^2, \tag{11}$$

where $m = m(A_i)$ is a long combination of A_i and $\dot{A}_i$. This case also turns out to be non-dynamical.

$\dot{\pi} = 0$:

Since $\dot{\pi} = 0$, only the summand G_4H remains from the condition $A_4 = 0$. Because the coefficient A_2 is equal to

$$A_2 = 2G_4 - 2G_{5X}\dot{\pi}^2\ddot{\pi} - G_{5\pi}\dot{\pi}^2, \tag{12}$$

condition $G_4 = 0$ leads to a strong coupling in the action for tensor perturbations, so it is necessary to impose the condition $H = 0$, which corresponds to the case of Minkowski space ($a(t) = \mathrm{const}$).

In this case, the action (6) is:

$$S^{(2)} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left(\mathcal{G}_S \left(\dot{\mathcal{X}}\right)^2 + m\mathcal{X}^2 - \mathcal{F}_S \frac{\left(\overrightarrow{\nabla}\mathcal{X}\right)^2}{a^2} \right), \tag{13}$$

where

$$\mathcal{G}_S = \mathcal{F}_S = \frac{1}{2G_4}\left(3G_{4\pi}^2 + 2F_X G_4 - 2K_\pi G_4\right), \tag{14a}$$

$$m = \frac{1}{2}F_{\pi\pi}. \tag{14b}$$

This case corresponds to a non-minimally coupled scalar field in Minkowski background. We can see that in this case the speed of sound squared is $c_\infty^2 = 1$. However, this does not mean that the scalar mode propagates at the speed of light, since it has mass, but at higher momentum ($k \to \infty$) the speed of propagation will tend to c_∞.

3.1 Brief summary

Here's a table which is summarizing the results of the previous sections:

$A_4 \neq 0$	$c_\infty^2 = \mathcal{F}_S/\mathcal{G}_S$(9)	
$A_4 \equiv 0$	$\dot{\pi} \neq 0$	no dynamics in scalar sector
	$\dot{\pi} = 0$	$c_\infty^2 = 1$

Thus, we obtained that $A_4 = 0$ everywhere, always leads to a stable solution in the scalar perturbation sector. In the case of non-trivial field π there are no dynamical scalar perturbations, and thus the stability condition does not arise at all, and in the case of a static background field π, we obtain a scalar perturbation with the sound speed squared $c_\infty^2 = 1$.

In the following section we construct specific examples of the bouncing scenario in Horndeski theory imposing $A_4 \equiv 0$

4. Bounce solution

Without loss of generality we choose the following form of the scalar field

$$\pi(t) = t, \tag{15}$$

so that $X = 1$. To reconstruct the theory which corresponds some solution we use the following ansatz for the Lagrangian functions

$$F(\pi, X) = f_0(\pi) + f_1(\pi)\cdot X, \tag{16a}$$

$$K(\pi, X) = k_1(\pi)\cdot X, \tag{16b}$$

$$G_4(\pi, X) = \frac{1}{2}. \tag{16c}$$

We are interested to consider the case $G_4 = \text{const}$, which corresponds to GR.

Hubble parameter can be choosen in the following form for the case of the bounce:

$$H(t) = \frac{t}{3\left(\tau^2 + t^2\right)}, \tag{17}$$

so that

$$a(t) = \left(\tau^2 + t^2\right)^{\frac{1}{6}}, \tag{18}$$

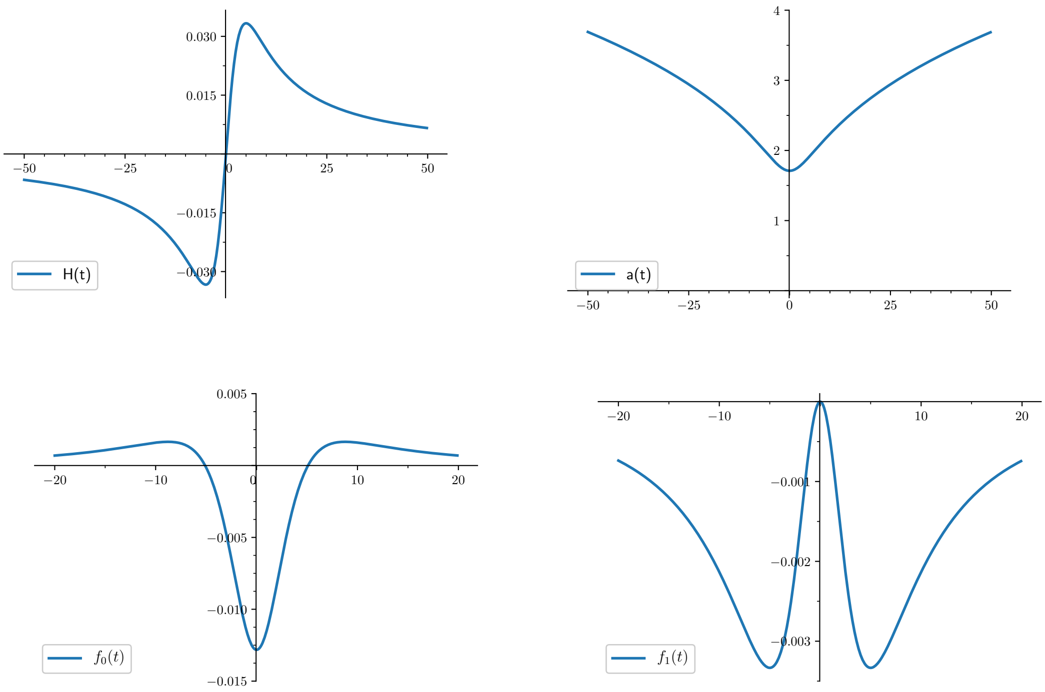

Figure 1: Hubble parameter $H(t)$, scale factor $a(t)$ and the Lagrangian functions $f_0(t)$, $f_1(t)$ of the bouncing scenario with parameter $\tau = 25$ (recall that $k_1(t) = H(t)$).

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and the bounce occurs at $t = 0$. In what follows we take $\tau \gg 1$ to make this scale safely greater than Planck time. The parameter τdetermines the duration of the bouncing stage.

Corresponding Lagrangian reads

$$\mathcal{L} = \frac{\pi^2 - \tau^2}{3\left(\tau^2 + \pi^2\right)^2} - \frac{\pi^2 X}{\left(\tau^2 + \pi^2\right)^2} + \frac{\pi X}{3\left(\tau^2 + \pi^2\right)}\Box\pi + \frac{1}{2}R. \tag{19}$$

You can see the graphs at 1.

5. Deviation from isotropic background

The next part of our study was to check whether the stability of the solution we found is a consequence of the isotropy of the background. For this purpose, we considered the background as Bianchi I type metric:

$$ds^2 = dt^2 - \left(a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2\right). \tag{20}$$

In this case, the analogous (4) action takes the form

$$S^{(2)} = \int \mathrm{d}x\ abc \left(\frac{1}{6} A_1 \sum_{i \neq j} \dot{\Psi}_i \dot{\Psi}_j + \frac{A_2}{2} \sum_{\substack{i=a,b,c \\ i\neq j\neq k}} \Delta_i \Psi_j \Delta_i \Psi_k + A_3 \Phi^2 \right.$$
$$+ \Phi \left(A_4^i \Delta_i^2 \beta \right) + A_5 \sum_{\substack{i=a,b,c \\ i\neq j\neq k}} \dot{\Psi}_i \left(\Delta_j^2 \beta + \Delta_k^2 \beta \right) + \Phi \left(A_6^i \dot{\Psi}_i \right) + \frac{A_7}{2} \Phi \sum_{\substack{i=a,b,c \\ i\neq j\neq k}} \Delta_i^2 \left(\Psi_j + \Psi_k \right)$$
$$+ \Phi \left(A_8^i \Delta_i^2 \chi \right) + \dot{\chi} \left(A_9^i \Delta_i^2 \beta \right) + \chi \left(A_{10}^i \ddot{\Psi}_i \right) + A_{11} \Phi \dot{\chi} + \chi \left(A_{12}^i \Delta_i^2 \beta \right)$$
$$+ \chi \frac{1}{2} A_{13}^{ij} \left(\Delta_i^2 \Psi_j + \Delta_j^2 \Psi_i \right) + A_{14} (\dot{\chi})^2 + A_{15}^i \left(\Delta_i \chi \right)^2 + A_{17} \Phi \chi + \chi \left(A_{18}^i \dot{\Psi}_i \right)$$
$$+ A_{20} \chi^2 + \frac{1}{2} \sum_{\substack{i,j=a,b,c \\ i\neq j}} B^{ij} \Psi_i \dot{\Psi}_j - \Psi_a \left(B^{ab} \Delta_b^2 \beta + B^{ac} \Delta_c^2 \beta \right) + \Psi_b \left(B^{ab} \Delta_a^2 \beta + B^{bc} \Delta_c^2 \beta \right)$$
$$\left. + \Psi_c \left(B^{ac} \Delta_a^2 \beta - B^{bc} \Delta_b^2 \beta \right) \right). \tag{21}$$

Here dot denotes the derivative with respect to the cosmic time t, $\Delta_a = a^{-1}\partial_x, \Delta_b = b^{-1}\partial_y, \Delta_c = c^{-1}\partial_z$, $\Psi_i = \bar{H}_i \Psi$ and $\bar{H}_i = H_i/H$ and we assume summation by dummy indices.

To further analyze the theory, we consider the action (4) in the unitary gauge $\chi = 0$ and direct the momentum $\vec{k}$ along the x-axis, so $\vec{k} = (k_x, 0, 0)$. Then by integrating out all constrains we obtain the following action for one dynamical scalar mode:

$$S^{(2)} = \int \mathrm{d}t\, \mathrm{d}^3x\, abc \left(\mathcal{G}_S \left(\dot{\Psi}\right)^2 + M\Psi^2 + \mathcal{F}_S \frac{k_x^2}{a^2} \Psi^2 \right), \tag{22}$$

where

$$\mathcal{G}_S = \frac{2}{9} \frac{A_3 {A_1}^2}{\left(A_4^x\right)^2} \left(\bar{H}_b + \bar{H}_c\right)^2 - \frac{2}{3} \frac{A_1}{A_4^x} \left(A_4^y \bar{H}_b + A_4^z \bar{H}_c\right) \left(\bar{H}_b + \bar{H}_c\right) + \frac{2}{3} A_1 \bar{H}_b \bar{H}_c, \tag{23a}$$

$$\mathcal{F}_S = -2A_2 \bar{H}_b \bar{H}_c - \frac{1}{9a^3} \left(\bar{H}_b + \bar{H}_c\right)^2 \frac{d}{dt} \left[\frac{A_1^2 a^3}{A_4^x} \right] + \frac{{A_1}^2}{9A_4^x} \left(\bar{H}_b^2 - \bar{H}_c^2 \right) \left(H_b - H_c \right), \tag{23b}$$

$$c_S^2 = \frac{\mathcal{G}_S}{\mathcal{F}_S}, \tag{23c}$$

the explicit value of M is not important to us now, we can use the expression for the square of the speed of sound c_S^2 to check the stability of the solution(17).

Let us consider an anisotropic bounce - deviate from the isotropic case in two directions:

$$H_a = \frac{t}{\left(\tau^2 + t^2\right)} + \frac{\alpha}{\left(\tau^2 + t^2\right)^{3/2}}, \quad H_b = \frac{t}{\left(\tau^2 + t^2\right)} - \frac{\alpha}{\left(\tau^2 + t^2\right)^{3/2}}, \quad H_c = \frac{t}{\left(\tau^2 + t^2\right)}. \tag{24}$$

Here the parameter τ defines the bounce amplitude and α the degree of deviation from the isotropic case(See Fig.2).

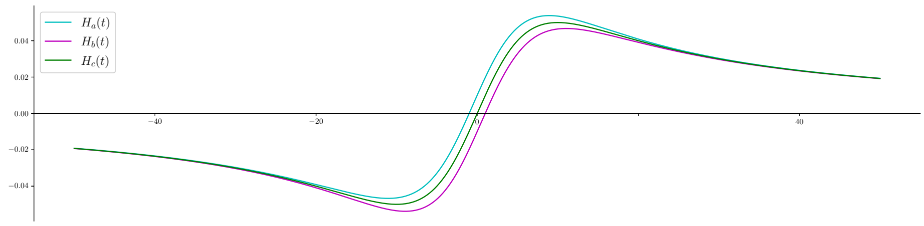

Figure 2: Hubble parametrs $H_a(t), H_b(t), H_c(t)$, when we choose $\alpha = 10, \tau = 10$.

To analyze the stability of the scalar field, we numerically plot the square of the speed of sound c_S^2(Fig.3):

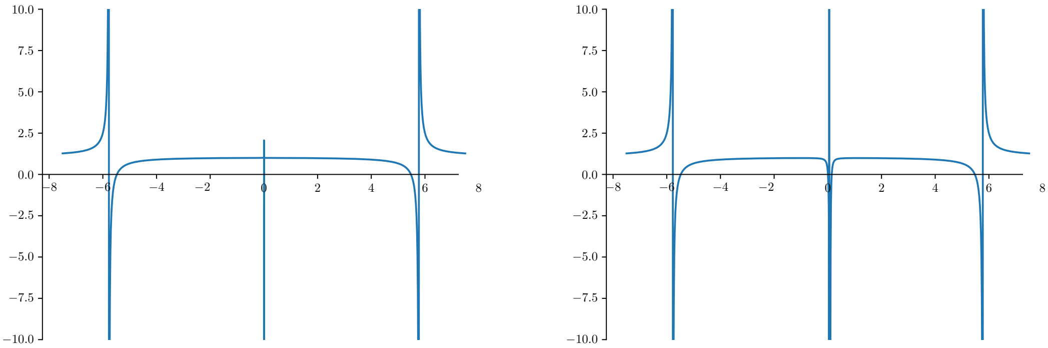

Figure 3: The square of the speed of sound c_S^2, when we choose $\alpha = 0.1, \tau = 10$ (left panel) and $\alpha = 1, \tau = 20$ (right panel). In this case, the square of the speed of sound will have at least 2 symmetric singular points and tends to 0 as univerce becomes isotropic.

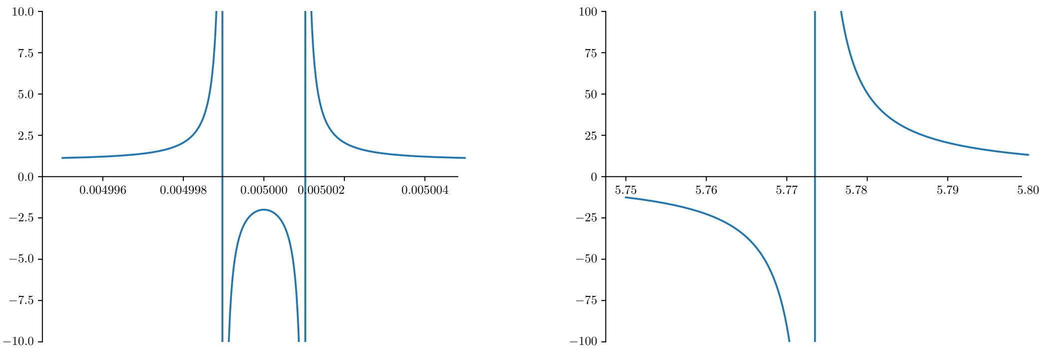

Figure 4: Zoom of the neighborhood of singular points of the square of the speed of sound c_S^2 for parameters $\alpha = 0.1, \tau = 10$.

The plots Fig.4 show that in the theory (19) the scalar field becomes unstable even with a small deviation from the isotropic background. This tells us that the result obtained in the previous sections is a very special case directly related to the background isotropy.

6. Conclusion

In this paper, we propose a method for constructing stable solutions in Horndeski theory. By reducing the second-order action to an explicitly gauge-invariant form, we consider scenarios where the previously used unitary gauge turns out to be singular. This not only of academic interest but also allows us to construct cosmological models that have previously suffered from the presence of singular points. As demonstrated by the example of the bouncing universe model, these solutions present opportunities for studying new cosmological dynamics.

The absence of dynamical scalar modes of perturbation can be compensated by introducing matter. In addition, we develop an action for the scalar modes of the metric and scalar field over an anisotropic background. We then investigate whether the previously derived solution for a universe with a bounce remains stable.

Our findings show that the stability of perturbations in the Universe with a bounce is closely linked to its isotropy. Even small deviations from isotropy can lead to the divergence and eventual negative value of the square of the sound speed. This emphasizes the delicate balance needed to ensure stability in such cosmological models.

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