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Massive Gravitons in Very Special Relativity: Theory and Observations

Alessandro Santoni,[a,b,*] **Jorge Alfaro**[a] **and Alex Soto**[c]

[a] *Instituto de Física, Pontificia Universidad de Católica de Chile, Avenida Vicuña Mackenna 4860, Santiago, Chile*

[b] *Institut für Theoretische Physik and Atominstitut, Technische Universität Wien, Wiedner Hauptstrasse 8–10, A-1040 Vienna, Austria*

[c] *School of Mathematics, Statistics and Physics, Newcastle University, Newcastle upon Tyne, NE1 7RU, UK*

E-mail: asantoni@uc.cl, jalfaro@uc.cl, alex.soto@newcastle.ac.uk

Recently, a new linearized gravity model, Very Special Linear Gravity (VSLG), has been developed within the framework of Very Special Relativity (VSR). One of its peculiar feature is to mantain gauge invariance while introducing a mass m_g for the gravitational degrees of freedom. Therefore, the consistency of VSLG has been put to test by calculating the gravitational radiation produced by a keplerian binary system, exploiting Effective Field Theory's techniques. The resulting energy loss and period decay rate show a continous limit to the General Relativity result when sending $m_g \to 0$. Finally, we were able to constrain the VSLG mass parameter by using astronomical data from binary pulsars. The obtained upperbound of $m_g \sim 10^{-21} eV$ is still slightly weaker than the kinematical one obtained by the GW170817 detection, which should still hold in VSLG.

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1. Introduction

Since the pioneering work of Fierz and Pauli in 1939 [1], where they presented for the first time a lagrangian for a massive spin-2 particle (analogous to Proca theory for the photon), massive gravity theories have had their ups and downs. In fact, it was later discovered that a discontinuity with General Relativity (GR) emerged in the massless limit [2, 3], due to the presence of additional degrees of freedom (d.o.f.) with ghost-like behaviour [4]. A first step to the solution of those problems came with the discovery of screening mechanisms, like the Vainshtein one [5], which have afterwards been implemented in ghost-free models like DGP [6, 7] and dRGT [8]. For more details on this topic see [9] and references therein.
While all these models involve breaking GR gauge invariance to allow a mass term for gravitons, recently an alternative proposal, named Very Special Linear Gravity (VSLG) and based on Lorentz violation, has been put forward [17]: in particular VSLG is a linearized massive gravity model constructed in the framework of Very Special Relativity (VSR), a peculiar Lorentz-violating theory presented for the first time by Cohen and Glashow in 2006 [10], to propose an alternative mechanism for neutrino masses [11]. Interestingly enough, VSR effects are directly related to the breaking of discrete symmetries, like CP. Therefore, they could become relevant in the gravitational context since, due to Sakharov conditions, these symmetries must be violated in cosmology [12].
The fundamental and unique feature of VSR is that, by reducing the spacetime symmetry group to $SIM(2)$ [10], it allows a preferred spacetime null direction. Labeling this direction with the lightlike fourvector n^μ, the main consequence is that we can now assemble ratios of scalar products involving n^μ both in the numerator and denominator, which are invariant under the subgroup $SIM(2)$ but not under the full Lorentz group. Thus, in VSR we are generally able to build additional terms for the usual field Lagrangians. Obviously that freedom comes at the price of Lorentz invariance and non-locality of the novel operators.

2. Very Special Linear Gravity

Let's quickly deep dive in the VSLG model: the fundamental building blocks of the quadratic Lagrangian of the spin-2 $h_{\mu\nu}$−field

$$\mathcal{L}_g = \frac{1}{2} h^{\mu\nu} O_{\mu\nu\alpha\beta} h^{\alpha\beta} \,, \tag{1}$$

are the Minkowski tensor $\eta_{\mu\nu}$, the derivative ∂_μ and the VSR peculiar operator $N^\mu = \frac{n^\mu}{n\cdot\partial}$. The exact expression of the quadratic operator $O_{\mu\nu\alpha\beta}$, included in [17, 18], is lenghty and intricated. However, at the moment of fixing a suited gauge, the equations of motion (e.o.m) simply becomes

$$(\partial^2 + m_g^2)\, h_{\mu\nu} = 0 \,, \tag{2}$$

where m_g is the only new parameter introduced by VSLG and can be seen to play the role of a graviton mass. Furthermore, it can be rigorously proved that no new degree of freedom are present in VSLG, apart from the two of GR, which now adquire a mass m_g. For more details on this, refere to [17]. Let us also stress that the addition of a mass to the gravitational d.o.f. would be of crucial relevance in many astrophysical and cosmological contexts, like for example the accelerated expansion of the Universe and dark matter [13].

3. Energy loss rate and Period decrease

The first step to calculate the rate of period decrease rate $\dot{P}$ due to gravitational waves (GW) emission in VSLG is to couple the gravitational d.o.f. to the energy-momentum tensor (EMT) $T_{\mu\nu}$ of the classical source, which in our case is a binary system. Since we are working in a linear framework and we want to preserve gauge invariance, the only term we can write for this purpose is

$$\mathcal{L}_{int} = -\frac{\kappa}{2} T_{\mu\nu}\, h^{\mu\nu}\,, \quad \kappa = \sqrt{32\pi G}\,. \tag{3}$$

Within an Effective field theory approach [19, 20], we can interpret the above Lagrangian as a vertex for our theory and, therefore, calculate the infinitesimal emission probability $d\Sigma$ of a GW as

$$d\Sigma = \frac{\kappa^2}{8(2\pi)^3} \sum_\lambda |\,\tilde{T}_{\mu\nu}\, \epsilon_\lambda^{\mu\nu}|^2\, \frac{d^3k}{\omega}\,, \tag{4}$$

where $\tilde{T}_{\mu\nu}$ is the momentum space version of the EMT, $\epsilon_\lambda^{\mu\nu}$ is the graviton polarization tensor and $\lambda = 1, 2$ runs over the two physical polarization states in VSLG. Thus, defining the polarization sum tensor as $S^{\mu\nu\alpha\beta}(k) = \sum_\lambda \epsilon_\lambda^{\mu\nu}(k)\epsilon_\lambda^{*\,\alpha\beta}(k)$, the total energy loss rate $\dot{E}$ for such a process will be

$$\dot{E} = \frac{dE}{dt} = \int \frac{\omega}{T} d\Sigma = \frac{\kappa^2}{8(2\pi)^3 T} \int \rho(\omega)\, \omega^2\, \tilde{T}^*_{\mu\nu}\, \tilde{T}_{\alpha\beta} S^{\mu\nu\alpha\beta}\, d\omega\, d\Omega_k\,. \tag{5}$$

There are many technicalities involved with the calculation of the above quantity, starting from the calculation of the polarization sum to the realization of the angular integral. Many of these complications, however, get simplified when considering both the conservation equation for the EMT and its (quasi-)periodicity. All the needed details can be found in [18].

3.1 Energy-Momentum Tensor of Binaries

Let's spend a few words on the EMT of a binary system. Working in the non-relativistic limit, we can make use of the known EMT's formula for a keplerian binary of total mass $M = m_1 + m_2$

$$T^{\mu\nu}(t, \vec{x}) = \mu\, U^\mu U^\nu \delta^3(\vec{x} - \vec{r}(t))\,, \tag{6}$$

where $\vec{r}(t)$ and $U^\mu = (1, \dot{r}_x, \dot{r}_y, 0)$ are the trajectory and non-relativistic four-velocity in the $x - y$ orbital plane of the reduced mass $\mu = m_1 m_2/M$. Defining b and e as the semi-major axis and the eccentricity of the orbit, we parametrize the motion in function of the eccentric anomaly $\phi(t)$ [21]

$$\vec{r}(t) = b\left(\cos\phi - e, \sqrt{1-e^2}\sin\phi, 0\right)\,, \quad \Omega\, t = \phi - e\sin\phi\,, \quad \Omega = \sqrt{\frac{GM}{b^3}}\,, \tag{7}$$

with Ω representing the fundalmental frequency defined by the revolution period $P_b \rightarrow \Omega = 2\pi/P_b$.

3.2 Period Time Derivative

In keplerian binaries, the energy loss and period decrease rates are related in the following way

$$\dot{P} = -6\pi \frac{b^{\frac{5}{2}} G^{-\frac{3}{2}}}{m_1 m_2 \sqrt{m_1 + m_2}} \dot{E}\,. \tag{8}$$

Therefore, after many computations and manipulations, the rate of period decrease in VSR can be rewritten in the following (experimentally) convenient form

$$\dot{P}_{VSR} = -\frac{192\pi T_{\odot}^{\frac{5}{3}}}{5} \frac{\tilde{m}_1 \tilde{m}_2}{\tilde{M}^{\frac{1}{3}}} \left(\frac{P_b}{2\pi}\right)^{-\frac{5}{3}} \sum_{N_{min}} f(N, e, \delta, \hat{n}), \tag{9}$$

with $\delta \equiv \frac{m_g}{\Omega}$, $T_{\odot} \equiv GM_{\odot}/c^3 = 4.925490947\,\mu s$ [22] and the “tilde"-masses defined as $\tilde{m} \equiv m/M_{\odot}$, with $M_{\odot}$ being the solar mass. The functions $f(N, e, \delta, \hat{n})$ [18], in general, are cumbersome combinations of the Bessel functions J_N depending on the VSR mass and preferred direction $\hat{n}$. We do not include here their expressions for reasons of space. However, an important property is that they recover the GR limit [23] when $m_g \to 0$

$$\lim_{\delta \to 0} \sum_{N} f(N, e, \delta, \hat{n}) = (1 - e^2)^{-\frac{7}{2}}(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4)\,. \tag{10}$$

4. Upperbounds for VSLG mass

Finally, we use all the previous results and data from two of the most well-studied pulsar binaries: the Hulse-Taylor binary PSR B1913+16, the first binary pulsar ever discovered [24], and the Double Pulsar PSR J0737-3039A/B [25], to experimentally constrain the VSLG mass parameter, restricting ourself to the case $\hat{n}//\hat{z}$, which produce the largest contribution possible in our case.
To estimate the m_g−upperlimit is sufficient to increase the value of m_g to the point where we saturate the maximal discrepancy allowed by a 95% confidence interval around the experimental rate of period decrease $\dot{P}_{exp}$ due to GW emission.. See Fig.1 for a graphical representation.

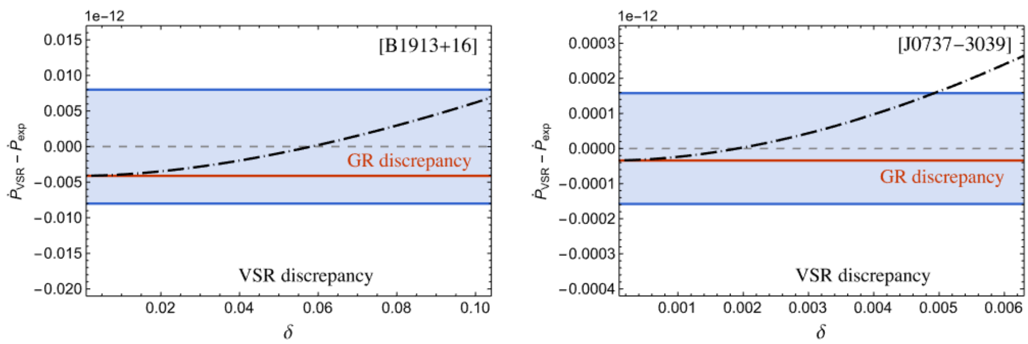

Figure 1: Period decrease rate predicted by GR and VSR for the Hulse-Taylor and the Double Pulsar. The measured value is used here as the offset. The light-blue band represents the experimentally allowed region.

For the binary pulsars took into consideration, we have found an upperbound of $m_g \sim 10^{-21} eV$, which is almost as strong as the kinematical bound from GW170817 [26]. The values we obtained are comparable to other graviton mass upperlimits from binary systems found in literature, like [20, 27, 28]. For some particular models, e.g. [29], much stronger bounds are obtained due to the additional presence of dipolar modes. In conclusion, we hope to improve the constraints on VSLG obtained here not only by increasing experimental precision, but also exploiting direct GW detections, for which further study in the VSLG framework should be carried on.

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