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Axion Dark Matter Simulations with Adaptive Mesh Refinement

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In this contribution, we give a brief overview for the current state-of-the-art of axion dark matter simulations in the post-inflationary scenario, which allow for a precise prediction of the axion dark matter mass.
The study of post-inflationary axions poses severe computational challenges due to the presence of large hierarchies in the network of topological defects, i.e. strings and domain walls, which inevitably forms if the Peccei-Quinn (PQ) symmetry is broken after inflation. This leads to strong limitations for the potential reach of such simulations, and predictions for the axion dark matter mass usually involve extrapolations over several orders of magnitude in the string tension.
We comment on different approaches to increase the dynamical range of the simulations, with a focus on Adaptive Mesh Refinement (AMR) and Effective Models that allow us to simulate networks at higher string tensions.
As an outlook, we briefly present some details of our current comprehensive study on the radiation emitted by isolated global axion string loops.

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1. Introduction

The QCD axion is amongst the most prominent candidates to explain dark matter in the Universe, with a plethora of ongoing research activities, both on the theory and experimental side, cf. [1, 2]. Originally proposed as a consequence of the dynamical Peccei-Quinn (PQ) solution to the *strong-CP problem* of QCD [3], axions are gaining increased attention due to their distinctive features, namely their exceptionally low masses and the feebleness of their interactions with SM particles. This results from values of $f_a > 10^7$ GeV (for the high energy scale of the PQ symmetry breaking), motivated by the absence of their observation to date. A central open question that requires theory input is an exact prediction of the axion mass, which, even accounting for all currently available constraints and bounds from astrophysics and cosmology, spans more than ten orders of magnitude. Accurate predictions of the final axion dark matter abundance, $\Omega_a = \Omega(m_a)$, together with the assumption that axions constitute the entire dark matter, enable tuning haloscope experiments to a specific, unique mass value, allowing for a drastic speed up for experimental detection.

Such predictions for the axion mass can only be realised in certain scenarios. For the case where the PQ symmetry is broken *after* inflation, the initial axion field values θ_i cannot be coherent beyond the causal horizon H^{-1}. This leads to the formation of a network of global cosmic strings as explained by the Kibble mechanism during the PQ phase transition [4]. The strings form loops that shrink and intersect, resulting in a *scaling* regime, with a string length per Hubble volume of $\xi \sim O(1)$. Around the QCD phase transition, nonperturbative effects (instantons) become important and the strings are forced to annihilate in the presence of ($N_{\rm DW} = 1$) domain walls.

Resolving the complex dynamics of the string network is highly intricate and dedicated numerical simulations are difficult to realise due to the large hierarchy of scales in the system. This manifests in the energy (per unit length) stored in the strings, $\mu \sim \pi f_a^2 \log(f_A/H)$, which is logarithmically corrected by $\kappa \equiv \log(f_A/H) \approx \log(10^{30}) \approx 70$. This energy is partially released into axions, providing an additional production mechanism next to the standard *misalignment* case.

Up-to-date numerical simulations, such as performed with the `jaxions`[1] code, can use a maximum of $N \lesssim 10^4$ grid points, effectively only reaching smaller values of f_a. Predictions of the final axion abundance rely on extrapolations from $\kappa \lesssim 10$ to 70 [5–7]. Usually, two different approaches are distinguished. In *direct* simulations, one evolves the full field equations and determines the axion yield at the end of the simulation, which then gets extrapolated to the cosmological value of $\kappa \approx 70$. The direct simulations from Ref. [8] suggest $m_a \simeq 17\mu$eV. In *indirect* simulations one models the axion emission from strings by extrapolating the instantaneous emission spectrum $\mathcal{F} \sim k^{-q}$, with a strong dependence on the *spectral index* q, whose precise value is a subject of an ongoing debate within the community. Depending on the concrete value of q, the mass predictions range from $m_a \simeq 110 \pm 70\ \mu$eV with $q \simeq 1$ [9] to $m_a \geq 500\ \mu$eV with $q \gg 1$ [5]. In order to extend the dynamical range of the simulations, besides the obvious "brute force" way of simply running bigger simulations on more powerful HPC infrastructures, the two most promising approaches are *Adaptive Mesh Refinement* (AMR, cf. next section) and *Effective Models*, such as the one developed by Klaer and Moore [10]. They realise higher string tensions by adding new degrees of freedom to the UV theory, one additional $U(1)$ gauge field A_μ and two complex scalar fields $\phi_{1,2} = |\phi_{1,2}|e^{i\theta_{1,2}}$ with charges $q_{1,2}$ under the gauge field. The cosmic strings then acquire a

[1]Publicly available on GitHub: https://github.com/veintemillas/jaxions

global axion charge, but with an Abelian-Higgs-type core, rendering the string tension effectively tunable as a function of $O(1)$ discrete charges,

$$\kappa \simeq \frac{\pi\left(v_1^2+v_2^2\right)}{\pi f_a^2} = \frac{\left(v_1^2+v_2^2\right)\left(q_1^2 v_1^2 + q_2^2 v_2^2\right)}{v_1^2 v_2^2} \overset{(v_1=v_2)}{=} 2\left(q_1^2+q_2^2\right). \tag{1}$$

Simulations for this concrete model predict $m_a \simeq 26\ \mu$eV and contribute valuable insight into the dynamics of string networks at high tension.

2. Potential Improvements with Adaptive Mesh Refinement

The idea of AMR is to focus the available computational power around the strings by dynamically adding ℓ new grids/levels with finer resolution ($N_\ell/N_0 = r^\ell$) in regions that satisfy specific refinement criteria. These techniques have recently gained attention in the context of axion string simulations, and first studies of individual strings [11] and networks [9] have demonstrated their huge potential. The network simulations reached values of $\kappa \sim 10$, with much smaller grids, $N_0 = 2048$ with $\ell \leq 5$, than for the best currently available static-grid string network simulations, $N \simeq 10^4$ [7]. We are currently performing AMR simulations using `axionyx` [12], which has an interface with `jaxions` and therefore provides valuable input for optimising the AMR routines.
To quantify the potential improvement for string network simulations, we can estimate the needed RAM to perform a `jaxions`/complex scalar simulation with AMR as

$$\text{RAM} = 2\times 2\times 4 \text{ bytes } \times \left(N_0^3 + \frac{\pi n_c n_r^2}{4}\frac{r^\ell - 1}{r-1}N_p\right), \tag{2}$$

if we refine only around the strings, evaluate the string length ξ as a function of time and multiply it by the amount of RAM per unit length. Here, $N_p = \xi \times 6\,(L/(N_0\tau))^2 \times N_0^3$, is the number of string plaquettes pieced by strings in the base grid, following the algorithm of [13]. Additionally, $n_r = 2^3$ is the diameter around the string that needs to be refined and $n_c = r^3$ is the number of new cells in each refined cell[2]. This results in values of log $\approx 13, 16, 18$ for base grids with $N_0 = 2048, 4096, 8192$ and $\ell = 9, 11, 13$ for $r = 2$, for a total estimated improvement of $O(2)$.

3. Towards the Continuum Limit of Global Axion String Loop Decays

In order to gain more insights about the constituents of the string network, we are currently studying the dynamics of isolated global string loops. This is realised by making use of a duality between the axion field θ and the antisymmetric Kalb-Ramond 3-form, $f_A \partial_\mu \theta = \frac{1}{6}\epsilon_{\mu\nu\alpha\beta}F^{\nu\alpha\beta}$, as exemplified in [13]. This allows us to compute the axion field around arbitrarily shaped string configurations, in close analogy to the computation of the magnetic field around a current-carrying conductor via the Biot-Savard law. Numerically, this is realised by constructing the string initial conditions as a collection of points and computing the axion field for this specific configuration by integrating over the respective links in the whole coordinate plane,

$$\theta_{\mathbf{x}+\mathrm{d}x} - \theta_{\mathbf{x}} = \int_x^{x+\mathrm{d}x} \mathrm{d}^3\mathbf{x}\cdot\nabla\theta = -\frac{1}{2}\int_x^{x+\mathrm{d}x}\mathrm{d}^3\mathbf{x}\cdot\int \mathrm{d}\sigma \frac{(\mathbf{x}-\mathbf{X}(\sigma))\times\mathbf{X}'}{|\mathbf{x}-\mathbf{X}(\sigma)|^3}. \tag{3}$$

[2]Additional considerations here include for example RAM balancing between the base and the refined grids, which suggests a time-dependent number of refinement levels, i. e. $\ell + 7 \simeq \log_2\left(N_0^3/(\pi N_p)\right) = \log_2\left(N_0^2\tau^2/(\pi 6\xi L^2)\right)$.

More details about the explicit derivation will be provided in [14]. We perform various static-grid and AMR simulations of different characteristic loop shapes, e.g. kinks and long strings, and investigate their continuum limit by simultaneously decreasing volume and discretisation effects. For the exemplary case of a circular loop with radius R in the simulation volume L^3, these effects can be tuned by decreasing the ratio R/L, while increasing the resolution of the simulation, controlled by $m_s a = m_s L/N$, the parameter that controls the resolution of the string cores $\sim m_s^{-1}$. The ultimate goal of this study is the determination of the axion emission for all kinds of strings in order to gain insight about their respective contribution to the full network spectrum.

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References

[1] C. B. Adams et al., *Snowmass 2021*, *FERMILAB-CONF-22-996-PPD-T* (2022)

[2] I. G. Irastorza, J. Redondo, *Prog. Part. Nucl. Phys.* **102**, 89–159 (2018)

[3] R. Peccei, H. R. Quinn, *Phys. Rev.* **D16**, 1791–1797, *Phys. Rev. Letter* **38**, 1440–1443 (1977)

[4] Kibble, T. W. B., *Phys. Rept.* **67**, 183 (1980)

[5] M. Gorghetto, E. Hardy, G. Villadoro, *JHEP* **07**, 151 (2018), *SciPost Phys.* **10**, 050 (2021)

[6] M. Buschmann, J. W. Foster, B. R. Safdi, *Phys. Rev. Lett.* **124**, 161103 (2020)

[7] J. Redondo, K. Saikawa, A. Vaquero, *In preparation*, (2024)

[8] A. Vaquero, J. Redondo, J. Stadler, *JCAP* **04**, 012 (2019)

[9] M. Buschmann et al., *Nature Commun.* **13**, 1049 (2022)

[10] V. B. Klaer, G. D. Moore, *JCAP* **10**, 043 (2017), *JCAP* **11**, 049 (2017), *JCAP* **06**, 021 (2020)

[11] A. Drew, E. P. S. Shellard, *Phys. Rev. D* **105**, 063517 (2022), *Phys. Rev. D* **107**, 043507 (2023)

[12] B. Schwabe et al., *Phys. Rev. D* **102**, 083518 (2020)

[13] L. Fleury, G. D. Moore, *JCAP* **01**, 004 (2016)

[14] M. Kaltschmidt, J. Redondo, *In preparation*, (2024)