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AdS/CFT, Wilson loops and M2-branes

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We discuss testing AdS/CFT correspondence between $U(N)_k \times U(N)_{-k}$ Chern-Simons-matter 3d gauge theory and M-theory in $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$ background. We show that the quantum M2 brane partition function expanded near the corresponding classical solution matches the localization predictions on the gauge theory side in the case of BPS Wilson loop expectation value and instanton corrections to free energy.

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1. Introduction

While the critical first-quantized string is described by an effectively Gaussian path integral, this is not so for the membrane which has a highly non-linear 3d action

$$S = -T_2 \int d^3\sigma \sqrt{-\det\gamma_{\alpha\beta}}\,, \qquad \gamma_{\alpha\beta} = \eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu \ . \tag{1.1}$$

The theory is formally non-renormalizable, and while UV finite at 1-loop order logarithmic divergences proportional to powers of curvature should be expected to appear at higher loops. While the existence of a consistent quantum theory of bosonic membranes may be in doubt, it may happen to be well defined for the 11d supermembrane or M2 brane theory [1–3]. The large amount of supersymmetry and possibly some unknown hidden symmetries may lead to its UV finiteness despite formal power-counting nonrenormalizability.

This may be true, in particular, for the supermembrane in the maximally supersymmetric $AdS_4 \times S^7$ or $AdS_7 \times S^4$ backgrounds [4–6]. M-theory in the orbifold $AdS_4 \times S^4/\mathbb{Z}_k$ background should be dual to the $\mathcal{N} = 6$ supersymmetric 3d $U_k(N) \times U_{-k}(N)$ Chern-Simons-matter theory [7, 8] (ABJM theory).

Recent work [9, 10] provided a remarkable evidence that direct semiclassical quantization of the M2 brane in $AdS_4 \times S^7/\mathbb{Z}_k$ background reproduces the results of large N localization computations [11–13] of the $\frac{1}{2}$-BPS Wilson loop and instanton contributions to free energy in the ABJM gauge theory.

Expanded near a classical solution with non-degenerate induced 3d metric M2 brane action can be quantized in a static gauge. Then the leading 1-loop result for its partition function is UV finite and thus unambiguous [2, 9, 10, 14–16]. As the $1/N$ expansion of the localization results on the gauge theory side have the form of an expansion in the inverse of the effective M2 brane tension $\mathrm{T}_2 = \frac{\sqrt{2k}}{\pi}\sqrt{N}$, this suggests that matching with the 1-loop M2 brane computations [9, 10] should, in fact, extend also to 2-loop and higher orders.

This requires the corresponding quantum M2 brane theory to be UV finite at higher loops despite its apparent non-renormalizability. One may hope that this may happen due to high degree of underlying supersymmetry and hidden symmetries of the M2 brane theory in $AdS_4 \times S^7$ or $AdS_7 \times S^4$ backgrounds that remain to be uncovered.

The M2 brane action in 11d background is related to the type IIA string in the corresponding 10d background by a double dimensional reduction [17, 18]. Considering M2 brane world volume of topology $\Sigma^2 \times S^1$ and expanding 3d fields in Fourier modes in S^1 coordinate one gets an effective 2d string action on Σ^2 coupled to an infinite tower of massive 2d fields. Choosing a static gauge in the M2 brane action one gets a static gauge Nambu-Goto action for the massless transverse string modes coupled to a tower of the massive "Kaluza-Klein" 2d modes. This "effective string" 2d action is essentially equivalent to the original M2 brane action and thus may inherit some of its hidden symmetries. Examples of such semiclasical computations will be provided below.

Let us first recall some basic M-theory relations. The action of the 11d supergravity is

$$S_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\,\sqrt{-G}\,\Big(R - \frac{1}{2\cdot 4!}F_{mnk\ell}F^{mnk\ell} + \cdots\Big), \qquad 2\kappa_{11}^2 = (2\pi)^8\,\ell_P^9\,, \tag{1.2}$$

The M2-brane tension is

$$T_2 = \frac{1}{(2\pi)^2\, \ell_P^3}\ . \tag{1.3}$$

The M2 brane wrapped on the x^{11} circle gives the fundamental string action with the standard tension

$$2\pi\, \mathrm{R}_{11}\, T_2 = T_1\,, \qquad\qquad T_1 = \frac{1}{2\pi\alpha'}\ . \tag{1.4}$$

If we specialize to the $\mathrm{AdS}_4 \times S^7$ space supported by the 4-form flux with $\hat{N}$ units of charge (which is the near-horizon limit of the background sourced by multiple M2-branes then

$$ds^2_{11} = L^2_{11}\Big(\frac{1}{4}ds^2_{\mathrm{AdS}_4} + ds^2_{S^7}\Big), \qquad ds^2_{\mathrm{AdS}_4} = dr^2 + \sinh^2 r\, d\Omega_3^2, \qquad F_4 = dC_3 \sim \hat{N}\,\epsilon_4. \tag{1.5}$$

Considering $\mathbb{Z}_k$ quotient of S^7 we get [7]

$$ds^2_{S^7/\mathbb{Z}_k} = ds^2_{\mathrm{CP}^3} + \frac{1}{k^2}(d\varphi + kA)^2\,, \qquad \varphi \equiv \varphi + 2\pi \tag{1.6}$$

$$ds^2_{\mathrm{CP}^3} = \frac{dw^s d\bar{w}^s}{1+|w|^2} - \frac{w_r \bar{w}_s}{(1+|w|^2)^2} dw^s d\bar{w}^r, \qquad dA = i\Big[\frac{\delta_{sr}}{1+|w|^2} - \frac{w_s\bar{w}_r}{(1+|w|^2)^2}\Big] dw^r \wedge d\bar{w}^s,$$

and thus

$$R_{11} = g_s^{2/3}\, \mathrm{R}_{11} = \frac{L_{11}}{k}\,, \qquad \hat{N} = Nk\,, \qquad \frac{L_{11}}{\ell_P} = \big(2^5\pi^2 Nk\big)^{1/6}\ . \tag{1.7}$$

Upon dimensional reduction we get the metric and parameters of 10d string theory

$$ds^2_{10} = L^2\Big(\frac{1}{4}ds^2_{\mathrm{AdS}_4} + ds^2_{\mathrm{CP}^3}\Big), \qquad L_{\mathrm{AdS}_4} = \frac{1}{2}L\,, \tag{1.8}$$

$$L = g_s^{1/3}\, L_{11}\,, \qquad\qquad g_s = \big(\frac{L_{11}}{k\,\ell_P}\big)^{3/2}\ . \tag{1.9}$$

Expressed in terms of the dual gauge-theory parameters N and k the string coupling and the effective dimensionless string tension are

$$g_s \equiv \sqrt{\pi}\,\big(\frac{2}{k}\big)^{5/4} N^{1/4} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \qquad\qquad \lambda = \frac{N}{k}\,, \tag{1.10}$$

$$T \equiv L^2_{\mathrm{AdS}_4} T_1 = \frac{L^2}{8\pi\alpha'} = g_s^{2/3}\frac{L^2_{11}}{8\pi\alpha'} = \frac{\sqrt{\lambda}}{\sqrt{2}}\,, \tag{1.11}$$

$$\frac{g_s^2}{8\pi T} = \frac{\lambda^2}{N^2} = \frac{1}{k^2}\ . \tag{1.12}$$

The M-theory perturbative expansion corresponds to large curvature scale or large effective M2 brane tension for fixed parameter k of the background

$$\mathrm{L} \equiv \frac{L_{11}}{\ell_P} \gg 1\,, \qquad \mathrm{T}_2 \equiv T_2 L^3_{11} \gg 1\,, \qquad k = \text{fixed}\,, \tag{1.13}$$

i.e. to the large N limit with fixed k. The 10d string perturbative expansion corresponds to $g_s \ll 1$ for fixed T, i.e. to the the 't Hooft expansion in the large N, large k limit with fixed $\lambda = \frac{N}{k}$.

2. Expectation value of $\frac{1}{2}$-BPS Wilson loop and free energy in ABJM theory from localization

The $\mathrm{AdS}_4/\mathrm{CFT}_3$ duality between the $U(N)_k \times U(N)_{-k}$ ABJM theory [7] and M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ provides a remarkable opportunity to shed light on the properties of quantum M2 brane theory by testing its predictions against exact results in 3d superconformal gauge theory.

In the large N limit with k fixed, the holographic dual of a Wilson loop in the fundamental representation is expected to be an M2 brane wrapping the M-theory circle direction. This limit is different from the standard large N 't Hooft limit, where N and k are taken to be large with $\lambda = N/k$ fixed, and in which Wilson loops are described by fundamental strings in type IIA string theory in $\mathrm{AdS}_4 \times \mathrm{CP}^3$.

Our first example will be the $\frac{1}{2}$-BPS Wilson loop. For fixed k, the large N expansion of the Wilson loop operator in the ABJM theory corresponds to the expansion in the large effective M2 brane tension $R^3 T_2 \sim \sqrt{Nk}$, where R is the curvature radius of $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ and $T_2 = \frac{1}{(2\pi)^2 \ell_P^3}$. The analytic expression for the expectation value of the $\frac{1}{2}$-BPS circular Wilson loop (WL) in the ABJM theory derived using supersymmetric localization in [13]

$$\langle W_{\frac{1}{2}} \rangle = \frac{1}{2\,\sin(\frac{2\pi}{k})} \frac{\mathrm{Ai}\left[C^{-\frac{1}{3}}\left(N - \frac{k}{24} - \frac{7}{3k}\right)\right]}{\mathrm{Ai}\left[C^{-\frac{1}{3}}\left(N - \frac{k}{24} - \frac{1}{3k}\right)\right]} \,. \tag{2.1}$$

Here $\mathrm{Ai}(z)$ is the Airy function, and $C = 2/(\pi^2 k)$. This expression resums all of the perturbative $1/N$ corrections at fixed k (we ignore instanton corrections, see below).

In order to compare to the semiclassical expansion in the M2 brane world-volume theory, one is to expand (2.1) at large N with fixed k, which gives

$$\langle W_{\frac{1}{2}} \rangle = \frac{1}{2\sin(\frac{2\pi}{k})} e^{\pi\sqrt{\frac{2N}{k}}} \left[1 - \frac{\pi\left(k^2+32\right)}{24\sqrt{2}\,k^{3/2}} \frac{1}{\sqrt{N}} + O(\frac{1}{N})\right] . \tag{2.2}$$

The WL has a dual description in terms of an M2 brane wrapped on $\mathrm{AdS}_2 \times S^1$ [22] in the M-theory background $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. The exponential factor in (2.2) is reproduced by the classical action of the M2 brane with $\mathrm{AdS}_2 \times S^1$ world-volume, while the k-dependent prefactor $(2\sin\frac{2\pi}{k})^{-1}$ is matched precisely by the 1-loop correction coming from the functional determinants of the quantum fluctuations around this M2 brane solution [9].

Next, let us review the localization result for the partition function $Z(N,k)$ of the $U(N)_k \times U(N)_{-k}$ ABJM theory on S^3. We shall assume that $k > 2$. As a function of N the partition function can be represented as a sum of a perturbative part (given by a series in $\frac{1}{\sqrt{N}}$) and a non-perturbative part involving factors like $e^{-h(k)\sqrt{N}}$ that are exponentially suppressed at large N, i.e.

$$Z = Z^{\mathrm{p}}(N,k) + Z^{\mathrm{np}}(N,k)\,, \tag{2.3}$$

$$Z^{\mathrm{p}}(N,k) = C(k)^{-\frac{1}{3}}\, e^{A(k)}\, \mathrm{Ai}\Big[C(k)^{-\frac{1}{3}}\big(N - B(k)\big)\Big]\,, \qquad C(k) = \frac{2}{\pi^2 k}, \quad B(k) = \frac{k}{24} + \frac{1}{3k} \tag{2.4}$$

$$A(k) = -\frac{\zeta(3)}{8\pi^2}\left(k^2 - \frac{16}{k}\right) + \frac{k^2}{\pi^2}\int_0^\infty dx\, \frac{x}{e^{kx}-1}\,\log(1-e^{-2x}), \tag{2.5}$$

Then the free energy is $F \equiv -\log Z = F^{\rm p} + F^{\rm np}$ where the large N expansion of the perturbative part follows from (2.4) (see [27] for details)

$$F^{\rm p} = -\log Z^{\rm p} = \tfrac{1}{3}\sqrt{2}\pi k^{1/2} N^{3/2} - \tfrac{\pi}{24\sqrt{2}}\big(k^2+8\big)\,k^{-1/2}\,N^{1/2} + \tfrac{1}{4}\log\tfrac{32N}{k} - A(k) + O(N^{-1/2})\,, \quad (2.6)$$

and the non-perturbative part is [12, 28]

$$F^{\rm np} = \sum_{n_{\rm I},n_{\rm II}=0}^{\infty} f_{n_{\rm I},n_{\rm II}}(N,k)\,\exp\Big[-2\pi\sqrt{N}\,\Big(n_{\rm I}\sqrt{\frac{2}{k}} + n_{\rm II}\sqrt{\frac{k}{2}}\Big)\Big]\,. \quad (2.7)$$

In the type IIA string theory regime (i.e. for large N and k with $\lambda = \frac{N}{k}$=fixed) these may be interpreted as the contributions of the string world-sheet instantons (wrapping CP^1 in CP^3 [19]) and of the D2-brane instantons (wrapping a 3-cycle $\mathbb{RP}^3 = S^3/\mathbb{Z}_2$ in CP^3) respectively [12]. In the M-theory regime (i.e. for large N with fixed k), the world-sheet instantons correspond to the M2 brane instantons wrapping the 11d circle and a CP^1 in CP^3, i.e. $S^3/\mathbb{Z}_k \subset S^7/\mathbb{Z}_k$, while the D2 instantons correspond to the M2 instantons wrapping the $\mathbb{RP}^3$ 3-cycle in the CP^3 part of $S^7/\mathbb{Z}_k$.

The corresponding term in the non-perturbative part of free energy is then (cf. (2.4),(2.7)) $F^{\rm np}(N,k) = F^{\rm inst}(N,k) + \cdots$

$$\begin{aligned} F^{\rm inst}(N,k) &= -d_1(k)\frac{\mathrm{Ai}[C(k)^{-\frac{1}{3}}(N - B(k) + \frac{4}{k})]}{\mathrm{Ai}[C(k)^{-\frac{1}{3}}(N - B(k))]} \\ &= F_1^{\rm inst}(N,k)\Big[1 + \tfrac{\pi}{\sqrt{2k}}\tfrac{k^2-40}{12k}\tfrac{1}{\sqrt{N}} + \ldots\Big]\,, \end{aligned} \quad (2.8)$$

$$F_1^{\rm inst}(N,k) = -d_1(k)\,e^{-2\pi\sqrt{N}\sqrt{\frac{2}{k}}} = -\frac{1}{\sin^2(\frac{2\pi}{k})}\,e^{-2\pi\sqrt{\frac{2N}{k}}}\,. \quad (2.9)$$

Here $F_1^{\rm inst}$ is the leading large N term in the 1-instanton contribution.

Below we will discuss how to reproduce (2.9) on the dual M-theory side by a quantum M2 brane computation following [10]. There is a close analogy to how that was done in [9] for the leading term in the Wilson loop expression in (2.2). In the instanton prefactor in the localization result for the leading large N non-perturbative contribution to the ABJM free energy $F^{\rm inst}(N,k) = -\frac{1}{\sin^2(\frac{2\pi}{k})}\,e^{-2\pi\sqrt{\frac{2N}{k}}} + \ldots$ the exponent comes from the action of an M2 brane instanton with $S^3/\mathbb{Z}_k$ world-volume geometry. The M2 instanton wraps the 11d circle S^1 and a CP^1 in CP^3, and it represents the M-theory uplift of the CP^1 instanton in type IIA string theory on $\mathrm{AdS}_4\times\mathrm{CP}^3$ [19].

3. Wilson loop expectation value from M2-brane path integral

The world-volume action for a probe M2 brane in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ background background is given by [1, 6]

$$S_{\rm M2} = T_2\int d^3\sigma\sqrt{-\det g} + T_2\int C_3 \ + \ \text{fermionic terms}\,, \qquad T_2 = \frac{1}{(2\pi)^2}\frac{1}{\ell_P^3}\,. \quad (3.1)$$

The action (3.1) admits a simple classical solution given by the M2 brane wrapping the M-theory circle direction and occupying the AdS_2 subspace of AdS_4 spanned by the coordinates t, z. The resulting membrane has the $AdS_2 \times S^1$ world-volume geometry and is dual to the $\frac{1}{2}$-BPS Wilson loop along the t direction at the boundary of AdS_4. The value of the classical action (3.1) for this $AdS_2 \times S^1$ solution is (that there is no contribution from the Wess-Zumino term involving the 3-form field in (3.1))

$$S^{\rm cl.}_{\rm M2} = T_2 R^3 \frac{1}{4} {\rm vol}({\rm AdS}_2) \frac{2\pi}{k} = -\pi \sqrt{\frac{2N}{k}}\,. \qquad (3.2)$$

Thus $e^{-S^{\rm cl.}_{\rm M2}}$ precisely matches the exponential in the localization prediction (2.2).

Next, let us compute the 1-loop correction to the M2 brane partition function due to the quantum fluctuations about this classical solution. Starting with the action (3.1) one may expand it near a classical solution to quadratic order fixing a 3d reparametrization and κ-symmetry gauge to get an action for 8+8 physical 3d fluctuation fields. The resulting spectrum of the quantum fluctuations around the above $AdS_2 \times S^1$ solution was obtained in ref. [22]. It is natural to chose a static gauge identifying two membrane coordinates σ_1, σ_2 in (3.1) with the AdS_2 directions and the third σ_3 with the S^1 angle φ. After a Kaluza-Klein (Fourier) expansion of the 3d fields in the periodic coordinate σ_3, one obtains a tower of bosonic and fermionic fluctuations that can be viewed as 2d fields propagating on the AdS_2 background. Thus one gets an equivalent 2d theory with an infinite number of fields.

The bosonic fluctuations in the two transverse directions within AdS_4 give a tower of complex scalar fields η_n (two real scalars for each n) with masses $m^2_{\eta_n} = \frac{1}{4}(kn-2)(kn-4)$, $\quad n \in \mathbb{Z}$, while from the fluctuations in the six CP^3 directions one finds a tower of 3 complex fields ζ^s_n $(s = 1, 2, 3)$ with masses $m^2_{\zeta^s_n} = \frac{1}{4}kn(kn+2)$. For the fermionic fluctuations, the KK reduction leads to a tower of eight two-component spinors ϑ^A_n $(A = 1, \dots, 8)$ for each value of the KK mode number n, with masses given by $m_{\vartheta^a_n} = \frac{kn}{2} \pm 1$, $\ m_{\vartheta^i_n} = \frac{kn}{2}$ For $n = 0$, this spectrum coincides with the spectrum of bosonic and fermionic fluctuations around the corresponding AdS_2 string solution in the type IIA superstring theory on $AdS_4 \times CP^3$ [21, 29]: we get 2 scalars of $m^2 = 2$, 6 scalars of $m^2 = 0$, 3+3 fermions of $m = \pm 1$ and 2 fermions of $m = 0$.

Using the above spectrum, we can derive the 1-loop correction to the partition function of the M2 brane theory expanded around the Euclidean $AdS_2 \times S^1$ solution with circular boundary

$$Z_{\rm M2} = Z_1 e^{-S^{\rm cl.}_{\rm M2}} \Big[1 + O\Big(\frac{1}{R^3 T_2}\Big)\Big]\,, \qquad (3.3)$$

where the 1-loop term Z_1 is the ratio of the determinants of the fluctuation operators

$$Z_1 = \prod_{n\in\mathbb{Z}} \frac{\big[\det(-\nabla^2 + \frac{R^{(2)}}{4} + (\frac{kn}{2}+1)^2)\big]^{\frac{3}{2}} \big[\det(-\nabla^2 + \frac{R^{(2)}}{4} + (\frac{kn}{2}-1)^2)\big]^{\frac{3}{2}} \det(-\nabla^2 + \frac{R^{(2)}}{4} + (\frac{kn}{2})^2)}{\det\big(-\nabla^2 + \frac{1}{4}(kn-2)(kn-4)\big)\big[\det\big(-\nabla^2 + \frac{1}{4}kn(kn+2)\big)\big]^3} \qquad (3.4)$$

Here $R^{(2)} = -2$ is the curvature of AdS_2. The $n = 0$ factor in (3.4) is the same as the 1-loop partition function [21, 29] for the fluctuations near the corresponding type IIA AdS_2 string worldsheet ending on a circle at the boundary of $AdS_4 \times CP^3$.

The functional determinants in (3.4) may be computed by the standard AdS_d spectral zeta-function techniques (as was done in the similar AdS_2 string case in e.g. [21, 30, 31]). One can first verify the cancellation of the logarithmically divergent part of the 1-loop free energy $\Gamma_1 = -\log Z_1$

in (3.4):

$$\zeta_{\rm tot}(0) = \frac{1}{2}\sum_{n\in\mathbb{Z}}\big(-2+4\big) = \sum_{n\in\mathbb{Z}} 1 = 1 + 2\zeta_R(0) = 0\,, \tag{3.5}$$

where we have used the Riemann zeta-function regularization to evaluate the sum. Note that the contribution of all massive KK modes at non-zero n levels cancels 1 coming from the $n = 0$ modes, i.e. cancels the logarithmic UV divergence that was present in the similar computation in the $\mathrm{AdS}_4 \times \mathrm{CP}^3$ superstring regime [21].

The vanishing of the logarithmic divergence in the free energy was actually expected, as the M2 brane theory is 3d one, and there are no logarithmic divergences in the corresponding functional determinants in 3d. The reduction to 2d with all KK modes included cannot produce logarithmic divergences that were not present in the 3d formulation. The 1-loop free energy is thus finite and is given by the sum of the bosonic and fermionic contributions

$$\Gamma_1 = -\log Z_1 = -\frac{1}{2}\zeta'_{\rm tot}(0)\,, \qquad \zeta'_{\rm tot}(0) = \sum_{n\in\mathbb{Z}}\zeta'_{\rm tot}(0;n) \tag{3.6}$$

$$\zeta'_{\rm tot}(0;n) = 2\zeta'_{_B}(0;\frac{1}{4}(kn-2)(kn-4)) + 6\zeta'_{_B}(0;\frac{1}{4}kn(kn+2)) \tag{3.7}$$

$$+\,3\zeta'_{_F}(0;\frac{kn}{2}+1) + 3\zeta'_{_F}(0;\frac{kn}{2}-1) + 2\zeta'_{_F}(0;\frac{kn}{2})\,. \tag{3.8}$$

Summing up the bosonic and fermionic contributions, some remarkable simplifications occur

$$\Gamma_1 = \sum_{n=1}^{\infty}\log\Big(\frac{k^2n^2}{4}-1\Big) = 2\sum_{n=1}^{\infty}\log(\frac{kn}{2}) + \sum_{n=1}^{\infty}\log\Big(1-\frac{4}{k^2n^2}\Big)\,. \tag{3.9}$$

Using again the Riemann zeta-function regularization ($\zeta_R(0) = -\frac{1}{2},\ \zeta'_R(0) = -\frac{1}{2}\log(2\pi)$) we get

$$\sum_{n=1}^{\infty}\log\Big(1-\frac{4}{k^2n^2}\Big) = \log\Big[\prod_{n=1}^{\infty}(1-\frac{4}{k^2n^2})\Big] = \log\Big[\frac{k}{2\pi}\sin\big(\frac{2\pi}{k}\big)\Big]\,, \tag{3.10}$$

where we used that $\sin(\pi x) = \pi x\prod_{n=1}^{\infty}(1-\frac{x^2}{n^2})$. The final result for the 1-loop partition function for $k > 2$

$$Z_1 = e^{-\Gamma_1} = \frac{1}{2\sin(\frac{2\pi}{k})}\,. \tag{3.11}$$

This is in precise agreement with the localization result in (2.2).

4. M2 brane computation of instanton contrubution to free energy

Motivated by the expected duality between the ABJM theory and M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ it is natural to expect that the perturbative part of the free energy (2.6) should be reproduced by some higher derivative extension of the 11d supergravity action evaluated on the $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ background [27, 32]. Indeed, it was found in [11] that the leading $N^{3/2}$ term in (2.6) is matched by the on-shell value of the Euclidean 11d supergravity action.

More generally, we shall conjecture that the gauge theory free energy should be reproduced by some properly defined supermembrane partition function,

$$F \sim Z_{\rm M2}\,, \qquad Z_{\rm M2} = \int [dx\, d\theta]\, e^{-S_{\rm M2}(x,\theta)}\,, \tag{4.1}$$

where $S_{\rm M2}$ is the M2 brane action on $\rm AdS_4 \times S^7/\mathbb{Z}_k$ with the dimensionless coefficient of the effective tension (here again R is the radius of S^7 or twice the radius of $\rm AdS_4$)

$$\mathrm{T}_2 \equiv R^3 T_2 = \frac{1}{(2\pi)^2}\frac{R^3}{\ell_P^3} = \frac{\sqrt{2k}}{\pi}\sqrt{N}\,, \qquad\qquad \frac{R}{\ell_P} = (32\pi^2 N k)^{1/6}\,. \tag{4.2}$$

Then for fixed k (or fixed radius of the 11d circle) the semiclassical large T_2 expansion of $Z_{\rm M2}$ should be equivalent to the large N expansion on the gauge theory side.

One may further conjecture that the perturbative part of $Z_{\rm M2}$ in the large $\mathrm{T}_2 \sim \sqrt{N}$ limit may be captured by an expansion near "point-like" M2 branes or, more precisely, degenerate 3-surfaces with a topology of S^1 times a point which have zero 3-volume. At the same time, the non-perturbative $e^{-a\mathrm{T}_2} = e^{-a\frac{\sqrt{2k}}{\pi}\sqrt{N}}$ contributions may come from saddle points with non-vanishing 3-volumes, e.g. from M2 branes wrapping the M-theory circle and a $\mathrm{CP}^1 \subset \mathrm{CP}^3$, or a 3-cycle in CP^3 (and their superpositions), $Z_{\rm M2} = Z^{(0)}_{\rm M2} + Z^{\rm inst}_{\rm M2} + \ldots$. Here the first term (coming from contributions of "degenerate" M2 brane surfaces) when expanded at large k should represent the sum of all perturbative tree level plus higher loop type IIA string corrections to the on-shell value of the partition function.

Below we will perform a semiclassical computation in the case of the M2 brane instanton wrapping $S^3/\mathbb{Z}_k$, reproducing the $\frac{1}{\sin^2\frac{2\pi}{k}}$ prefactor in (2.9) from the corresponding 1-loop fluctuation determinants. We are interested in the M2 brane configuration with $S^3/\mathbb{Z}_k$ world-volume, that is wrapped on the 11d circle φ of radius R/k and on $\mathrm{CP}^1 \subset \mathrm{CP}^3$. This is the M2 uplift of the IIA string CP^1 instanton of [19]. The CP^1 will be chosen as the $w^2 = w^3 = 0$ surface in CP^3. We fix the world-volume reparametrization invariance using the static gauge: we identify $(w_1, \bar w_1, \varphi)$ with the 3 real world-volume coordinates $\xi^i = (u, v, s)$ according to

$$w^1 \equiv z = u + iv\,, \qquad \bar w^1 = \bar z = u - iv\,, \qquad \varphi = s, \qquad\qquad s \in (0, 2\pi]\,. \tag{4.3}$$

As the C_3 potential has only the $\rm AdS_4$ components the bosonic part of the corresponding Euclidean M2 brane action is given by $S_{\rm cl} = T_2 \int d^3\xi\,\sqrt{g}$, where g_{ij} is the induced world-volume metric ($\kappa = (1 + u^2 + v^2)^{-1}$)

$$ds_3^2 = g_{ij} d\xi^i d\xi^j = R^2 \frac{dz\, d\bar z}{(1 + |z|^2)^2} + \frac{R^2}{k^2}\Big[ds + kA(z,\bar z)\Big]^2, \quad A = \kappa(-v du + u dv)\,. \tag{4.4}$$

This is the metric of $S^3/\mathbb{Z}_k$ (for $k = 1$ this is the standard Hopf metric of S^3 with radius R). The resulting classical value of the action (4) is

$$S_{\rm cl} = T_2 R^3 \,\mathrm{vol}(S^3/\mathbb{Z}_k) = \frac{1}{k} T_2 R^3\, \mathrm{vol}(S^3) = \frac{2\pi^2}{k}\mathrm{T}_2 = 2\pi\sqrt{\frac{2N}{k}}\,. \tag{4.5}$$

Here the effective dimensionless tension is

$$\mathrm{T}_2 \equiv R^3 T_2 = \frac{1}{(2\pi)^2}\frac{R^3}{\ell_P^3} = \frac{1}{\pi}\sqrt{2Nk}\,. \tag{4.6}$$

This is also the same as the value of the classical action of the string world sheet wrapped on CP^1 in $\mathrm{AdS}_4 \times \mathrm{CP}^3$ [19], i.e. $S_{\rm cl} = 2\pi\sqrt{2\lambda}$.

Next, we are to compute the 1-loop prefactor Z_1 in the corresponding 1-instanton contribution to the M2 brane partition function

$$Z^{\rm inst}_{\rm M2} = Z_1\, e^{-S_{\rm cl}} + \dots \,. \tag{4.7}$$

The factor Z_1 will be expressed in terms of the determinants of operators of the bosonic and fermionic fluctuations which will be functions of the 3d coordinates (u, v, s) in the static gauge. In this static gauge we will have 8 real bosonic fluctuations: 4 in the AdS_4 directions and 4 in the 2 complex transverse CP^3 directions w_2, w_3. Fixing a κ-symmetry gauge, we will also have 8 fermionic fluctuations. Expanding the M2 action one finds that the fluctuations of w_2 and w_3 decouple and their contributions are described by

$$L_2(\phi) = \frac{R^2}{2}\sum_{i,j=1}^{3} g^{ij} D_i\bar\phi\, D_j\phi - \bar\phi\phi - \frac{i}{2}k(\bar\phi\partial_s\phi - \partial_s\bar\phi\,\phi)\,, \tag{4.8}$$

$$D_i\phi = (\partial_i - i\,A_i)\,\phi, \qquad D_i\bar\phi = (\partial_i + i\,A_i)\,\bar\phi\,. \tag{4.9}$$

Here $\partial_i = (\partial_u, \partial_v, \partial_s)$ and $A_i = (A_u, A_v, 0)$ is the 3d gauge potential in (4.4). As s is a periodic coordinate we may interpret the corresponding 3d action $\int d^3\xi\sqrt{g}\, L_2$ as a 2d action for an infinite tower of the Fourier modes of ϕ by setting $\phi(u, v, s) = \sum_n \phi_n(u, v)\, e^{ins}$. This 2d action will be defined on CP^1 with the metric g_{ab} of a 2-sphere of radius $R/2$. This leads to the corresponding Lagrangian for a tower of 2d charged massive complex scalars ϕ_n on the 2-sphere coupled to abelian gauge field potential A_a of a magnetic monopole, $\Delta = -D^2 + M^2$, $D_a = \partial_a - iqA_a$. Measuring the masses in terms of the radius $L = R/2$ of S^2 we thus get the following bosonic spectrum: 2 towers of complex ϕ_n modes and 4 towers of $\eta_n = \bar\eta_{-n}$ modes with

$$\phi_n:\quad m^2 \equiv L^2M^2 = -\frac{3}{4} + \frac{1}{4}(1+nk)^2\,, \quad q = 1+nk; \qquad \eta_n:\quad m^2 = \frac{1}{4}(nk)^2\,, \quad q = nk\,. \tag{4.10}$$

The detailed structure of the quadratic fermionic Lagrangian in the type IIA string limit [20] shows that it is equivalent to the sum of 2d fermionic terms $\bar\psi D\psi$ where D is the standard 2d Dirac operator on the 2-sphere of radius $L = R/2$ in the monopole background with a particular mass term $D = i\,\hat D + M_1\sigma_3 + M_2$, $\hat D = \sigma^{\rm a} e^a_{\rm a}(\partial_a + \frac{i}{2}\omega_a\sigma_3 - iqA_a)$. The explicit values of the dimensionless mass parameters are $m_1 \equiv LM_1 = -\frac{1}{4}(u - u')$, $m_2 \equiv LM_2 = -\frac{1}{4} - \frac{3}{4}uu'$, $\quad u, u' \in \{1, -1\}$, where u, u' represent 4 independent sign factors arising from 10-d Gamma matrices in a suitable representation. Thus one finds 8 fermionic modes organized as 2d fermionic fields with 4 choices of mass parameters $m_1 = (-\frac{1}{2}, \frac{1}{2}, 0, 0)$, $m_2 = (\frac{1}{2}, \frac{1}{2}, -1, -1)$. In addition, the values of the charges are $q = (1, -1, 0, 0)$ [20]. Similarly, from from M2 brane action we find that the Lagrangian for the tower of the 2d fermionic modes originating from the quadratic fermionic part of the M2 brane action can be represented by a collection of 4 2d fermionic fields with the Dirac-like operators where the parameters depend on k as

$$m_1 = -\frac{1}{4}(u - u') - \frac{1}{2}nk, \qquad m_2 = -\frac{1}{4} - \frac{3}{4}uu', \qquad q = -2m_1\,. \tag{4.11}$$

For a massless scalar field of charge q on S^2 in the field of a monopole the spectrum of the corresponding Laplace operator was found in [33]. Its eigenvalues and degeneracies are given by

$$\lambda_\ell = \ell(\ell+1) - \frac{q^2}{4}, \qquad \ell - \frac{|q|}{2} = 0, 1, 2, \dots, \qquad \deg\lambda_\ell = 2\ell + 1\,. \tag{4.12}$$

Inclusion of mass term in the operator can be done by the obvious shift $\lambda_\ell \to \lambda_\ell + m^2$, $m \equiv LM$. Then the formal expression for the corresponding determinant may be written as

$$\log\det\left[L^2(-D^2+M^2)\right] = \sum_{\ell=\frac{|q|}{2}}^{\infty}(2\ell+1)\log\left[\ell(\ell+1)-\frac{q^2}{4}+m^2\right] = \sum_{\ell=\frac{|q|+1}{2}}^{\infty} 2\ell\,\log\left[\ell^2-\frac{1}{4}-\frac{q^2}{4}+m^2\right]. \tag{4.13}$$

The massless Dirac operator $i\hat{D}$ has eigenspinors with the following eigenvalues (normalized again to the radius L of the sphere) and degeneracies [34] $\lambda_\ell = \pm\sqrt{\ell^2-\frac{q^2}{4}}$, $\ell-\frac{|q|}{2} = 0, 1, 2, \ldots$, $\deg\lambda_\ell = 2\ell$. For the minimal value $\ell = \frac{|q|}{2}$ (assuming $|q| \geq 1$), we get $|q|$ zero modes with definite chirality. In the case of the massive operator $i\hat{D} + M_1\sigma_3 + M_2$ this spectrum leads to the following expression for the determinant (here $m_a = LM_a$)

$$\log\det\left[L(i\hat{D}+M_1\sigma_3+M_2)\right] = |q|\,\log\left|\text{sign}(q)\,m_1+m_2\right| + \sum_{\ell=\frac{|q|}{2}+1}^{\infty} 2\ell\log\left(\ell^2-\frac{q^2}{4}+m_1^2-m_2^2\right). \tag{4.14}$$

That the eigenvalues of all other modes contain the effective mass-squared parameter $m_1^2 - m_2^2$ (cf. (4.13)) follows from the direct evaluation of the determinant in (4.14) or can be seen from "squaring" the first-order operator.

The determinant of an elliptic 2nd order operator Δ can be expressed in terms of the spectral ζ-function $\zeta_\Delta(z) = \sum_\ell \lambda_\ell^{-z}$, $\lambda_\ell \neq 0$, as

$$\log\det\Delta = -\zeta_\Delta(0)\,\log(\Lambda^2L^2) + (\log\det\Delta)_{\rm fin}\,, \qquad (\log\det\Delta)_{\rm fin} = -\zeta'_\Delta(0)\,, \tag{4.15}$$

where Λ is a 2d UV cutoff. One finds

$$(\log\det\Delta)_{\rm fin} = -4\,\zeta'(-1,p) + \int_0^\mu dx\,\left[\psi(p+\sqrt{x}) + \psi(p-\sqrt{x})\right] \equiv s_p(\mu). \tag{4.16}$$

and similar expression in the fermionic case. Let us sum up the log det contributions of all 2d fluctuation fields. The corresponding 1-loop correction is

$$\Gamma = \Gamma_B - \Gamma_F = \frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_i(-1)^{F_i}\log\det{}'\Delta_i = -\zeta_{\rm tot}(0)\log(\Lambda L) - \frac{1}{2}\zeta'_{\rm tot}(0)\,. \tag{4.17}$$

We find that the coefficient $\zeta_{\rm tot}(0) = \sum_{n=-\infty}^{\infty}\zeta_{{\rm tot},\,n}(0) = \sum_{n=-\infty}^{\infty} 2 = 2 + 4\zeta_R(0) = 0$. Thus, as in the 1-loop correction in the case of the M2 brane wrapped on $\text{AdS}_2\times S^1$ discussed above, the UV divergences coming from the tower of $n \neq 0$ modes effectively cancel the non-zero contribution of the $n = 0$ string modes so that the full M2 brane 1-loop correction is UV finite.

For the sum of the finite parts we get (using again that $\zeta_R(0) = -\frac{1}{2}$, $\zeta'_R(0) = -\frac{1}{2}\log(2\pi)$):

$$\begin{aligned}\Gamma &= 4\sum_{n=1}^{\infty}\log\tfrac{nk}{2} + 2\sum_{n=1}^{\infty}\log\left(1-\tfrac{4}{n^2k^2}\right)\\ &= 4\log\tfrac{k}{2}\,\zeta_R(0) - 4\zeta'_R(0) + 2\log\left(\frac{k}{2\pi}\sin\frac{2\pi}{k}\right) = 2\,\log\left(2\,\sin\frac{2\pi}{k}\right).\end{aligned} \tag{4.18}$$

We conclude that the 1-loop instanton prefactor in (4.7) is

$$Z_1 = \gamma\, e^{-\Gamma} = \gamma\, \frac{1}{4\sin^2(\frac{2\pi}{k})}\,, \tag{4.19}$$

where we introduced a numerical (k-independent) factor γ to account for the contribution of the 0-modes that we omitted above and also of possible degeneracy of the instanton saddle contributions. In the large k limit this reduces to $Z_1 = \gamma \frac{k^2}{16\pi^2} + ... = \frac{2\gamma}{\pi}\frac{T}{g_s^2} + ...$, i.e. to the expression found in [20]. Here we get it directly as a limit of the UV finite M2 brane contribution, without need to fix the form of the overall factor by some indirect considerations as was done in [20] using an analogy with the string Wilson loop case computation in [21].

To determine the value of γ let us note first that we are to add a factor of 2 due to the equal instanton and anti-instanton contributions. We also need a further factor of 2 that was argued in [20] to represent the contribution of the 0-modes of the string fluctuations. It would be very interesting to derive this result systematically by introducing the collective coordinates for the bosonic and fermionic 0-modes and computing the volume of the corresponding supercoset. Thus using that $\gamma = 4$ we precisely match the 1-instanton prefactor in the localization result in (2.9).

5. Conclusion

We presented new remarkable tests of the AdS_4/CFT_3 duality between ABJM theory with large rank of the gauge group N and finite level k and M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$. In the Wilson loop case the prefactor in the localization result is reproduced by expanding near the $AdS_2 \times S^1$ M2 brane solution [9].

We also reproduced the leading (at large N and fixed $k > 2$) instanton prefactor in the localization result for the non-perturbative part of the ABJM free energy on S^3 in (2.9) from a quantum 1-loop correction to the classical action factor of the M2 brane $S^3/\mathbb{Z}_k$ instanton [10]. This generalizes to finite k the analysis of the string $CP^1 \subset CP^3$ instanton contribution in type IIA string theory [19, 20]. One subtle issue that would be interesting to clarify further is the factor 2 associated with the zero mode contribution that originates from the string-level fluctuations.

There are several possible extensions. One may consider the leading perturbative $\frac{1}{\sqrt{N}}$ correction to the prefactor in (2.9),(2.8)

$$F^{\rm inst}(N,k) = -\frac{1}{\sin^2(\frac{2\pi}{k})}\Big[1 + \frac{1}{\sqrt{N}} h_1(k) + ...\Big]\, e^{-2\pi\sqrt{\frac{2N}{k}}} + ...\,, \qquad h_1(k) = \frac{\pi}{\sqrt{2k}}\frac{k^2-40}{12k}\,, \tag{5.1}$$

and try to reproduce the coefficient $h_1(k)$ from the 2-loop M2 brane correction, which should come with a factor of the inverse of the effective M2 brane tension in (4.2), i.e. $T_2^{-1} = \frac{\pi}{\sqrt{2k}}\frac{1}{\sqrt{N}}$.

As was suggested in [9], a similar 2-loop computation in the case of the $AdS_2 \times S^1$ M2 brane surface should reproduce the coefficient of the $\frac{1}{\sqrt{N}}$ correction to the prefactor of the Wilson loop expectation value in (2.2).[1] Such a 2-loop calculation would require the use of the quartic bosonic

[1] Note that the string theory values of the coefficients of these $\frac{1}{\sqrt{N}}$ corrections in (5.1) and (2.2) are sensitive to the precise form of the relation between the string theory parameters in (4.2) and gauge theory parameters N, k, i.e. to the shift $N \to N - \frac{1}{24}(k - k^{-1})$ suggested in [38].

and fermionic terms in the corresponding supermembrane action [6]. It would be important to check if the 2-loop M2 brane contribution is, in fact, UV finite, despite the apparent non-renormalizability of the membrane action.

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