Testing universal dark-matter caustic rings with galactic rotation curves

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Infall of cold dark matter on a galaxy may result in caustic rings where the particle density is enhanced. They may be searched for as features in the galactic rotation curves. Previous studies suggested the evidence for these caustic rings with universal, that is common for different galaxies, parameters. Here we test this hypothesis with a large independent set of rotation curves by means of an improved statistical method. No evidence for universal caustic rings is found in the new analysis.

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1. Introduction

Dark matter is an invisible substance not accounted for by the Standard Model of modern particle physics, see e.g. Ref. [1] and references therein. One aspect of this challenge involves the accumulation of cold dark matter in galaxies, which can lead to the formation of "infall caustics." These are concentric structures of increased dark matter density [2–6]. Notably, inner caustics often appear as ring-like structures [7], especially in galaxies containing axion dark matter [8].

The presence of these caustic structures should be observable as specific features in the rotation curves of galaxies. However, galaxy rotation curves are generally smooth and universal Refs. [9, 10], making it challenging to identify caustic-caused variations. These variations tend to be subtle and difficult to distinguish from other factors like local streams or statistical fluctuations.

The positions of these caustics are determined by the distribution of angular momenta of the dark matter particles falling into the galaxy. Based on observations in specific galaxies like NGC 3198 [11] and the Milky Way [12], Kinney and Sikivie proposed [13] that the distribution of these angular momenta is universal. Consequently, the positions of caustic rings should be similar across different galaxies, allowing for a standard scaling adjustment. An analysis of the authors [13] of 32 high-quality galactic rotation curves indicated the potential existence of two such caustic-associated features, with a statistical significance ranging between 2.4σ and 3.0σ.

1.1 Purpose

Building on the previously discussed research about the presence of caustic rings in galaxies, this text highlights that the earlier findings have not yet been re-evaluated using newer data. Despite the availability of newer and similar quality rotation curves, these results remain untested. The current work aims to address this gap by testing the hypothesis of universal caustic rings using a large independent set of galaxy rotation curves that have been published since the original study [13].

2. Data on rotation curves

We use the *Spitzer Photometry and Accurate Rotation Curves* (SPARC) database of rotation curves[1] for 175 galaxies with various morphologies and luminosities [14]. This is a significant expansion from the previous study [13] that used a sample of 32 rotation curves taken from Refs. [15, 16]. To ensure an independent evaluation, the current study excludes the 29 galaxies from the SPARC database that were also part of the earlier sample of 32 galaxies. Additionally, 25 more rotation curves are removed due to specific quality criteria detailed in a later section of the study. As a result, the main sample, independent from Ref. [13], used for this independent test consists of 121 rotation curves.

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[1]Data available from `http://astroweb.cwru.edu/SPARC`.

3. Data analysis

3.1 Individual rotation curves

The focus is on identifying caustic rings in galaxies, which are areas where dark matter concentration is higher. The presence of these rings is expected to cause an increase in rotation velocity in these specific regions, leading to distinct features or bumps in the galaxy's rotation curve. However, it is important to note that galaxy rotation curves are inherently complex and not uniformly flat. This complexity is further increased by the presence of individual gas clouds within a galaxy, which can have their own peculiar velocities, independent of any caustic structures. To quantify the presence or lack of features, the authors of Ref. [13] proposed to use the procedure for separation the background rotation of the galaxy and for quantifying the "noise". This separation is crucial for isolating and confirming the presence of features specifically associated with caustic rings.

The hypothesis of Refs. [11, 13] predicts caustic rings at

$$\tilde{r} = a_n \frac{j_{\max}}{0.25} \frac{0.7}{h},$$

where the rescaled radius is defined as

$$\tilde{r} \equiv r \left(\frac{220 \text{ km/s}}{v_{\rm rot}} \right), \tag{1}$$

$n = 1, 2, 3 \ldots$ enumerates the caustic rings formed by the particles experiencing the nth infall,

$$a_n = \{39, 19.5, 13, 10, 8, \ldots\} \text{ kpc}$$

are the universal rescaled positions of these caustics, $j_{\max}$ is the peak of the distribution of dimensionless angular momenta of dark-matter particles and h is the dimensionless local Hubble constant. The universality hypothesis of Ref. [13] is that for all galaxies, $j_{max} = 0.27$ and therefore the rotation-curve bumps of various galaxies should coincide in terms of $\tilde{r}$, Eq. (1).

In practice, the rotation curves are never exactly flat, and the plateau regions, if any, are reached at various radial distances. Ref. [13] suggested to use the average rotation velocity in the outer part of the galaxy as v_{rot}. We follow the same prescription and determine the fitting parabola $\bar{v}(\tilde{r}) = c_0 + c_1 \tilde{r} + c_2 \tilde{r}^2$, then the average value $v_{\rm rot} = \langle v(\tilde{r}) \rangle_{\tilde{r} \geq 10 \text{ kpc}}$ is used in Eq. (1).

As a result, we obtain a set of rescaled data points, $(\tilde{r}_i, v_i \pm \Delta v_i)$, where all $\tilde{r}_i \geq 10$ kpc, and the corresponding fitting parabola $\bar{v}(\tilde{r})$.

3.2 Ensemble of rotation curves

Continuing from the previous discussion on identifying caustic rings in galaxies, we outline the specific method used in the current study to search for these features. We adopt the unbinned likelihood method, in which a continuous function $L(\tilde{r})$ is constructed from the entire data set of all galaxies. To assess the statistical significance of any identified features, including those that might be universal caustic rings, we use the Monte-Carlo method. This involves creating the same function L from simulated data and comparing it to the observed data. This comparative analysis

allows us to determine whether the features observed in the actual data are statistically significant or if they could be attributed to random variations.

The averaged likelihood function $L(\tilde{r})$ defined as

$$L(\tilde{r}) = - \sum_{j=1}^{N(\tilde{r})} \frac{\log[p_j(\tilde{r})]}{N(\tilde{r})}, \tag{2}$$

where a function $p_j(\tilde{r})$ was obtained by the linear interpolation of the corresponding points $(\tilde{r}_i, p_i)$ for each galaxy, and $N(\tilde{r})$ is the number of galaxies, for which p_j is determined at the point $\tilde{r}$.

3.3 Monte-Carlo estimate of significance

To estimate the significance of potential universal features in the rescaled rotation curves, we follow the standard parametric bootstrap approach, see e.g. Ref. [17]. We use the maximal value L_{max} of $L(\tilde{r})$ over the interval of interest, 10 kpc $\leq \tilde{r} \leq$ 75 kpc, which covers well the first two caustic rings discussed in Ref. [13].

Within some assumptions, we obtain a simulated data set and process it in the same way as the real data, see Sec. 3.2. We repeat M times the same procedure with simulated rotation-curve measurements and obtain a set of simulated $L_{\text{max}}^{(k)}$, $k = 1, \ldots, M$. The significance of the strongest universal feature in the set of rescaled rotation curves is determined by the p-value, counting how often the observed or larger value of L_{max} happens by chance in the simulated sets, that is the number of cases for which $L_{\text{max}}^{(k)} > L_{\text{max}}$ divided by M.

4. Results and discussion

4.1 Results for the main sample

The application of the procedure described in Sec.3.1, 3.2 to the main data set of 121 rotation curves not used in previous search for caustics, described in Sec.2, results in the $L(\tilde{r})$ function presented in Fig. 1. No significant peaks of $L(\tilde{r})$ are observed at the expected positions of caustic rings found in Ref. [13], nor elsewhere. Indeed, Figure 2 presents the distribution of $L_{\text{max}}^{(k)}$ for $k = 1, \ldots, M = 1000$ Monte-Carlo simulated data sets assuming no universal caustics and only statistical fluctuations in the rotation-curve measurements. The observed $L_{\text{max}} \approx 1.83$ or larger was found 202 times out of 1000, resulting in the p-value of 0.2 for the null hypothesis of the absence of universal caustic rings.

4.2 Results for the previously used sample

The results obtained above for the statistically independent sample of rotation curves do not support the indications to the presence of universal caustic rings claimed in Ref. [13]. It is therefore instructive to repeat our analysis for the original sample used in that study. Figure 3 presents the results of this analysis. Interestingly, it also does not reveal any significant excesses at the expected positions of the universal caustics. We find the p-value of 0.98 for the null hypothesis, which means that the consistency between data and expectations is "too good", which may happen for overestimated statistical errors in the initial data.
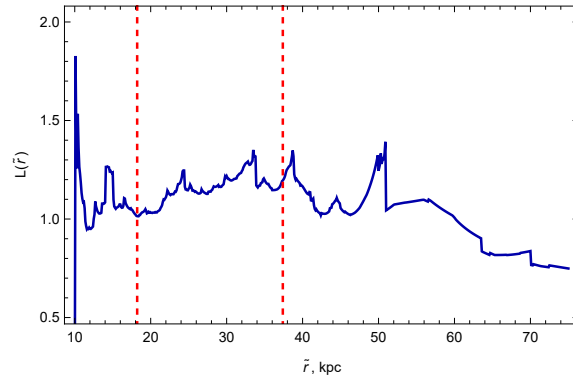
Figure 1: The function $L(\tilde{r})$ for the main sample of 121 galaxies which does not include those studied in Ref. [13]. The vertical dashed lines indicate the expected positions of the $n = 1, 2$ caustic rings claimed in Ref. [13], corrected for the slightly different value of h.
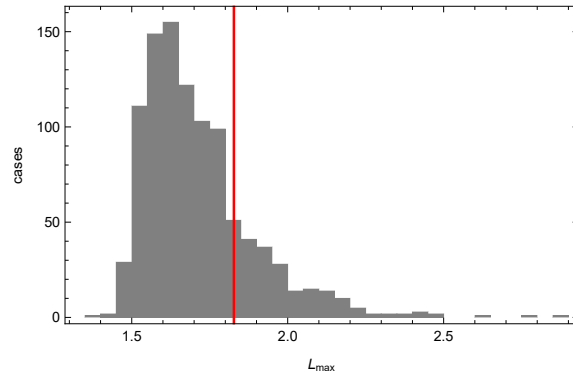

Figure 2: The distribution of the maxima of 1000 Monte-Carlo simulated $L(\tilde{r})$ functions for the main sample of rotation curves. The vertical line indicates the value of $L_{\max}$ obtained for the data.
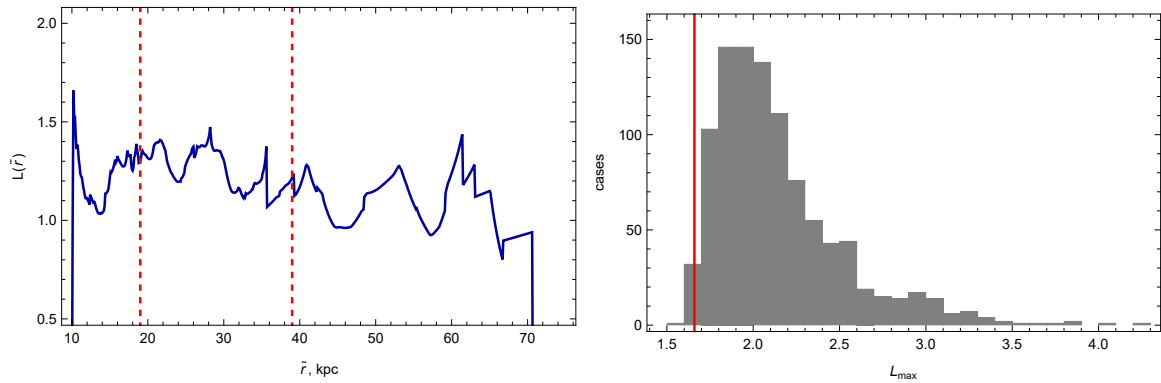

Figure 3: Results for the sample of 32 rotation curves studied in Ref. [13]. Left: the function $L(\tilde{r})$. The vertical dashed lines indicate the expected positions of the $n = 1, 2$ caustic rings claimed in Ref. [13]. Right: the distribution of the maxima of 1000 Monte-Carlo simulated $L(\tilde{r})$. The vertical line indicates the value of $L_{\max}$ obtained for the data.
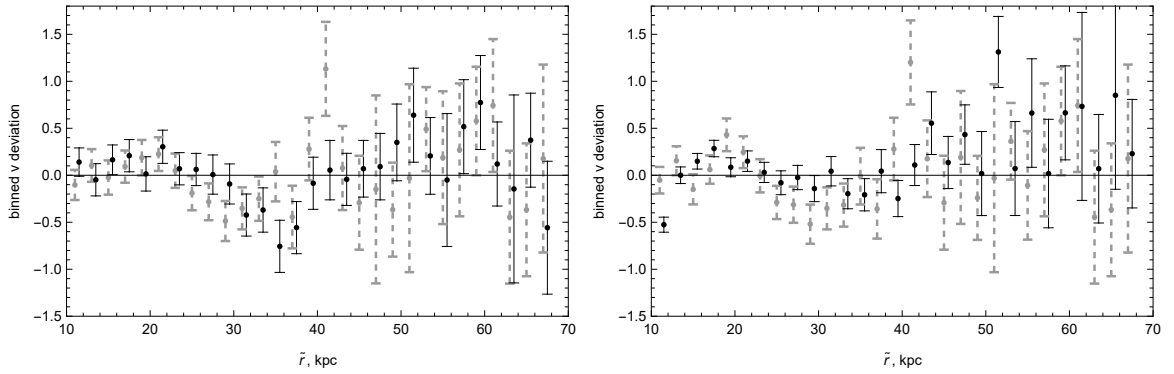
Figure 4: Comparison of the new results with those of Ref. [13] by the binned method of Ref. [13], see the text for details. Grey points with dashed error bars: the rotation curves used in Ref. [13]; black points with full error bars: new rotation curves from the SPARC database. Left panel: 29 galaxies present in both samples. Right panel: 32 galaxies in the old sample and 121 other galaxies in the new sample.

4.3 Discussion: comparison with the previous result

There are two main differences between the statistical analysis used in Ref. [13] and that of the present work. Firstly, we use the unbinned likelihood while the analysis of Ref. [13] was based on binning. Secondly, we use Monte-Carlo simulations based on the null hypothesis to estimate the significance, while Ref. [13] assigned statistical errors to the binned data by hand. We note that the resulting significance reported in Ref. [13] does not account for the selection of the bin with the strongest deviation (the look-elsewhere effect), nor for the choice of the bin size. For the 30 bins, Ref. [13] reports a 2.6σ) significance, that is the p-value of 0.01; since the data in the bins are independent, a reasonable estimate of the look-elsewhere correction would be to multiply the p-value by the number of bins, which would result in the post-trial significance fully consistent with the null hypothesis.

To provide a comprehensive comparison with the previous study Ref. [13], the current research also analyzes the new data using the same methodology as that earlier study. Fig.4 shows normalized deviations of observed velocities from fitted ones, averaged over bins in the rescaled radius $\tilde{r}$, similar to what was done in Ref. [13].

5. Conclusions

In this paper, the hypothesis of universal caustic rings in galaxies, a concept originally proposed in Ref. [13], is tested. To conduct this test, we use the SPARC database of galactic rotation curves [14], deliberately excluding the galaxies that were part of the original study [13]. After applying additional criteria to ensure the quality of the data, we focus on a main sample consisting of 121 rotation curves.

We apply the unbinned likelihood method for analysis and use Monte-Carlo simulations to evaluate the statistical significance of any potential universal features in these rotation curves. Our calculations do not support the presence of universal caustic rings. The data agrees with the

expectations of the null hypothesis with the p-value of 0.20, suggesting no significant evidence of such features.

We discuss the reasons for the discrepancy between our results and the original findings of Ref. [13]. We conclude that the most likely cause of this discrepancy is the statistical method used in the original study which overestimates the significance. We demonstrate that even when applying the methodology of Ref. [13] to the new data, the conclusions of the original study are not supported. This comprehensive analysis thus casts doubt on the earlier claims of universal caustic rings in galaxies.

However, the stacking method, used in Ref. [13] and in the present work, tests only the universality of the caustics. The locations of the caustic rings may vary from one galaxy to another even in terms of the rescaled coordinate $\tilde{r}$, indicating a scatter in the angular momentum distributions of infalling dark matter. To find or exclude such non-universal caustics would be a much more difficult observational task because the effect of the matter overdensity on the rotation curve should be disentangled from local variations in the velocity field of the galactic gas.

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