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On thermal false vacuum decay around black holes

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We study false vacuum decay in scalar field theories in the presence of black holes and a heat bath. We assume that a black hole is in thermal equilibrium with the heat bath (the Hartle-Hawking state). Combining analytical and numerical methods, we show that there are no regular Euclidean time-dependent and spherically-symmetric solutions around many types of black holes in this state. This is in contrast with the flat spacetime where such solutions describe tunneling at low temperature. We conclude that, for many black hole metrics in various dimensions, the Euclidean solution that is relevant for the decay of the Hartle-Hawking vacuum is always static.

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1. Introduction

Consider a theory of scalar field φ in flat spacetime and in contact with an external heat bath. Let the scalar field potential $V(\varphi)$ have a local metastable minimum $V = 0$ at $\varphi = 0$, separated from the global minimum by the barrier. At low ambient temperature T, the escape from the potential well around $\varphi = 0$ is driven by quantum tunneling. On the other hand, at high temperature the decay of the metastable state proceeds via classical thermal jumps of the field over the barrier.

In the semiclassical approximation, the vacuum decay probability is saturated by the bounce — a regular solution of Euclidean equations of motion satisfying vacuum boundary conditions [1, 2]. The low-temperature tunneling corresponds to bounces which are periodic in Euclidean time with the period $1/T$ and have nontrivial time dependence [3, 4]. The high-temperature thermal activation is described by the Euclidean time-independent solution referred to as sphaleron. Importantly, the relevant Euclidean solution must have exactly one negative mode — a linear perturbation reducing its Euclidean action [1, 2].

In this note we study how this picture of thermal vacuum decay is modified in the vicinity of black holes. The latter are assumed to be in equilibrium with the surrounding heat bath. This is known as the Hartle-Hawking vacuum [5], and its decay can be studied in the Euclidean time formalism.

False vacuum decay catalysed by black holes is a long-standing problem [6–9] and a subject of recent research [10–25]. The Euclidean time approach was adopted, e.g., in Refs. [8, 10, 13] which study spherically-symmetric classical configurations in the thin-wall approximation around the Schwarzschild black hole. Interestingly, regular Euclidean solutions found in [8] are all time-independent. Likewise, Refs. [10, 13] found that the dominant contribution to the vacuum decay probability around a static black hole is always provided by the static configuration. The absence of periodic bounces at all temperatures is surprising, and one may wonder if it is an artifact of the thin-wall approximation.

In this note we present arguments that this is not the case. We perform both analytical investigation and numerical search for spherically-symmetric periodic bounces around many types of black holes in various dimensions. Our result is that, in most cases, the sphaleron is indeed the only relevant Euclidean solution at all T. We find one exception to this rule (Reissner-Nordström black hole near criticality) and discuss its possible physical interpretation.

First, we develop an analytical argument for the absence of periodic bounces in the vicinity of low-temperature black holes (sec. 2). It is based on the observation that one can build a family of Euclidean configurations that resemble exact tunnelling solutions ("Rindler valley"). The Euclidean action computed along this family achieves a local minimum on a static configuration, corresponding to the sphaleron. Next, we perform a numerical search for periodic bounces which is not confined to the black hole vicinity and spans all values of T (sec. 3). This includes the most physically interesting region $T \sim \ell^{-1}$, where ℓ is the size of the flat-space vacuum bounce, for which one expects the largest catalysing effect [16, 21]. Finally, in sec. 4, following [24], we present a general analytical argument for the absence of periodic bounces around black holes, which is valid for a general multiscalar theory and a wide class of black holes in d dimensions. It amounts to counting the number of $O(d-1)$-symmetric negative modes of the sphaleron. Contrary to the flat space case, we find exactly one such mode at any temperature. In sec. 5 we conclude. Throughout our analysis we neglect the back-reaction of the scalar field on the metric.

2. Rindler valley

We start with the argument that works near the black hole horizon, $r - r_h \ll r_h$, where r, r_h are the physical distance and the Schwarzschild radius, correspondingly, and at small temperature, $T \ll \ell^{-1}$. Going beyond these limitations is the subject of secs. 3, 4. The analysis is done in two dimensions; the generalisation to $d > 2$ is straightforward and discussed in the end of this section.

As discussed in [21], only the exterior region of a black hole is relevant for the computation of the vacuum decay probability. In this region, the metric can be written in the form

$$\mathrm{d}s_2^2 = \Omega(\mathrm{d}\tau^2 + \mathrm{d}x^2) \ . \tag{1}$$

The Euclidean action of the field φ with the potential $V(\varphi)$ reads,

$$S = \int \mathrm{d}\tau \mathrm{d}x \left\{ \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \Omega V(\varphi) \right\} . \tag{2}$$

For definiteness, consider the background of a $2d$ dilaton black hole [26]

$$\Omega = (1 + \mathrm{e}^{-2\lambda x})^{-1} \ , \tag{3}$$

where λ is associated with the temperature as $\lambda = 2\pi T$. Near horizon, the conformal factor (3) is expanded as $\Omega = \mathrm{e}^{2\lambda x} - \mathrm{e}^{4\lambda x} + \ldots$ The change of variables

$$\mathcal{T} = \frac{1}{\lambda}\mathrm{e}^{\lambda x} \sin\lambda\tau \ , \quad X = \frac{1}{\lambda}\mathrm{e}^{\lambda x} \cos\lambda\tau \tag{4}$$

maps the near-horizon region $x < 0$ to the vicinity $\lambda R < 1$ of the point $(\mathcal{T}, X) = (0, 0)$, where we denote $R^2 = \mathcal{T}^2 + X^2$. The Euclidean line element (1) becomes

$$ds_2^2 = F(\mathrm{d}\mathcal{T}^2 + \mathrm{d}X^2) \ , \tag{5}$$

and the function $F = \Omega \mathrm{e}^{-2\lambda x}$ behaves as $F = 1 - \lambda^2 R^2 + \ldots$ near the origin. We see that close to the horizon, $\lambda R \ll 1$, the spacetime geometry is approximately flat. In fact, the transformation (4) of the Cartesian coordinates $(\mathcal{T}, X)$ brings the flat metric to the Rindler metric. The latter is the metric of uniformly accelerating observers in flat spacetime. The line of constant x represents a trajectory of an observer moving with the acceleration $\lambda \mathrm{e}^{-\lambda x}$. Thus, the black hole vicinity looks like the Rindler wedge—the portion of the flat spacetime accessible to the observer, separated from the rest of the world by the horizon [22, 27].

Let $\varphi_0 = \varphi_0(R)$ be the vacuum (zero-temperature) bounce in flat space. Denote

$$\varphi_{X_0} \equiv \varphi_0\left(\sqrt{(X - X_0)^2 + \mathcal{T}^2}\right) \ . \tag{6}$$

The corresponding Euclidean action B_0 is clearly independent of X_0. Switching to the Rindler metric converts φ_0 into a static solution $\varphi_0(R) = \varphi_0(\mathrm{e}^{\lambda x}/\lambda)$, while the bounces φ_{X_0} with $X_0 > 0$

become periodic solutions with the period $2\pi/\lambda = 1/T$. [1] Thus, the decay of the Minkowski vacuum in the Rindler frame is mediated by periodic bounces or a sphaleron. This is to be expected, since the Minkowski vacuum contains a heat bath from the viewpoint of the accelerating observer.

Crucially, the Euclidean action of the Rindler bounces φ_{X_0}, computed in the Rindler frame, is independent both of X_0 and λ, and equals B_0. Thus, the Rindler space possesses a degenerate family of periodic Euclidean solutions parameterised by $X_0 > 0$, with the limit $X_0 = 0$ corresponding to the sphaleron.

Consider the black hole corrections $\propto \lambda^2 R^2$ to function F in eq. (5). The Rindler bounces (6) are no longer solutions of the equation of motion in this background, and the parameter X_0 no longer corresponds to a zero-mode. Hence, the Euclidean action $S[\varphi_{X_0}]$ of the Rindler bounces in the background (5) depends on X_0: the Rindler valley is tilted by the deviation of the black hole metric from the Rindler metric. Near horizon the deviation is small, and we can write

$$S[\varphi_{X_0}] = B_0 + \Delta S(X_0) \ . \tag{7}$$

Let $X_0 = \bar{X}_0$ be the local minimum of this action. The bounce $\varphi_{\bar{X}_0}$ provides the closest approximation to the true Euclidean solution. To the leading order in X_0 and λ, the correction in eq. (7) is given by

$$\Delta S(X_0) = X_0^2 \lambda^4 \int \mathrm{d}^2 X \, R^2 V(\varphi_0) + \text{const} \ , \tag{8}$$

where const denotes the X_0-independent term. The integral in eq. (8) is positive. This follows from the monotonicity of the flat vacuum bounce and from the fact that, by the Derrick's scaling argument [28], the integral of $V(\varphi_0)$ equals 0 in two dimensions. Hence, $\bar{X}_0 = 0$, i.e., the action is minimized on the Rindler sphaleron.

Thus, we argue that the bounce in the black hole background is itself static. The validity of the argument is based on the assumption that the correction to B_0 in eq. (7) is small. This is true as long as the bounce fits the near-horizon region, requiring

$$\lambda \ell \ll 1 \ , \quad \lambda X_0 \ll 1 \ . \tag{9}$$

The first of these conditions means that the black hole is cold compared to the characteristic energy scale of the flat-space vacuum bounce, which is given, typically, by the mass of the field φ. The second condition restricts the analysis to the black hole vicinity $x < 0$ or, equivalently, $r - r_h \ll r_h$.

The above analysis is generalised straightforwardly to other black hole metrics and $d > 2$. As an example, consider the Schwarzschild black hole in four dimensions. We write the metric as

$$\mathrm{d}s_4^2 = \Omega(\mathrm{d}\tau^2 + \mathrm{d}x^2) + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \ , \tag{10}$$

where

$$\Omega(x) = 1 - \frac{r_h}{r(x)} \ , \quad x = r + r_h \log[2\lambda r - 1] \ . \tag{11}$$

The Euclidean action of φ, restricted to $O(3)$-symmetric configurations, takes the form

$$S = 4\pi \int \mathrm{d}\tau \mathrm{d}x \, r^2 \left\{ \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \Omega V(\varphi) \right\} \ . \tag{12}$$

[1] It is sufficient to consider positive X_0, since the bounces with $X_0 < 0$ can be transformed to those with $X_0 > 0$ by a shift along the τ-axis.

Near horizon, we expand $2\lambda r = 1 + \mathrm{e}^{2\lambda x} + \ldots$ and $\Omega = \mathrm{e}^{2\lambda x} + \ldots$ and observe that the dimensionally-reduced theory (12) lives in the Rindler wedge of the flat $2d$ space. The corrections to Ω and the function $r(x)$ spoil this correspondence. Proceeding as before, we arrive at eq. (7), where φ_{X_0} are Rindler bounces from the flat-space limit of the theory (12) and B_0 is the action of the flat-space vacuum bounce in that theory. The correction is readily computed and equals

$$\Delta S(X_0) = \pi X_0^2 \int \mathrm{d}^2 X \, (\varphi_0')^2 + \text{const} \; . \tag{13}$$

The integral is manifestly positive, hence the Rindler valley is tilted towards the sphaleron.

The validity conditions (9) limit the significance of the argument based on the Rindler valley in $4d$. At small λ, the Schwarzschild black hole is large, and the Euclidean action of the sphaleron around the black hole is enhanced by a big factor $\propto r_h^2$. Clearly, the tunneling away from the black hole is preferred in this case. It is desirable to extend the search for periodic bounces to the physically interesting region $\lambda \gtrsim \ell^{-1}$, and we do this below.

3. Neumann mirror

Here our goal is to probe numerically the region $\lambda \sim \ell^{-1}$ and look for a possible location of the periodic bounce anywhere at $x \gtrsim 0$. For the numerical analysis we choose the theory of the massive field with the quartic self-interaction:

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 - \frac{1}{4} g \varphi^4 \; , \tag{14}$$

where $g > 0$. We explain our analysis using again the $2d$ dilaton black hole as an example and comment on its generalisation and on the $4d$ Schwarzschild black hole in the end.

In fact, it is natural to expect that the periodic bounce, if it exists at all, is located at the border between the near-horizon and the asymptotically-flat regions, at $x \approx 0$ or, equivalently, $r - r_h \approx r_h$. To see this, we analyse the asymptotically-flat region by the method of sec. 2. Away from the black hole, the flat-space periodic bounce φ_{th} is a near-solution to the equation of motion. The leading correction to its action comes from the deviation of the metric from the Euclidean one and is captured by expanding Ω in eq. (2) at large λx. We obtain

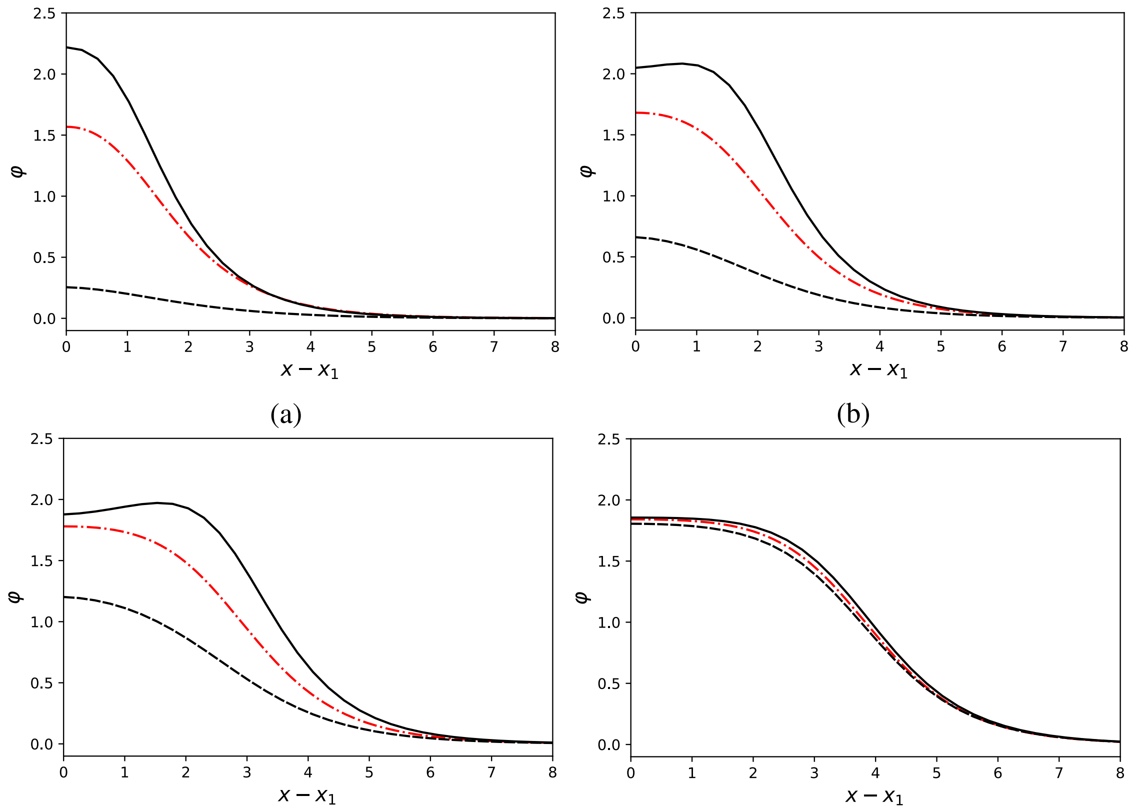
$$S(x_0) = B_{th} - \mathrm{e}^{-2\lambda x_0} \int_{-\pi/\lambda}^{\pi/\lambda} \mathrm{d}\tau \int_{-\infty}^{\infty} \mathrm{d}x \, V(\varphi_{th}) \; , \tag{15}$$

where x_0 is the position of the center of the bounce, and we assume λx_0, $m x_0 \gg 1$. One can check numerically that the integral in eq. (15) is positive. Furthermore, in the limit of vanishing temperature, $S(x_0) - B_{th} \to 0$.

The periodic bounce φ_{th} exists as long as $\lambda < \lambda_c \approx 1.73m$ [24]. At larger temperatures, the only Euclidean solution is the sphaleron φ_s for which we have

$$\int_{-\pi/\lambda}^{\pi/\lambda} \mathrm{d}\tau \int_{-\infty}^{\infty} \mathrm{d}x \, V(\varphi_s) = \frac{\pi}{\lambda} \int_{-\infty}^{\infty} \mathrm{d}x \, \varphi_s'^2 > 0 \; . \tag{16}$$

Thus, at any λ the "Minkowski valley" is tilted towards the black hole. This is just another manifestation of the catalysing effect of the curved geometry of the black hole. One might,



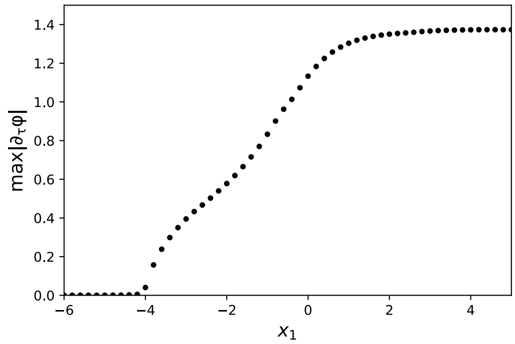
(a) (b) (c) (d)

Figure 1: Euclidean solutions of the two-dimensional model (2), (14) in the black hole background (3), attached to the Neumann mirror located at $x = x_1$. We take $\lambda = 0.7m$. Black solid and black dashed lines show $\tau = 0$ and $\tau = \pi/\lambda$ slices of the periodic solutions. Red dash-dot lines show the slice of the static solutions. x and φ are measured in units of m^{-1} and $m/\sqrt{g}$, correspondingly. **(a)** $x_1 = -1$. The mirror has just entered the near-horizon region. The periodic solution still closely resembles the flat-space thermal bounce. **(b)** $x_1 = -2$. As the mirror advances towards the horizon, the maximum of the periodic solution detaches from the mirror and stays at the border of the near-horizon region. **(c)** $x_1 = -3$. The maximum of the periodic solution persists at $x \approx 0$, but its overall time-dependence is smoothed out gradually. **(d)** $x_1 = -4$. The periodic solution is on the verge of merging with the corresponding solution from the static branch.

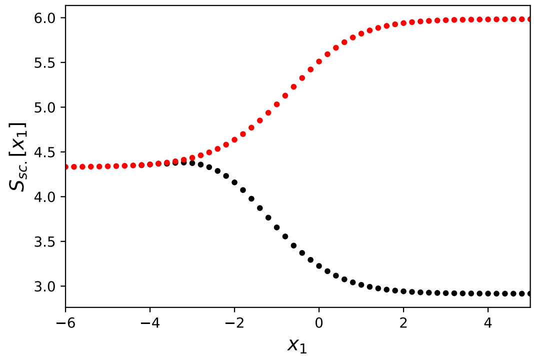
therefore, expect that the endpoint of the valley at $x = 0$ corresponds to a periodic configuration that approximates the true periodic bounce. The associated tunneling suppression may, furthermore, be less than that for the black hole sphaleron, at least in a certain range of temperatures.

To check this possibility, we will modify our system in the way that forces the periodic bounce to exist at a given position $x = x_1$, and we will see if the time-dependence of the bounce persists once the modification is gradually removed.

Consider the regime $\lambda < \lambda_c$, in which the flat thermal bounce φ_{th} is almost the solution far from the black hole. To convert it to the true solution, we put a shell with the reflective boundary



(a)



(b)

Figure 2: (a) Maximum absolute value of the time-derivative of the Euclidean periodic solutions on the mirror, as a function of the position of the mirror x_1. We take $\lambda = 0.7m$. x_1, τ and φ are measured in units of m^{-1} and $m/\sqrt{g}$, correspondingly. We see that at $x_1 \approx -4$ the time-dependence of the mirror solutions disappears, cf. Fig. 1. **(b)** The action of the periodic solutions on the mirror (black) and the action of the static solutions on the mirror (red) at the same temperature. We see again that the two branches of the solutions merge at a finite value of x_1. Note that the periodic solution action is always smaller than that of the static solution.

condition— the "Neumann mirror"—at $x = x_1$ such that $x_1 m, x_1\lambda \gg 1$. The half of the bounce φ_{th} is attached to the side $x > x_1$ of the mirror. As can be checked numerically, the reflective condition on the mirror counter-reacts the tilt induced by the black hole, and the "half-bounce" becomes the valid solution at $x \geqslant x_1$. [2]

Next, we start moving the mirror towards the black hole. By continuity, the Euclidean solution will exist for all positions of the mirror, and we would like to see how it changes once the mirror enters the near-horizon region. This procedure is illustrated in Figs. 1, 2, where we take $\lambda = 0.7m$. We see that at positive x_1, the solution attached to the mirror is similar to the flat-space thermal bounce. When x_1 becomes negative, the solution develops a local maximum at $x \approx 0$. However, as the mirror moves further, the maximum disappears. At the same time, the time-dependence of the solution gradually diminishes (Fig. 2 (a)). Finally, at some $x_1 = x_1^*$, the solution becomes static. It remains static for $x_1 < x_1^*$ and evolves towards the black hole sphaleron.

For completeness, we repeat the procedure with the flat-space "half-sphaleron" as an initial guess. It gives rise to a branch of static solutions parameterized by the position of the mirror. They are also shown in Fig. 1. The branch of periodic solutions merges with the branch of static solutions at $x_1 = x_1^*$. It is interesting to compare the Euclidean actions (2) of the solutions from the two branches. We plot them in Fig. 2 (b). Note that the space integral in (2) is restricted to $x > x_1$. We see that the action of the periodic solution is always smaller than that of the static solution.

We search for the periodic bounce with the Neumann mirror at various temperatures below λ_c. In the small-temperature limit, the point x_1^* of merging of the two branches of solutions remains finite but tends to $-\infty$. As the temperature grows, x_1^* grows as well. Thus, we have another argument against periodic bounces in the black hole background that works in the physically most interesting

[2]The half-space $x < x_1$ is excluded from consideration.

range of temperatures, $\lambda \sim m \sim \ell^{-1}$, and at an arbitrary distance from the black hole.

The above analysis can be repeated with any black hole metric and in $d > 2$. Let us comment on the case of the $4d$ Schwarzschild black hole (eqs. (10) and (11)). At small temperature, the vacuum decay channel associated with the $O(3)$-symmetric Euclidean configurations, including the black hole sphaleron, is suppressed due to the large size of the black hole. On the other hand, at large temperature, the Euclidean solutions are insensitive to the small-size black hole, and the vacuum decay proceeds via the flat-space sphaleron [24]. The physically interesting range of temperatures to look for periodic bounces is where, on the one hand, the flat thermal bounce still exists and, on the other hand, the black hole sphaleron already dominates vacuum transitions. For the theory (12), (14) this range is $0.60 < \lambda/m < 1.73$.

We replace the black hole horizon with the $O(3)$-symmetric Neumann mirror located at $x = x_1$, and study periodic configurations attached to the mirror in outer region $x > x_1$. Once the seed time-dependent solution is found at some values of λ, x_1, we start collapsing the mirror by decreasing x_1. [3] We find again a critical value, $x_1 = x_1^*$, at which the time-dependence of the solutions disappears. The latter evolve towards the black hole sphaleron as $x_1 \to -\infty$.

4. Negative mode

Finally, we present the argument based on counting negative modes of the black hole sphaleron [24]. As discussed in introduction, the relevant bounce, being a saddle point of the action, must have exactly one such mode. We give the proof using the $2d$ model (2), (3) and comment on the generalisation in the end.

In the Euclidean action (2), make the change of variables

$$\sigma = \lambda^{-1}\,\mathrm{arcsh}(\mathrm{e}^{\lambda x})\ , \quad \theta = \lambda\tau\ . \tag{17}$$

In the new variables, the equation of motion for the sphaleron reads

$$-\frac{\mathrm{d}^2\varphi_s}{\mathrm{d}\sigma^2} - \frac{2\lambda}{\mathrm{sh}\,2\lambda\sigma}\frac{\mathrm{d}\varphi_s}{\mathrm{d}\sigma} + V'(\varphi_s) = 0\ . \tag{18}$$

Consider the eigenmodes of perturbations around φ_s. They have angular dependence $\mathrm{e}^{\pm in\theta}$. An eigenfunction $\chi_\mu(\sigma)$ corresponding to the eigenvalue μ in the n-th sector obeys a Schrödinger-type equation

$$H_n\chi_\mu = \mu\,\chi_\mu \tag{19}$$

with the Hamiltonian

$$H_n = -\frac{\mathrm{d}^2}{\mathrm{d}\sigma^2} - \frac{2\lambda}{\mathrm{sh}\,2\lambda\sigma}\frac{\mathrm{d}}{\mathrm{d}\sigma} + U_n(\sigma) \tag{20}$$

and

$$U_n(\sigma) = V''\big(\varphi_s(\sigma)\big) + \frac{n^2\lambda^2}{\mathrm{th}^2\,\lambda\sigma}\ . \tag{21}$$

Note that the Hamiltonian (20) is Hermitian with respect to the positive measure $\int \mathrm{d}\sigma\,\lambda^{-1}\,\mathrm{th}\,\lambda\sigma$ and the potential (21) is positive at $\sigma \to \infty$. Hence, negative modes ($\mu < 0$) of H_n, if any, form a discrete spectrum.

[3] Unlike the $2d$ case, the seed solution does not coincide with the flat-space thermal bounce.

Note that in the monopole sector ($n = 0$) the sphaleron has exactly one negative mode. Indeed, monopole perturbations correspond to static configurations whose Euclidean action is given simply by their energy times $2\pi/\lambda$. But the sphaleron is, by definition, the saddle-point of the static energy functional and hence the minimum of the energy in all but one direction.

Now we show that the spectrum in the dipole sector ($n = 1$) is strictly positive. Differentiating eq. (18) with respect to σ gives

$$-\frac{\mathrm{d}^2\dot{\varphi}_s}{\mathrm{d}\sigma^2} - \frac{2\lambda}{\mathrm{sh}\,2\lambda\sigma}\frac{\mathrm{d}\dot{\varphi}_s}{\mathrm{d}\sigma} + \tilde{U}(\sigma)\dot{\varphi}_s = 0\,, \tag{22}$$

where $\dot{\varphi}_s \equiv \mathrm{d}\varphi_s/\mathrm{d}\sigma$ and

$$\tilde{U}(\sigma) = U_1(\sigma) - \lambda^2\,\mathrm{th}^2\,\lambda\sigma\,. \tag{23}$$

This implies that $\dot{\varphi}_s$ is a zero mode of the Hamiltonian

$$\tilde{H} = -\frac{\mathrm{d}^2}{\mathrm{d}\sigma^2} - \frac{2\lambda}{\mathrm{sh}\,2\lambda\sigma}\frac{\mathrm{d}}{\mathrm{d}\sigma} + \tilde{U}(\sigma)\,. \tag{24}$$

Since the sphaleron is a monotonic function of σ, its derivative $\dot{\varphi}_s$ has no nodes and, hence, it is the ground state of $\tilde{H}$. But $H_1 - \tilde{H} = \lambda^2\,\mathrm{th}^2\,\lambda\sigma > 0$, hence the discrete spectrum of H_1 is strictly larger than that of $\tilde{H}$. Thus, the ground state energy of H_1 is strictly positive. Also, since $U_n(\sigma) > U_1(\sigma)$ for $n > 1$, the spectra in higher multipoles are positive as well. We conclude that the only negative mode of the Euclidean time-independent bounce resides in the monopole sector.

The above proof is straightforwardly generalized to the theory of several scalar fields and to higher dimensions [24]. In d dimensions, consider the metric

$$\mathrm{d}s^2 = f(r)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Sigma^2_{d-2}\,, \tag{25}$$

where $\mathrm{d}\Sigma^2_{d-2}$ is the line element on the $(d-2)$-dimensional unit sphere, and $f(r_h) = 0$, $f'(r_h) > 0$. Repeating the analysis above yields the sufficient condition for the absence of the second $O(d-1)$-symmetric negative mode around the $O(d-1)$-symmetric sphaleron: [4]

$$\frac{(f'(r_h))^2 - (f')^2}{4} - \frac{(d-2)f^2}{r^2} + \frac{(d-2)ff'}{2r} + \frac{ff''}{2} > 0 \tag{26}$$

at $r > r_h$. It is easy to see that this condition is satisfied for many common black hole metrics, including the Schwarzschild and Schwarzschild-(anti) de Sitter metrics. On the other hand, for the Reissner-Nordström metric, the condition is violated if the charge Q of the black hole is close to critical, $Q^2 > Q_*^2$, where in $4d$, $Q_*^2 \simeq 0.83$ (in units of black hole mass). In this case the time-independent bounce *does* have a negative dipole mode. Hence, for a nearly critical black hole, it is not a valid solution for the false vacuum decay and we expect existence of periodic bounces centered on the black hole.

[4]Our results do not exclude existence of extra negative modes which break this symmetry. In fact, their presence is expected for large enough black holes.

5. Conclusion

We presented the arguments, both analytical and numerical, for the absence of Euclidean time-dependent and spherically-symmetric periodic bounces around many types of black holes in various dimensions. This suggests to interpret thermal false vacuum decay around such black holes as driven by thermal activation at any temperature. This is consistent with the fact that, from the viewpoint of an asymptotic observer, the temperature in the vicinity of the black hole is blue-shifted and leads to enhanced thermal fluctuations, sufficient to overcome the barrier between the vacua.

However, it appears that there is no clear distinction between quantum tunneling and thermal activation in curved spacetime, as these concepts are frame-dependent. An example is Coleman-De Luccia [29] and Hawking-Moss instantons [30] in de Sitter spacetime, which can be interpreted either as vacuum bounces in inflationary coordinates [31], or as thermal (sphaleron) transitions in the static patch [32]. Similarly, the black hole sphaleron in Schwarzschild metric becomes Euclidean time-dependent in, e.g., Kruskal coordinates, and in this frame it corresponds to a vacuum bounce.

Interestingly, in the case of a near-critical Reissner-Nordström black hole we do expect the existence of periodic bounces centered on the black hole. An explanation of this fact may be that, in the critical limit, such black hole has zero temperature, hence the state around it appears as vacuum even for a static observer. The true bounce in this case must be localized in the Euclidean time, as in Minkowski space, whereas the time-independent bounce has infinite suppression.

It would be interesting to extend our analysis by including gravitational back-reaction of the bounce on the metric. Also, it would be interesting to see if one can use similar methods to get insight into the structure of complex bounces describing false vacuum decay in non-equilibrium situations such as a black hole emitting Hawking radiation into empty space (Unruh vacuum) [21, 22].

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