PROCEEDINGS OF SCIENCE

New Euclidean axion wormholes

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In my presentation I discuss recent progress in search for new Euclidean wormhole solutions, a topic which is based on research started with Valery Rubakov and Peter Tinyakov back in 1988 [1]. In current study we investigate extension of axion gravity (studied by Giddings and Strominger), with an extra massive dilaton field or scalar field with symmetric double well potential. We found that in both theories new types of wormhole solutions exist and investigated their properties as a function of parameters of the model (axion charge, dilatonic coupling constant etc.) Euclidean actions of these new wormholes are typically comparable with actions of Giddings-Strominger type wormholes. These new types of wormholes are leading to expanding baby universes after analytic continuation back to Minkowski signature, whereas Giddings-Strominger wormholes are leading to contracting baby universes. In addition rich structures of generalized Giddings-Strominger wormholes are found in axion gravity with massive dilaton. Presentation is based on a recent new results obtained in collaboration with Caroline Jonas and Jean-Luc Lehners [2].

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1. Introduction

It is very emotional for me to speak at Valery Rubakov's memorial conference, since I have known him for more than 40 years. I was Valery's second PhD student, but actually I met him for the first time at the "Quarks" conference in 1982 in Sukhumi, even before I started my PhD study in Moscow. Valery was not only my teacher, but a good friend as well. He has always been and will be for me the standard of dedication to science, integrity and high moral values. I'm extremely sorrowful that he passed away so early. What I will be talking about in this contribution is continuation of research, which we started together with Valery [1]. It is very sad that there is no longer an opportunity to discuss new results with him and hear his valuable opinion.

Before presenting the main results, let me make four remarks in order.

1.1 Three types of Euclidean solutions

Let's start with a celebrated instanton solution. For simplicity let's consider the quantum mechanic in symmetric double well potential $V(x)$ shown in Fig. (1) left panel. It is well known that a classical particle placed initially in the left well with the energy less than potential barrier cannot move to the right well. In quantum mechanics, if the potential barrier would be infinite,

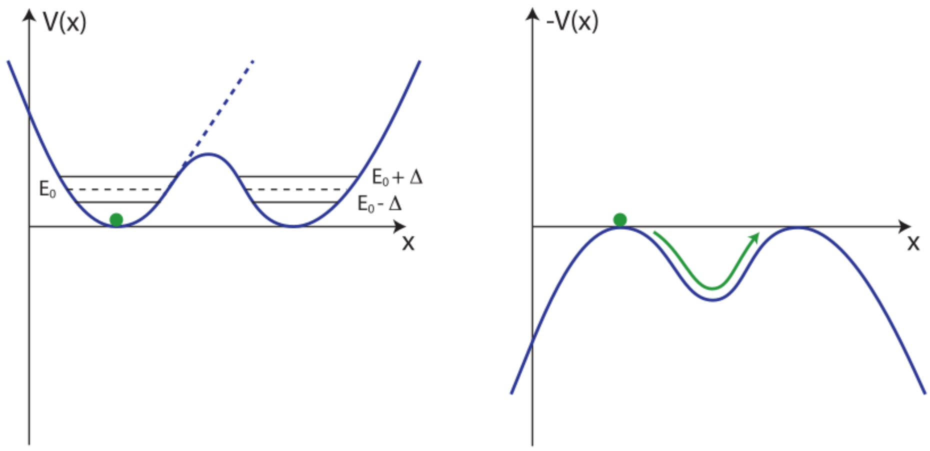


Figure 1: Simplest instanton in symmetric potential in quantum mechanics.

in each well one gets a ground state with the energy E_0. But due to the finiteness of the barrier, quantum tunneling happens and leads to splitting of what would be ground state energy. This can be described with standard Schroedinger equation calculations, or functional integral approach. In the later classical solution of equations of motion in the inverted potential, Fig. (1) right, i.e. Euclidean equations of motion with the finite action - instanton - is a saddle point of functional integral and leads to this splitting,

$$E = E_0 \pm \Delta \,, \quad \Delta = \mathcal{A} \ \mathrm{e}^{-S(x_{inst}(\tau))} \,, \tag{1}$$

where the leading exponential factor in energy splitting Δ is given by the Euclidean action of the instanton solution $x_{inst}(\tau)$. Here τ denotes Euclidean time. Pre-exponential factor $\mathcal{A}$ is given by (gaussian) integration of quadratic action of linear perturbations around the instanton solution [3] and has the form of the square root of the determinant of the corresponding operator. It is important that there are at most zero modes in the spectrum of linear perturbations about instanton solutions, which guarantees that $\mathcal{A}$ and correspondingly correction Δ are real.

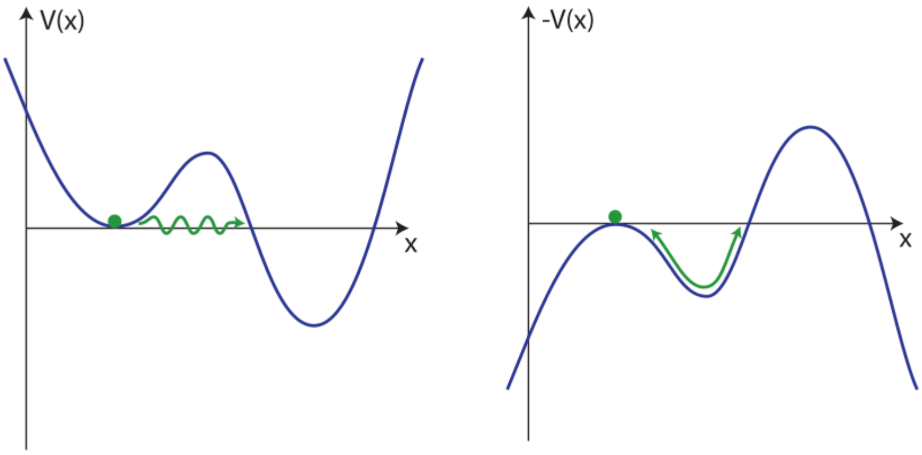


Figure 2: Bounce solution descibing metastable decay in asymmetric double well potential in flat space-time.

Situation changes when potential is asymmetric double well Fig. (2), left panel. Minimum of potential with higher energy is called metastable or false vacuum and one with lover energy true vacuum. In quantum theory metastable vacuum decays and this decay can be described by a bounce [4], solution of Euclidean equations of motion starting at false vacuum, rolling down towards metastable vacuum, bouncing at some point and at $\tau \to +\infty$ reaching false vacuum again, Fig. (2), right panel. There is exactly one negative mode in spectrum of linear perturbations about bounce solution, and correspondingly what would be ground state energy gets pure imaginary correction (square root of negative determinant!), justifying decay picture

$$E = E_0 + i\Gamma \ , \quad \Gamma = \mathcal{A} \ \mathrm{e}^{-S[x_{bounce}(\tau)]} \ , \tag{2}$$

where Γ is the decay width of a metastable state. When gravity is included, decay of metastable vacuum of a scalar field with asymmetric double well potential is described by the Coleman-de Luccia (CdL) solution [5]. CdL bounce is O(4)-symmetric and involves both: scalar field $\phi(\tau)$ and scale factor $a(\tau)$, Fig. 3 left panel.

Furthemore, it was found [6] that quadratic action about CdL bounce has schematically the form

$$S^{(2)} = \frac{1}{2} \int \mathrm{d}\tau \left(\frac{1}{Q(a,\phi)} \dot{\mathcal{R}}^2 + U(a,\phi)\mathcal{R}^2 \right) , \tag{3}$$

where $\mathcal{R}$ is a single physical perturbation after proper reduction and a dot denotes a Euclidean time derivative. The factor Q in the kinetic term as well as the potential U depend on background quantities, scalar field ϕ and scale factor a. When $Q > 0$ everywhere, this quadratic action leads to a regular Sturm-Liouville problem and one finds exactly one negative mode about the CdL bounce [7]. But for some bounces factor Q can be negative along the bounce, leading to catastrophic instability with infinitely many negative modes. In spite of much work in this direction [6–21] this negative mode problem did not get fully satisfying resolution up to now. Observation is that the exact interval where Q becomes negative depends on choice of perturbation variable $\mathcal{R}$, (see e.g.

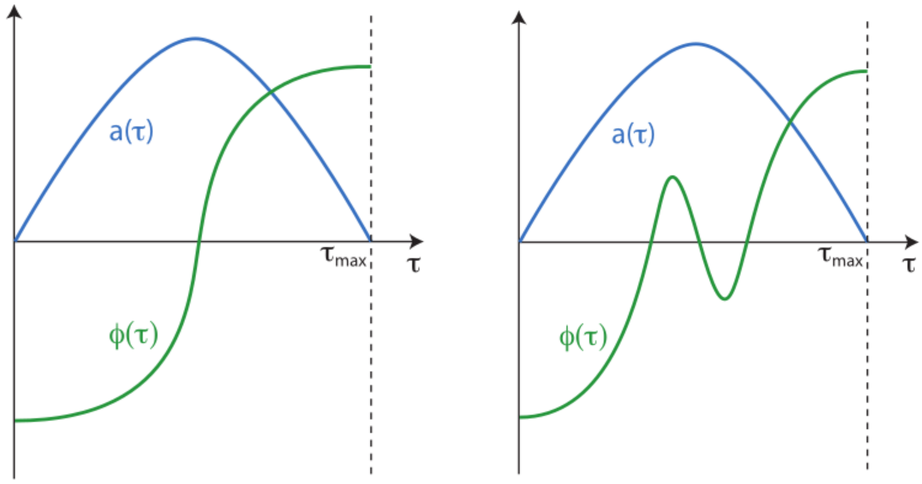


Figure 3: Coleman - De Luccia bounce and oscillating bounce in metastable vacuum decay with gravity.

comprehensive review [21]). This fact most probable indicates that existence of infinitely many negative modes is not physical, but rather an artifact of concrete description.

In addition to CdL bounce oscillating bounces were discussed in the literature, where scalar field goes back and forth between vacua several times, [22], Fig. (3) right panel. It was shown [13] that oscillating bounce with K oscillations (nodes) of scalar field has exactly K negative modes, thus doubting the physical relevance of such solutions with $K > 1$.

To summarize, we discussed three types of Euclidean solutions with the finite action known from metastable vacuum decay processes: (i) instanton-like, with at most zero modes, leading to the real corrections to the ground state energy, (ii) bounce like, with exactly one negative modes in spectrum of linear perturbations, leading to decay of metastable vacuum (iii) oscillating bounces, with the multiple negative modes, with obscure physical interpretation, subdominant saddle points.

1.2 Topology fluctuations / wormholes

Topology fluctuations have been discussed since long ago [23]. Idea is straightforward: when gravity in some regime (small distances/high energy) becomes quantum, the space-time metric should also be subject to fluctuations as are all other variables. Simplest topology fluctuation could be creation of the baby universe from the mother universe, $\Sigma_1 \rightarrow \Sigma_2 \oplus \Sigma_3$, Fig. 4 left panel. It is known that this process cannot be described by non-singular Lorentzian metric, but it can be represented by the Euclidean metric. This reminds us situation on Fig. (1) and Fig. (2), where classical motion in real time is impossible, but there are solutions of equations of motion in Euclidean time, relevant for quantum tunneling. From this analogy we can conclude that changes in topology could be also due to tunneling processes.

It is interesting to note that when we studied particle creation [24], [25] on this kind of Euclidean manifolds with Valery and Peter Tinyakov (also Rubakov's PhD student at that time), solutions with this kind of topology were unknown. Nevertheless, assuming that they exist, we

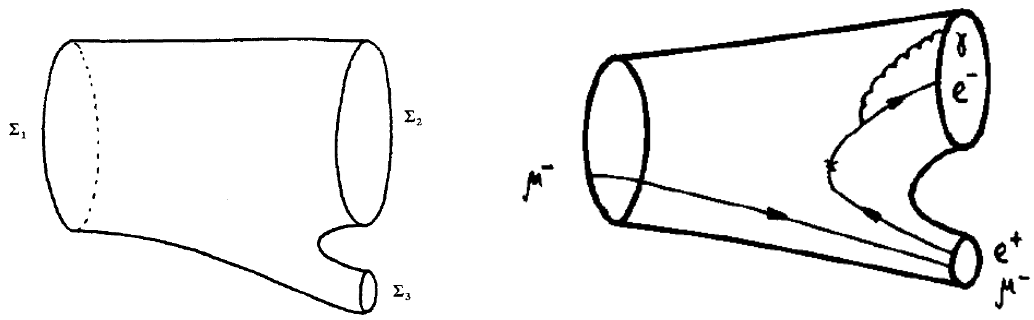


Figure 4: Baby universes and global quantum number non-conservation.

arrived at interesting conclusions.[1] Let me here mention just one possibility: global quantum number non-conservation. Muons from our universe could go into the baby universe, together with the positron, from a virtually created pair. An electron with the gamma quant goes to our universe. Effectively this leads to a rear process $\mu^- \to e^- + \gamma$, Fig. 4 fight panel, which is under experimental search. It came as a big surprise when in 1987 Valery came back to Moscow after short visit to US and told me and Peter that two guys in the US are working on a similar topic and they found solutions with such a topology in the Euclidean version of gravity-axion theory. These are Giddings-Stominger (GS) wormholes [27]. GS wormhole connects two asymptotically flat regions and half a wormhole describes creation of a baby universe, Fig. 5.

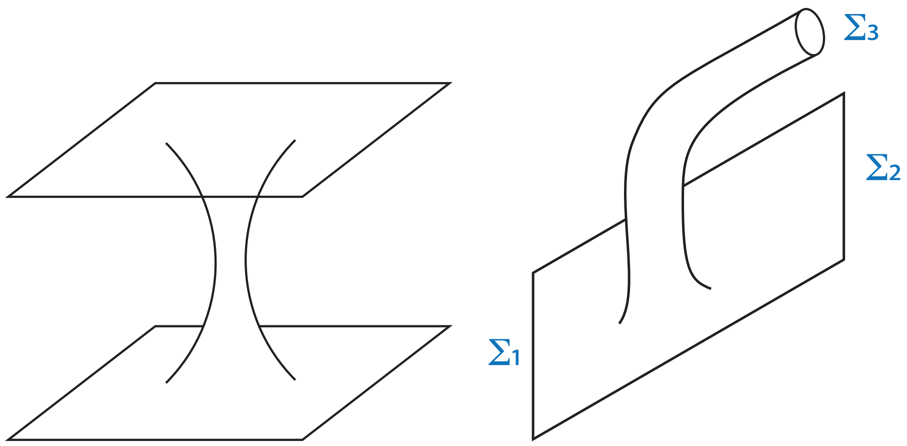


Figure 5: Common interpretations of Euclidean wormholes. On the left, a full wormhole connecting two asymptotic regions. On the right, a semi-wormhole, leading to the creation of a baby universe (Σ_3).

1.3 Baby universe interpretation

Euclidean wormholes can be interpreted as tunnelling events leading to the creation of baby universes [27, 28]. It was noted by Rubakov [1] that GS wormholes are leading to the materialisation

[1]First presentation of our results at the international event was at the 4th Seminar on Quantum Gravity in Moscow in May, 1987. Our plenary talk (Rubakov was presenting) was scheduled just after the S. Hawking's talk. Hawking's talk title was "TBA" until last moment and then surprisingly it turned out [26] that there was huge intersection between both investigations and conclusions!

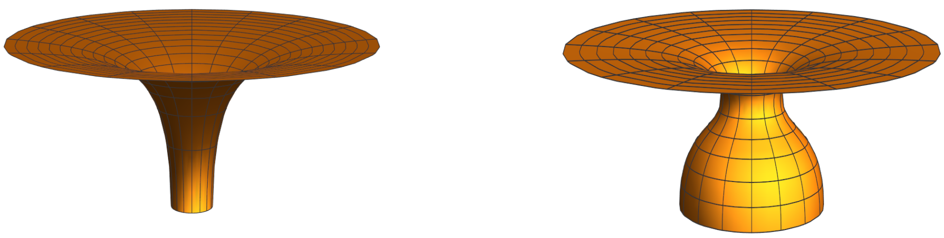
of baby universes which are *contracting* after analytic continuation to Minkowski time. Indeed, a regular wormhole at $\tau = 0$ has finite size $a(0) = a_0 \neq 0$, and zero derivative $\dot{a}(0) = 0$ such that for small τ we can expand

$$a(\tau) = a_0 + \frac{1}{2} a_2 \tau^2 + O(\tau^4) \ , \tag{4}$$

where the coefficient $a_2 = \ddot{a}(0)$. After analytic continuation to Minkowski time $t = -i\tau$ we get

$$a(t) = a_0 - \frac{1}{2} a_2 t^2 + O(t^4) \ . \tag{5}$$

Now it is clear that $a_2 > 0$ and $a_2 < 0$ correspond respectively to contracting and expanding small universes. The GS wormhole obviously has $a_2 = \ddot{a}(0) > 0$, since the neck of the wormhole is a minimum of $a(\tau)$. Instead, a wormhole leading to an *expanding* baby universe should have $a_2 = \ddot{a}(0) < 0$, *i.e.* the "neck" of such a wormhole should be a local maximum Fig. 6 b.



(a) Giddings-Strominger-type wormhole. **(b)** Wormhole leading to an expanding baby universe.

Figure 6: Visualisation of Euclidean wormholes. Wormholes of GS type lead to contracting baby universes upon analytic continuation, while wineglass shaped wormholes lead to expanding baby universes. Further details on embedding diagrams of the sort shown here can be found in [2].

1.4 Linear stability analysis

Stability analysis of GS wormholes and establishing their number of negative modes is quite an involved story.

First linear stability analysis of GS wormholes was undertaken by Rubakov with his student Shvedov back in 1996 [29]. Using Lagrangian formalism for derivation of quadratic action and after certain analytic continuation, they found a single negative mode about the GS wormhole in the O(4) symmetric (homogeneous) sector.

Later Alonso and Urbano [30], carefully reinvestigated this question and arrived at the conclusion that there are no physical degrees of freedom and correspondingly no negative modes in the homogeneous sector!

Next step was made by Hertog, Truijen and VanRiet [31], why claimed that Euclidean axion wormholes have multiple negative modes in non homogeneous (higher harmonics) sector, and therefore are not relevant saddle points of Euclidean quantum gravity.

But a few years later Loges, Shiu and Sudhir [32], using gauge invariant approach showed that Euclidean axion wormholes do not have any negative modes!

2. Wormhole solutions

Our aim is to extend axion-dilaton gravity and investigate wormholes in this broader theory. We will consider two such extensions: first addition of a massive dilaton field and the second addition of a scalar field with the symmetric double well potential. We show that in both theories there are new wormhole solutions leading to expanding baby universes and study their properties as a function of coupling constants of models (axion charge, dilatonic coupling constant etc.) On top of it in theory with the massive dilaton there are rich structures of generalized GS wormholes. Because of limited space, here we discuss only representative examples of our findings. Complete details can be found in [2].

2.1 Model

Our starting point is the Euclidean action for gravity coupled to an axion and a dilaton/scalar ϕ, which reads [33]:

$$S_{\rm E} = \int {\rm d}^4x\sqrt{g}\left(-\frac{1}{2\kappa}R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V(\phi) + \frac{1}{12f^2}e^{-\beta\phi\sqrt{\kappa}}H_{\mu\nu\rho}H^{\mu\nu\rho}\right), \tag{6}$$

where $\kappa \equiv M_{\rm Pl}^{-2} = 8\pi G$, the dilatonic coupling constant is denoted β and the potential $V(\phi)$, $H_{\mu\nu\rho}$ being the 3-form field strength of an axion field with coupling f. When $\beta \neq 0$, we refer to ϕ as a dilaton, while for $\beta = 0$ we simply call it a scalar.

We are interested in spherically symmetric, homogeneous solutions, described by the ansatz

$$\begin{cases} {\rm d}s^2 = h^2(\tau){\rm d}\tau^2 + a(\tau)^2{\rm d}\Omega_3^2\,, \\ \phi = \phi(\tau)\,, \\ H_{0ij} = 0\,,\ H_{ijk} = q\varepsilon_{ijk}\,, \end{cases} \tag{7}$$

In the gauge $h \equiv 1$, equations of motion are

$$\begin{cases} 2a\ddot{a} + \dot{a}^2 - 1 + \kappa a^2\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right) - \frac{\kappa N^2}{a^4}{\rm e}^{-\beta\phi\sqrt{\kappa}} = 0\,, \\ \dot{a}^2 - 1 = \frac{\kappa a^2}{3}\left(\frac{\dot{\phi}^2}{2} - V(\phi)\right) - \frac{\kappa N^2}{3a^4}{\rm e}^{-\beta\phi\sqrt{\kappa}}\,, \\ \ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} = \frac{{\rm d}V}{{\rm d}\phi} - \frac{\beta N^2\sqrt{\kappa}}{a^6}{\rm e}^{-\beta\phi\sqrt{\kappa}}\,, \end{cases} \tag{8}$$

where

$$N^2 \equiv \frac{q^2}{2f^2}\,, \tag{9}$$

and the overdot denotes a derivative with respect to the (Euclidean) time coordinate τ.

Proper wormhole should have $\dot{a}(0) = 0$ and $\dot{\phi}(0) = 0$ at the wormhole neck $\tau = 0$ as the initial conditions, and in the asymptotic region $\tau \to \infty$, should satisfy $\dot{a}(\infty) = 1$ and $\phi(\infty) = 0$, which imply that the asymptotic future is the flat Euclidean spacetime.

On the classical solution, we must also specify the initial values of the scale factor and scalar field. If the value of the scalar field $\phi(0) = \phi_0$ is left as a free parameter, then the throat size,

$a(0) = a_0$, is determined by the Friedmann constraint at $\tau = 0$:

$$1 = \frac{\kappa}{3}\left(a_0^2 V(\phi_0) + \frac{Q^2}{a_0^4}\right), \tag{10}$$

$$\Leftrightarrow \frac{\kappa}{3}V(\phi_0)x^3 - x^2 + \frac{\kappa Q^2}{3} = 0\,, \quad x = a_0^2\,, \tag{11}$$

where we defined

$$Q^2 = N^2 \mathrm{e}^{-\beta\phi_0\sqrt{\kappa}}\,. \tag{12}$$

In case of axion gravity, i.e. $V \equiv 0$, throat size was uniquely determined by the axion charge. Here we see that a_0 is obtained as a solution of cubic equation (11). It turns out that depending which root of this cubic equation we choose to determine a_0 as a function of ϕ_0, we obtain either GS type wormholes or expanding wormholes [1], [2].

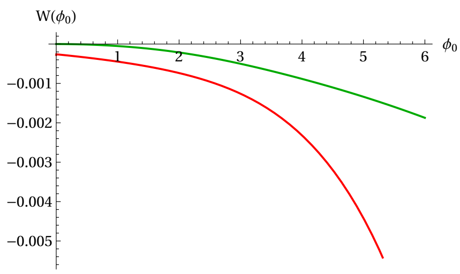
The equations of motions are systems of nonlinear ordinary differential equations - too complicated to be solved analytically for a massive dilaton or scalar field with symmetric double well potential. In both cases we find solutions numerically. Our strategy is as follows: we start integration at the neck of the wormhole with proper initial conditions and integrate until large Euclidean time τ, where asymptoticity is known analytically. In this setup the value of a scalar field at the neck of wormhole, ϕ_0 is a "shooting" parameter and should be adjusted to very high precision in order to obtain desired asymptotics for big values of Euclidean time τ. This way we obtain half a wormhole. Since a full wormhole is symmetric, it can easily be obtained by gluing together two half wormholes.

2.2 Axion wormholes with massive dilaton

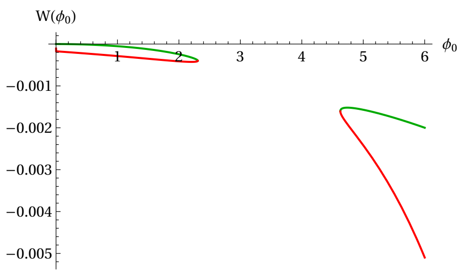
We consider simplest choice for the dilaton potential, just mass term

$$V(\phi) = \frac{1}{2}m^2\phi^2\,, \tag{13}$$

where m is the dilaton mass.



(a) Monotonic effective potential.



(b) Non-monotonic effective potential.

Figure 7: Effective potential $W(\phi)$ as a function of ϕ_0 for axion charge $N = 20000$ (left) and $N = 30000$ (right). The other parameters are the same for both plots: $m = 0.01$ and $\beta = 1.2$. The green line corresponds to the large root of the cubic equation, while the red line corresponds to the smaller positive root. The plot on the right contains a gap, which is caused by there existing no real solutions to the cubic equation (11) in that range.

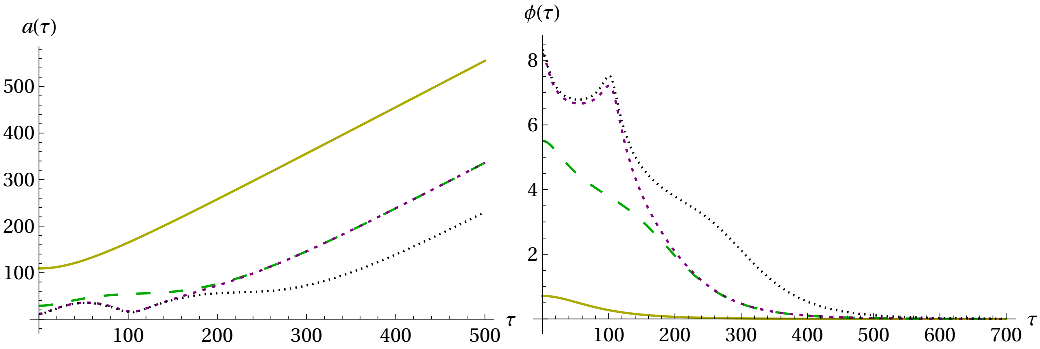


Figure 8: Wormhole solutions with a massive dilaton, with the scale factor shown on the left and the dilaton evolution on the right. All solutions have $\kappa = 1$, $\beta = 1.2$, $N = 30000$, $m = 0.01$. The individual solutions are characterised by the initial value of the dilaton, given respectively by the values $\phi_0 = 0.7118165858, 5.5075291704, 8.1964321797, 8.3116654157$ (we indicate a number of significant digits such that the action can be determined to better than percent level accuracy). Solutions with larger ϕ_0 display a more intricate field evolution, containing oscillations of the fields.

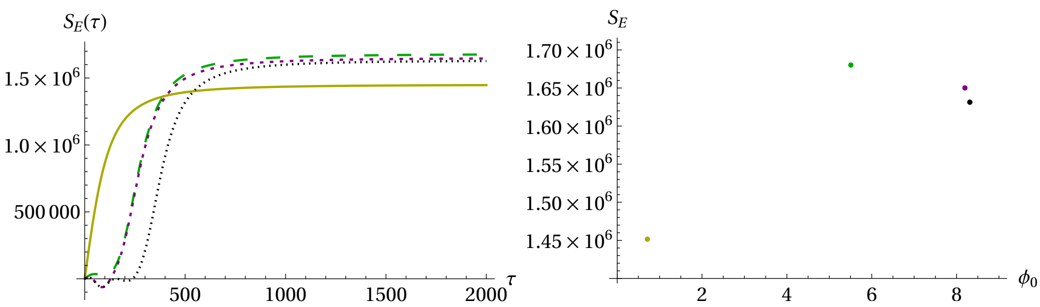


Figure 9: The Euclidean action, as a function of τ (left plot) and a graph with the asymptotic values (right plot), for the solutions shown in Fig. 8. Intriguingly, the action is not monotonic in ϕ_0, but starts decreasing as more oscillations are added.

The dilaton equation in (8) possesses a mechanical analogy as the motion of a *particle* $\phi(\tau)$ in an *effective potential* $W(\phi)$ (cf. [34]):

$$W(\phi) = -V(\phi) - \frac{N^2}{a^6} \mathrm{e}^{-\beta\phi\sqrt{\kappa}} , \tag{14}$$

$$\frac{\mathrm{d}W(\phi)}{\mathrm{d}\phi} = -m^2\phi + \frac{\beta N^2\sqrt{\kappa}}{a^6} \mathrm{e}^{-\beta\phi\sqrt{\kappa}} . \tag{15}$$

We see that the fate of the particle released at some point $\phi(0) = \phi_0 > 0$ with zero velocity $\dot{\phi}(0) = 0$ depends on the sign of $W_{,\phi}$ at this point. Depending on which term in the potential $W(\phi)$ dominates at this point, the particle either starts to move to the right (increasing ϕ) or to the left (decreasing ϕ). Since we want to obtain an asymptotically flat geometry, for $\tau \to \infty$ the dilaton field should eventually settle at its vacuum value $\phi = 0$. The shape of the effective potential crucially depends on the axion charge N, and also on which root is chosen in (11), see Fig. (7).

Choosing proper root in cubic Eq. (11) we obtain GS shape wormholes. Examples of generilized GS wormholes with massive dilaton are shown in Fig. 8. Euclidean actions of these solutions is shown in Fig. 9. Euclidean action has puzzling tendency that for more involved, complicated solutions it shows tendency to decrease, which is very cont-intuitive!

Note that original GS wormholes with the massless dilaton field exist up to critical value of dilaton coupling constant $\beta_c = 2\sqrt{\frac{2}{3}} \approx 1.63$. This fact actually was noticed by Valery during his 1987 trip to US and discussed with the authors and is properly acknowledged in [27]. That means non-singular massless dilaton wormholes does not exist in string motivated model with $\beta = 2 > \beta_c$. Inclusion of dilaton mass removes this obstacle and nonsingular wormholes exist for values $\beta \geq \beta_c$ as well, see Fig. 10, where different branches of generalized GS are shown for different values of dilatonic coupling constant β.

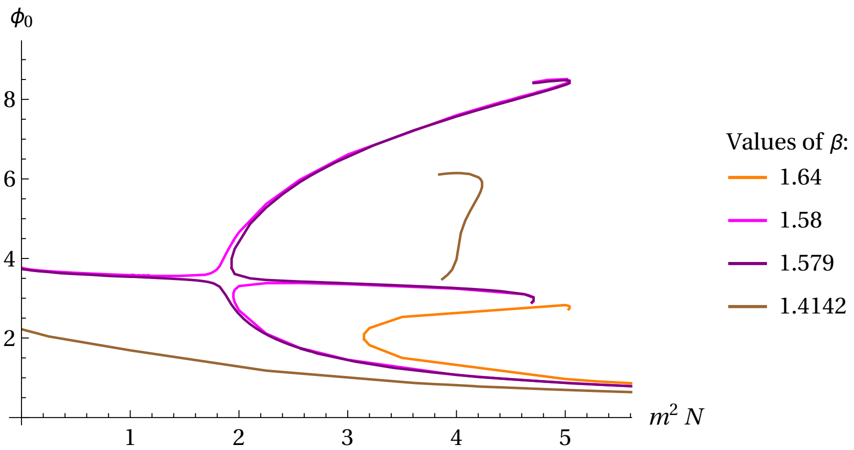


Figure 10: Existence of axion-dilaton wormhole solutions, as found in [2] for four typical values of β. The solutions are displayed via their initial dilaton value ϕ_0, as a function of their mass and charge combination $m^2 N$. Below the critical value $\beta_c = 2\sqrt{\frac{2}{3}} \approx 1.63$ solutions exist which are continuously connected to the massless case. In addition, for all dilaton couplings new bifurcating branches appear at sufficiently large values of $m^2 N$. Full details can be found in [2].

2.3 Axion dilaton wormholes leading to expanding baby universes

Choosing proper root of cubic equation in initial conditions we obtain wormholes with the wineglass shape, i.e. leading to creation of expanding baby universes upon analytic continuation. An example of such a wormhole with large dilaton coupling is shown in Fig. 11, whereas in Fig. 12 we compare GS-type wormholes properties with the expanding wormholes for the same parameter values. Among other things, we see that Euclidean actions of these solutions are comparable.

2.4 Axion scalar wormholes leading to expanding baby universes

We also investigated axion wormholes with extra scalar field. Whereas in [1] we studied wormholes with scalar field having asymmetric double well potential here we choose the scalar field potential to be of a simpler form, namely symmetric double well

$$V(\phi) = \frac{1}{4}\lambda(\phi^2 - v^2)^2 \,, \tag{16}$$

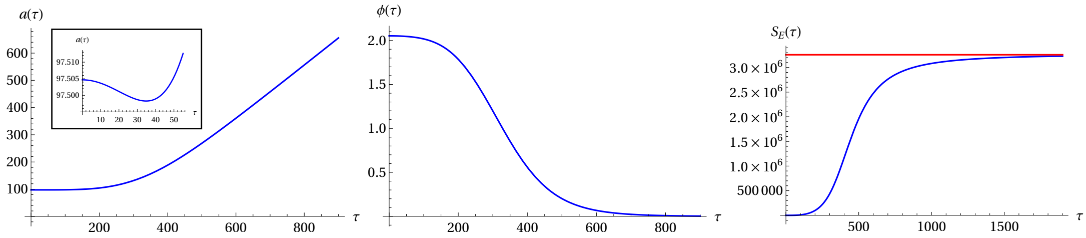


Figure 11: An example of a wormhole with large dilaton coupling, in this case $\beta = 2$. Shown are again the scale factor (left), dilaton (middle) and Euclidean action (right). The parameter values are $m = 0.01$, $\beta = 2$, $N = 73940$ and the initial dilaton value is $\phi_0 = 2.0522333714$.

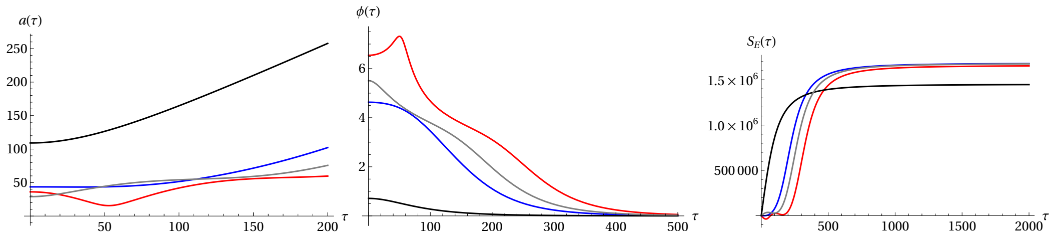


Figure 12: This figure compares GS-type and expanding wormholes, at the same parameter values $m^2 N = 3$ and dilaton coupling $\beta = 1.2$. The GS-type solutions are depicted by the black ($\phi_0 \approx 0.7$) and grey ($\phi_0 \approx 5.5$) lines. These solutions were already presented in Fig. 8. The blue ($\phi_0 \approx 4.6$) and red ($\phi_0 \approx 6.2$) curves correspond to expanding wormholes. Interestingly, the actions are seen to be quite close to each other, with the grey solution lying in between the two expanding wormhole solutions. It appears that overall the black GS-type solution is dominant, but a verification of this assertion would require an understanding of the infinite oscillation limit of expanding wormholes.

where λ is a dimensionless scalar field self-coupling and v is the vacuum expectation value. The self-coupling λ may be scaled to any convenient value using the rescalings detailed in [2]. With the simple arguments one can show that in such a potential there are no generalized GS solutions with non-trivial scalar field [2]. So, we concentrated on solutions leading to expanding baby universes. An example of such wormhole is shown in Fig. 13. One can clearly see that the scale factor starts to decrease first, reaches minimum value and starts increasing. So, corresponding wormhole indeed has wine-glass shape and leads to expanding baby universes after analytic continuation to Minkowski time.

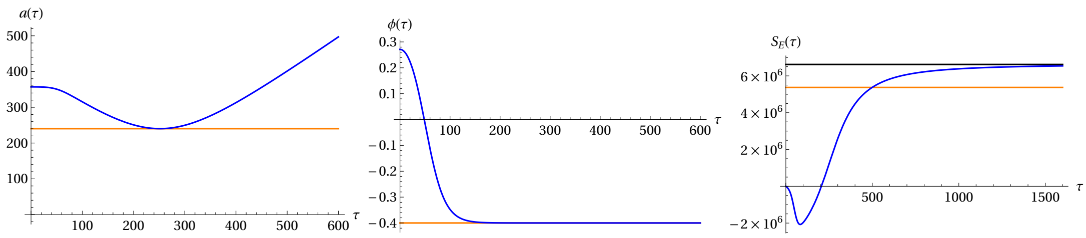


Figure 13: An example of an expanding wormhole supported by a large axionic charge, and a scalar field in a double well potential. The parameters used are $\lambda = 0.01$, $N = 100000$, $v = 0.4$. The initial scalar field value is $\phi_0 = 0.27112946714882599307$. The orange lines provide the GS wormhole values as reference.

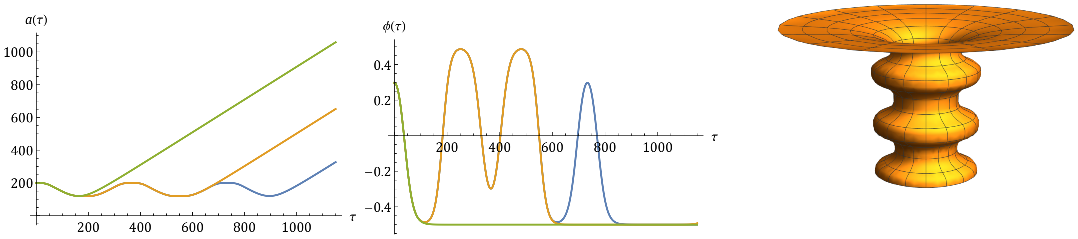


Figure 14: Comparison of solutions with one, two and three minima for the same theory parameters $N = 25000$, $\lambda = 0.01$ and $\nu = 0.5$. The solutions here are specified by initial scalar field values that lie very close to each other, respectively at ϕ_0 = 0.29769598017296931741454 0, 0.29753040978542151754655 8, 0.29753040964664825193709 1 (these solutions must be optimised to high accuracy in order to determine the action reliably).

2.5 Oscillating wormholes

We also studied oscillating wormhole solutions. It is interesting that whereas in case of metastable vacuum decay processes in oscillating solutions only scalar field is oscillating and geometry (scale factor) does not feel these oscillations, Fig. (3), here in the wormhole case both, scalar field and scale factor are oscillating, Fig. 14. Note that such oscillations of scale factor were first observed in [35]. Of course in order to clarify physical meaning of these oscillating wormholes and determine role which they play in Euclidean quantum gravity, one needs to study first their stability and determine the number of negative modes, which they possess.

3. Concluding remarks

• We discovered a whole zoo of Euclidean axionic wormholes in two different theories: axion gravity with massive dilaton and axion gravity with scalar field having a symmetric double well potential.

• Euclidean wormholes are very interesting and exciting, but the same time very confusing and obscure objects.

• Definitely more work needs to be done to understand deeply the role of wormholes in quantum gravity and their relevance to physical effects. Especially important is investigation of linear stability of these wormhole solutions in order to determine how many negative modes do they pocess and to which class of solutions they belong.

• My personal expectation is that the (lowest branch of) GS type wormholes are instanton like and have no negative modes whereas wormholes leading to expanding baby universes are of bounce like and have single negative mode.

4. N.B.

After presentation at the Rubakov's memory conference we continued our study of newly found wormhole solutions and analysed their linear stability [36]. It was striking when we once again faced negative mode problem - which was first noted in a different context with Valery almost 40 years ago [6]. Namely, for wormholes leading to expanding baby universes, the corresponding factor Q

was always negative somewhere or even singular, which prevented us from studying their stability. But at least in situations where the corresponding factor Q was positive everywhere we could arrive at reliable conclusions. This was the case for part of generalized GS wormholes. Namely, we find massless GS wormholes to always be perturbatively stable. In the phenomenologically more relevant case of a massive dilaton we find a wide variety of wormhole solutions, depending on the dilaton coupling and mass, and on the axion charge. We showed that the solutions with the smallest dilaton potential are perturbatively stable and dominant, even in cases where the wormhole solutions are not continuously connected to the massless case by decreasing the mass. For branches of solutions emanating from a bifurcation point, one side of the branch always contains a negative mode in its spectrum, showing that such solutions are unstable. The existence of classes of perturbatively stable wormhole solutions with massive dilaton sharpens the puzzles associated with Euclidean wormholes.

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