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Photon Polarization Operator in External Electromagnetic Field with Account of Virtual-Fermion AMM

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Two-point vector-tensor and tensor-tensor correlators of fermionic currents are calculated in a background filled with a constant homogeneous magnetic field. The tensor current is a fermionic part of the Pauli Lagrangian density describing the electromagnetic interaction of fermions through their anomalous magnetic moment (AMM). Under assumption that this interaction enters the effective QED Lagrangian, the contribution induced by AMM to the photon polarization operator is calculated and its impact is discussed.

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1. Introduction

Electron-positron colliders allow to get detail information about the photon propagation from the energy scan in the processes $e^+e^- \to$ hadrons and $e^+e^- \to \tau^+\tau^-$, for example, at BEPC-II in Beijing (China) or VEPP-4M in Novosibirsk. In this case, one needs to know the photon polarization function $\Pi^{\rm em}(Q^2)$, where $Q^2 = -q^2$ is the four-momentum carried by the photon, originated by the quark electromagnetic current $j^{\rm em}_\mu(x)$ (for its definition see, for example, [1, 2]). The Feynman diagram describing the $\gamma \to \gamma$ transition via virtual charged fermions in the one loop approximation is presented in Fig. 1. Many physical observables are connected with $\Pi^{\rm em}(Q^2)$ (see, for example, [3]). In addition to the vector-vector currents' correlator, massless diagonal vacuum correlators of scalar and tensor quark currents at higher orders in α_s are also known [4].

In the following, the photon polarization tensor and its eigenvalues are considered under an influence of an external constant homogeneous magnetic field. The Fock-Schwinger proper-time formalism is used in calculations. The extended version of the Lagrangian density in the spinor Quantum Electrodynamics (QED) is considered which contains, in addition to the standard vector interaction, a tensor one relevant for the electromagnetic interaction of a fermion due to its anomalous magnetic moment (AMM). It is well-known [5, 6] that for the electron AMM is suppressed by the fine structure constant, α, compared to the normal magnetic moment — the Bohr magneton, μ_B, but it can be enhanced substantially by "New Physics" contributions.

2. Photon Polarization Operator

The interaction Lagrangian density of the spinor QED has the form [7]:

$$\mathcal{L}_{\rm QED}(x) = -e \sum_f Q_f \left[\bar{f}(x)\gamma_\mu f(x)\right] A^\mu(x), \tag{1}$$

where summation is over all the charged fermions — charged leptons and quarks. For quarks, the sum over the color indicies is implicitly assumed. The matrix element of the $\gamma \to \gamma$ transition, the Feynman diagram of which is presented in Fig. 1, can be written as follows:

$$\mathcal{M}_{\gamma\to\gamma} = -i\,\varepsilon'^{*}_\mu(q)\,\mathcal{P}^{\mu\nu}(q)\,\varepsilon_\nu(q), \tag{2}$$

where $\varepsilon^{(\prime)}_\mu(q)$ is the polarization vector of the initial (final) photon with the four-momentum q^μ. The symmetric second-rank tensor, $\mathcal{P}^{\mu\nu}(q)$, in Eq. (2) is the Fourier-transform of a two-point vacuum correlator of two vector fermionic currents. Photon dispersion relations follow from the equations [5, 6]:

$$q^2 - \Pi^{(\lambda)}(q) = 0, \tag{3}$$

where $\Pi^{(\lambda)}(q)$ with $\lambda = 1, 2, 3$ are the eigenvalues of $\mathcal{P}^{\mu\nu}(q)$. In an external background electromagnetic field, the corresponding modification of the fermion propagator should be taken into account (see [8, 9] and references therein).

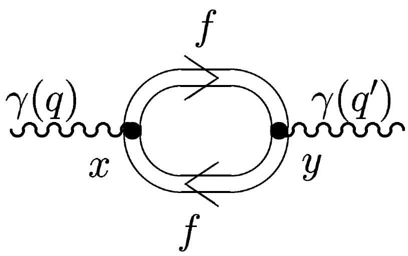

Figure 1: The Feynman diagram describing the $\gamma \to \gamma$ transition via virtual fermions. Double lines indicate that effects of an external electromagnetic field are taken into account exactly in the fermion propagators.

3. Basic Tensors in Presence of Magnetic Field

Minkowski space filled with external constant homogeneous magnetic field is effectively separated into two orthogonal subspaces: Euclidean with the metric tensor $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu} = \varphi_{\mu\rho}\varphi^{\rho}{}_{\nu}$ which is the plane orthogonal to the field strength vector $\mathbf{B}$ and pseudo-Euclidean with the metric tensor $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu} = \tilde{\varphi}_{\mu\rho}\tilde{\varphi}^{\rho}{}_{\nu}$ [8, 9]. Metric tensor of Minkowski space is the direct sum of them: $g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}$. The dimensionless tensor of the external magnetic field and its dual:

$$\varphi_{\alpha\beta} = \frac{F_{\alpha\beta}}{B}, \qquad \tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\,\varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}, \tag{4}$$

used above, are the Levi-Civita tensors in these subspaces. An arbitrary four-vector, $a^{\mu} = (a_0, a_1, a_2, a_3)$, can be decomposed into two orthogonal components:

$$a_{\mu} = \tilde{\Lambda}_{\mu\nu}a^{\nu} - \Lambda_{\mu\nu}a^{\nu} = a_{\parallel\mu} - a_{\perp\mu}. \tag{5}$$

For the scalar product of two four-vectors one has:

$$(ab) = (ab)_{\parallel} - (ab)_{\perp}, \tag{6}$$

where $(ab)_{\parallel} = (a\tilde{\Lambda}b) = a^{\mu}\tilde{\Lambda}_{\mu\nu}b^{\nu} = a_0b_0 - a_3b_3$ $(ab)_{\perp} = (a\Lambda b) = a^{\mu}\Lambda_{\mu\nu}b^{\nu} = a_1b_1 + a_2b_2$, and the explicit component combinations are valid for the strength vector $\mathbf{B} = (0, 0, B)$.

4. Magnetic-Field-Based Orthogonal Basis

Correlators having a non-zero rank can be decomposed in some orthogonal set of four-vectors. In the magnetic field, such a basis naturally exists [8, 9]:

$$\begin{aligned} b^{(1)}_{\mu} &= (q\varphi)_{\mu}, \qquad b^{(2)}_{\mu} = (q\tilde{\varphi})_{\mu} \\ b^{(3)}_{\mu} &= q^2\,(\Lambda q)_{\mu} - (q\Lambda q)\,q_{\mu}, \quad b^{(4)}_{\mu} = q_{\mu}. \end{aligned} \tag{7}$$

Note that these basic vectors are unnormalized. Moreover, while $b^{(1)}_{\mu}$ and $b^{(2)}_{\mu}$ are lying the Euclidean and pseudo-Euclidean subspaces, respectively, $b^{(3)}_{\mu} = q^2_{\perp}q_{\parallel\mu} - q^2_{\parallel}q_{\perp\mu}$ and $b^{(4)}_{\mu} = q_{\parallel\mu} - q_{\perp\mu}$ have non-trivial projections on both subspaces.

In this basis, an arbitrary vector a_{μ} can be written as follows:

$$a_{\mu} = \sum_{i=1}^{4} a_i\,\frac{b^{(i)}_{\mu}}{(b^{(i)}b^{(i)})}, \qquad a_i = a^{\mu}b^{(i)}_{\mu}. \tag{8}$$

The third-rank tensor, $T_{\mu\nu\rho}$, can be decomposed similarly as the direct product of three basic vectors:

$$T_{\mu\nu\rho} = \sum_{i,j,k=1}^{4} T_{ijk} \frac{b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}}{(b^{(i)} b^{(i)})\,(b^{(j)} b^{(j)})\,(b^{(k)} b^{(k)})}, \qquad T_{ijk} = T^{\mu\nu\rho} b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}. \tag{9}$$

The fourth-rank tensor, $T_{\mu\nu\rho\sigma}$, being also of our interest, is the direct product of four $b_\mu^{(i)}$.

Let us come back to the photon polarization tensor and assume that it is calculated in the magnetic field. Being the second-rank symmetric tensor, it has the following decomposition in the basis (7):

$$\mathcal{P}_{\mu\nu}(q) = \sum_{\lambda=1}^{3} \frac{b_\mu^{(\lambda)} b_\nu^{(\lambda)}}{(b^{(\lambda)})^2}\, \Pi^{(\lambda)}(q), \tag{10}$$

where $\Pi^{(\lambda)}(q)$ with $\lambda = 1,\ 2,\ 3$ are eigenvalues of the photon polarization operator:

$$\mathcal{P}_{\mu\nu}(q)\varepsilon^{(\lambda)\nu}(q) = \Pi^{(\lambda)}(q)\varepsilon_\mu^{(\lambda)}(q), \tag{11}$$

and $\varepsilon_\mu^{(\lambda)}(q)$ are the corresponding eigenvectors.

In the vacuum, $\mathcal{P}_{\mu\nu}(q)$ has two physical eigenmodes [5, 6]. In an external constant homogeneous magnetic field, their number remains the same and two eigenvectors are determined by the field strength tensor as follows [8, 9]:

$$\varepsilon_\mu^{(1)}(q) = b_\mu^{(1)}/\sqrt{q_\perp^2}, \qquad \varepsilon_\mu^{(2)}(q) = b_\mu^{(2)}/\sqrt{q_\parallel^2}. \tag{12}$$

In the magnetic field, $\Pi^{(\lambda)}(q)$ contains both vacuum, $\mathcal{P}(q^2)$, and field-induced parts. In particular, for the electron contribution it can be written in the form [8, 9]:

$$\Pi^{(\lambda)}(q) = -i\,\mathcal{P}(q^2) - \frac{\alpha}{\pi}\, Y_{VV}^{(\lambda)}, \tag{13}$$

where α is the fine-structure constant. Details on the field-induced term, $Y_{VV}^{(\lambda)}$, can be found, for example, in [8, 9].

5. Inclusion of Fermion AMM

Models beyond the SM can give a number of effective operators at energies accessible at present or future colliders and the Pauli Lagrangian density, in particular:

$$\mathcal{L}_{\mathrm{AMM}}(x) = \frac{1}{2}\, Q_f\, \mu_f\, a'_f\, \left[\bar{f}(x)\sigma_{\mu\nu} f(x)\right] F^{\mu\nu}(x), \tag{14}$$

where $\mu_f = e/(2m_f)$ is the magneton associated with a fermion with the mass m_f, a'_f is an additional contribution to the fermion anomalous magnetic moment induced by "New Physics", $\sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2$, and $F^{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the electromagnetic field strength tensor. For the electron with the charge $Q_e = -1$ the coupling in (14) includes the Bohr magneton, $\mu_B = e/(2m_e)$, accompined by the New Physics contribution, a'_e, in the electron AMM. The total interaction Lagrangian is as follows:

$$\mathcal{L}_{\mathrm{int}}(x) = \mathcal{L}_{\mathrm{QED}}(x) + \mathcal{L}_{\mathrm{AMM}}(x). \tag{15}$$

One should remember that such a theory is non-renormalizable but, being an effective theory, it works up to a scale at which it matches to some SM renormalizable extension with a larger gauge symmetry group. The presence of $\mathcal{L}_{\text{AMM}}(x)$ in (15) gives an additional contribution to the photon polarization operator. The contribution linear in AMM is related with the non-diagonal correlator of vector and tensor currents, $\Pi^{(\text{VT})}_{\mu\nu\rho}$, while the contribution quadratic in AMM is determined by the diagonal correlator of two tensor currents, $\Pi^{(\text{TT})}_{\mu\nu\rho\sigma}$. It should be noted that one can expect an appearance of quadratic divergence in the field-free part of the tensor-tensor correlator but, as explicit calculations show, there is a logarithmic divergence only, similar to the field-free part of vector-vector and vector-tensor correlators.

6. General Case of Two-Point Correlator

In this section we show the formalism used in calculations. Let us start from the generalized Lagrangian density of the local fermion interaction:

$$\mathcal{L}_{\text{int}}(x) = \left[\bar{f}(x)\Gamma^{A} f(x)\right] J_{A}(x), \tag{16}$$

where J_A is the generalized current which can be a photon, neutrino current, etc., Γ_A is a matrix from the complete set $\{1,\ \gamma_5,\ \gamma_\mu,\ \gamma_\mu\gamma_5, \sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2\}$ [6]. For simplicity, interaction constants are absorbed into the current J_A. With the Lagrangian (16) used, the two-point correlation function of a general form in the momentum space has the form [8, 9]:

$$\Pi_{AB}(q) = \int d^4X\, \mathrm{e}^{-i(qX)}\, \mathrm{Sp}\left\{S_{\mathrm{F}}(-X)\,\Gamma_A\, S_{\mathrm{F}}(X)\,\Gamma_B\right\}, \tag{17}$$

where $X^\mu = x^\mu - y^\mu$ is the integration variable and $S_{\mathrm{F}}(X)$ is the gauge and translationally invariant part of the fermion propagator in an external electromagnetic field. The corresponding Feynman diagram is shown in Fig. 2. Both diagonal and non-diagonal correlations of scalar, pseudoscalar, vector and axial-vector currents were studied previously [10]. Here, we consider correlations of the tensor fermionic current with the vector and tensor ones. Correlation functions of the tensor current with the scalar, pseudoscalar and axial-vector currents can be found in [11–15].

For the correlator representation, we choose a two-fold integral form. Such a form for correlators follows naturally after the Fock-Schwinger proper-time representation of fermion propagators is used. In the constant homogeneous electromagnetic field, the general form of the propagator was obtained in [16] and is as follows:

$$G_{\mathrm{F}}(x, y) = \mathrm{e}^{i\Omega(x,y)}\, S_{\mathrm{F}}(x - y). \tag{18}$$

It is written in a way that the gauge and translationally invariant, $S_{\mathrm{F}}(x - y)$, and non-invariant, $\mathrm{e}^{i\Omega(x,y)}$, parts are factorized. To see non-invariance, let us write explicit expression for the phase:

$$\Omega(x, y) = -eQ_f \int_y^x d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi - y)^\nu\right], \tag{19}$$

which is dependent explicitly on the four-potential, $A_\mu(\xi)$. In the two-point correlation function phase factors from two propagators cancel each other: $\Omega(x, y) + \Omega(y, x) = 0$, and this correlation

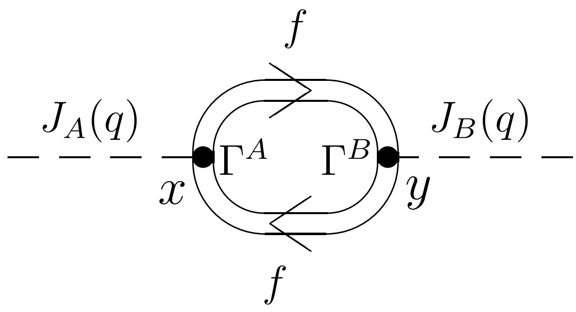

Figure 2: The Feynman diagram describing the generalized two-point correlator of two fermionic currents. Double lines indicate that effects of an external electromagnetic field are taken into account exactly in the fermion propagators.

function becomes gauge and translationally invariant. To perform calculations, the gauge and translationally invariant part of the fermion propagator needs to be known which has the following proper-time representation in the external constant homogeneous magnetic field [8, 9]:

$$\begin{aligned} S_{\mathrm{F}}(X) &= -\frac{i\beta}{2(4\pi)^2}\int\limits_0^\infty \frac{ds}{s^2}\,\exp\left\{-i\left[m_f^2\,s+\frac{1}{4s}\,(X\widetilde{\Lambda}X)-\frac{\beta\cot(\beta s)}{4}\,(X\Lambda X)\right]\right\} \qquad (20)\\ &\times\left\{(X\widetilde{\Lambda}\gamma)\cot(\beta s)-i(X\widetilde{\varphi}\gamma)\gamma_5-\frac{\beta s}{\sin^2(\beta s)}\,(X\Lambda\gamma)+m_f\,s\,\left[2\cot(\beta s)+(\gamma\varphi\gamma)\right]\right\}. \end{aligned}$$

where m_f is the fermion mass, $\mathbf{B}$ is the constant field strength, and $\beta = e|\mathbf{B}|Q_f$.

7. Correlator of Vector and Tensor Currents

The correlator of the vector and tensor fermionic currents is the rank-3 tensor:

$$\Pi^{(\mathrm{VT})}_{\mu\nu\rho}(q) = \int d^4X\,\mathrm{e}^{-i(qX)}\,\mathrm{Sp}\left\{S_{\mathrm{F}}(-X)\,\gamma_\mu\,S_{\mathrm{F}}(X)\,\sigma_{\nu\rho}\right\}, \qquad (21)$$

The vector-current conservation and antisymmetry of the tensor current reduce the number of independent components to 18. In the basis decomposition (9) of $\Pi^{(\mathrm{VT})}_{\mu\nu\rho}(q)$, four coefficients only are non-trivial. For them, it is convenient to use the following two-fold integral representation:

$$\begin{aligned} \Pi^{(\mathrm{VT})}_{ijk}(q^2,q_\perp^2,\beta) &= \frac{1}{4\pi^2}\int\limits_0^\infty\frac{dt}{t}\int\limits_0^1 du\,Y^{(\mathrm{VT})}_{ijk}(q^2,q_\perp^2,\beta;t,u) \qquad (22)\\ &\times\exp\left\{-i\left[m_f^2\,t-\frac{q_\parallel^2}{4}\,t\,(1-u^2)+q_\perp^2\,\frac{\cos(\beta tu)-\cos(\beta t)}{2\beta\sin(\beta t)}\right]\right\}, \end{aligned}$$

where $t = s_1 + s_2$ and $u = (s_1 - s_2)/(s_1 + s_2)$ are the integration variables expressed in terms of proper times, s_1 and s_2, entering two propagators and one keeps in mind the relation $q_\parallel^2 = q^2 + q_\perp^2$

between momenta squared. Non-trivial integrands in (22) are as follows:

$$Y^{(\mathrm{VT})}_{114}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{141}(q^2, q_\perp^2, \beta; t, u) = -m_f\, q_\perp^2\, q^2\, \frac{\beta t \cos(\beta t u)}{\sin(\beta t)}, \tag{23}$$

$$Y^{(\mathrm{VT})}_{223}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{232}(q^2, q_\perp^2, \beta; t, u) = m_f\, q_\perp^2\, (q_\parallel^2)^2\, \frac{\beta t}{\sin(\beta t)}\, [\cos(\beta t) - \cos(\beta t u)], \tag{24}$$

$$Y^{(\mathrm{VT})}_{224}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{242}(q^2, q_\perp^2, \beta; t, u) = m_f\, q_\parallel^2\, \frac{\beta t}{\sin(\beta t)}\, \left[q_\perp^2\, \cos(\beta t) - q_\parallel^2\, \cos(\beta t u)\right], \tag{25}$$

$$Y^{(\mathrm{VT})}_{334}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{343}(q^2, q_\perp^2, \beta; t, u) = -m_f\, q_\perp^2\, q_\parallel^2\, (q^2)^2\, \frac{\beta t \cos(\beta t u)}{\sin(\beta t)}. \tag{26}$$

The antisymmetry in the last two indices is due to the antisymmetric tensor current. The choice of basic vectors is optimal here because of the vector-current conservation — all the coefficients $Y^{(\mathrm{VT})}_{4jk}$ vanish in this basis.

8. Correlator of Two Tensor Currents

The correlator of two tensor fermionic currents is the rank-4 tensor:

$$\Pi^{(\mathrm{TT})}_{\mu\nu\rho\tau}(q) = \int d^4X\, \mathrm{e}^{-i(qX)}\, \mathrm{Sp}\left\{S_{\mathrm{F}}(-X)\, \sigma_{\mu\nu}\, S_{\mathrm{F}}(X)\, \sigma_{\rho\tau}\right\}, \tag{27}$$

The antisymmetry of the tensor currents leave 36 independent components in this correlator and only eight of them are non-vanishing. One can utilize again the two-fold integral representation of the coefficients, $\Pi^{(\mathrm{TT})}_{ijkl}(q^2, q_\perp^2, \beta)$, in the basis decomposition of the tensor (27):

$$\Pi^{(\mathrm{TT})}_{ijkl}(q^2, q_\perp^2, \beta) = \frac{1}{4\pi^2} \int_0^\infty \frac{dt}{t} \int_0^1 du\, Y^{(\mathrm{TT})}_{ijkl}(q^2, q_\perp^2, \beta; t, u) \tag{28}$$
$$\times \exp\left\{-i\left[m_f^2\, t - \frac{q_\parallel^2}{4}\, t\,(1-u^2) + q_\perp^2\, \frac{\cos(\beta t u) - \cos(\beta t)}{2\beta \sin(\beta t)}\right]\right\}.$$

The integrands entering the photon polarization operator are as follows:

$$\begin{aligned} Y^{(\mathrm{TT})}_{1414}(q_\parallel^2, q_\perp^2, \beta; t, u) = -q_\perp^2 \Bigg\{ & 2q_\perp^2\, (q_\perp^2 + q_\parallel^2) \frac{\cos(\beta t) - \cos(\beta t u)}{\sin^2(\beta t)} \\ & + 4 q_\perp^2 q_\parallel^2\, [\cos(\beta t u) - u \sin(\beta t u) \cot(\beta t)] - q_\parallel^2 \left[\left(1-u^2\right) q_\parallel^2 + 4m_f^2\right] \cos(\beta t u) \\ & - q_\perp^2 \left[\left(1-u^2\right) q_\parallel^2 - 4m_f^2\right] \cos(\beta t) + \frac{4i}{t} q_\parallel^2 \left[\cos(\beta t) - \frac{\beta t}{\sin(\beta t)}\right] \Bigg\}, \end{aligned} \tag{29}$$

$$\begin{aligned} Y^{(\mathrm{TT})}_{2424}(q_\parallel^2, q_\perp^2, \beta; t, u) = -q_\parallel^2 \Bigg\{ & 2q_\perp^2\, (q_\perp^2 + q_\parallel^2) \frac{\cos(\beta t) - \cos(\beta t u)}{\sin^2(\beta t)} \\ & + 4 q_\perp^2 q_\parallel^2\, [\cos(\beta t u) - u \sin(\beta t u) \cot(\beta t)] - q_\parallel^2 \left[\left(1-u^2\right) q_\parallel^2 + 4m_f^2\right] \cos(\beta t u) \\ & - q_\perp^2 \left[\left(1-u^2\right) q_\parallel^2 - 4m_f^2\right] \cos(\beta t) + \frac{4i}{t} q_\parallel^2 \left[\cos(\beta t) - \frac{\beta t}{\sin(\beta t)}\right] \Bigg\}. \end{aligned} \tag{30}$$

The other non-vanishing coefficients can be found in a forthcoming paper [17].

9. AMM Contribution to Photon Polarization Operator

After the Pauli term is added to the spinor QED Lagrangian, the field-induced part of $\Pi^{(\lambda)}(q)$ gets an additional contribution containing AMM, a'_f. If we consider the electron as an example, its contribution to the photon polarization operator can be written as the following sum:

$$\Pi^{(\lambda)}(q) = -i\,\mathcal{P}(q^2) - \frac{\alpha}{\pi}\,Y^{(\lambda)}_{VV} + 2\,\frac{\alpha}{\pi}\,a'_e\,Y^{(\lambda)}_{VT} + \frac{\alpha}{\pi}\,a'^2_e\,Y^{(\lambda)}_{TT}. \tag{31}$$

The last two terms can be presented in the form of the two-fold integrals:

$$Y^{(\lambda)}_{VT} = \int_0^\infty \frac{dt}{t}\int_0^1 du\left\{\frac{\beta t}{\sin(\beta t)}\,y^{(\lambda)}_{VT}\,e^{-i\Phi} - q^2\,e^{-i\Phi_0}\right\}, \tag{32}$$

$$Y^{(\lambda)}_{TT} = \int_0^\infty \frac{dt}{t}\int_0^1 du\left\{\frac{\beta t}{\sin(\beta t)}\,y^{(\lambda)}_{TT}\,e^{-i\Phi} - q^2\left[1+\frac{q^2}{4m_f^2}\left(1-u^2\right)\right]e^{-i\Phi_0}\right\}, \tag{33}$$

where notations are borrowed from [8, 9]. The part independent on the field, $\mathcal{P}(q^2)$, is subtracted. Integrands entering the vector-tensor part, $Y^{(\lambda)}_{VT}$, are as follows:

$$y^{(1)}_{VT} = y^{(3)}_{VT} = q^2\cos(\beta t u), \qquad y^{(2)}_{VT} = q^2_\parallel\cos(\beta t u) - q^2_\perp\cos(\beta t). \tag{34}$$

For the electron, the standard AMM $a_e = \alpha/(2\pi) + \cdots$ [5, 6] which is in good agreement with experimental data [7]. Nevertheless, a very small ”New Physics” contribution, a'_e, is still allowed but its possible manifestation in physical experiments is not obvious.

Integrands in the tensor-tensor part, $Y^{(\lambda)}_{TT}$, are more lengthy and can be expressed through the integrands entering the tensor-tensor coefficients (29) and (30):

$$y^{(1)}_{TT} = \frac{\sin(\beta t)}{\beta t}\,\frac{Y^{(TT)}_{1414}}{4m_e^2\,q^2_\perp}, \qquad y^{(2)}_{TT} = \frac{\sin(\beta t)}{\beta t}\,\frac{Y^{(TT)}_{2424}}{4m_e^2\,q^2_\parallel}. \tag{35}$$

For the electron, the tensor-tensor contribution, $Y^{(\lambda)}_{TT}$, gives an α-suppressed correction to $Y^{(\lambda)}_{VT}$ and its effect is numerically much smaller.

10. Conclusions

Two-point correlators of the tensor fermionic current with the vector and tensor ones are calculated in the background filled by the constant homogeneous magnetic field. Study of correlators of the tensor fermionic current with the others allows to investigate effects of the fermion anomalous magnetic moment in the one-loop approximation. Field-induced contributions to the photon polarization operator linear and quadratic in the electron anomalous magnetic moment are calculated and shown to be suppressed. Computer technique developed for two-point correlators is assumed to be applied for calculations of three-point ones.

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