PROCEEDINGS OF SCIENCE

Formation of ultralight dark matter solar halos

Marco Gorghetto[a,b,*]

[a] *Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Herzl St 234, Rehovot 761001, Israel*

[b] *Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany*

E-mail: marco.gorghetto@desy.de

This short contribution summarizes some recent results on the formation of ultra-light dark matter halos around the Sun and other massive astrophysical objects. These halos, resembling gravitational atoms, are formed quite generically via the capture of light scalar dark matter, mediated by its (weak) self-interactions. The capture process is effective whenever the dark matter waves in the galactic halo are gravitationally focused by an external gravitational potential. One of our most striking results is that for a dark matter boson with mass of order 10^{-14} eV, a halo around the Sun can form on a timescale comparable to the lifetime of the Solar System, with a density at the position of the Earth $O(10^4)$ times larger than that predicted in the standard galactic halo model. If the self-interactions are attractive, the halo collapses when its density is large, and this is likely to be associated with the emission of relativistic bosons, a 'Bosenova'.

1st General Meeting and 1st Training School of the COST Action COSMIC WSIPers (COSMICWISPers)
5-14 September, 2023
Bari and Lecce, Italy

*Speaker

PoS(COSMICWISPers)024

1. Introduction

New ultralight bosons (i.e. with mass $m \lesssim 1$ eV) are compelling new physics candidates because they can comprise the dark matter (DM) of our Universe, explain other open questions of the Standard Model (SM), and are generically predicted by String Theory [1–12]. Owing to their small mass, these DM particles have a macroscopic occupation number in galaxies such as the Milky Way. Their evolution is thus well approximated by their classical equations of motion, whose free solutions are waves. The parameters usually assumed in searches for these DM particles, derived from observations of our galaxy at large scales, are an energy density $\rho_{\rm dm} \simeq 0.4\,{\rm GeV/cm^3}$ and velocity $v_{\rm dm} \simeq 240\,{\rm km/s}$ [13–18]. For ultralight DM (ULDM), the coherence time is also important for the theoretical interpretation of experimental results [19–21]. However, overdensities at scales much smaller than the galaxy can modify these expectations. These can take the form of compact gravitationally bound objects, which can be either self-gravitating, e.g. 'boson stars' [22–30], or supported by external baryonic potentials. e.g. 'solar halos' [31, 32].

Following my recent work [33], in this contribution I will show that there is a generic class of theories where ULDM capture processes do become important in the background of the gravitational potential of baryonic sources. This capture leads to the formation of dark matter solar halos with density much larger than $\rho_{\rm dm}$ and a possibly modified coherence time. As I will discuss in section 2, the capture efficiency is determined by the amount of gravitational focusing of the galactic DM waves. I refer to [34–41] for other interesting observational effects of gravitational focusing.

2. Gravitational focusing and dark matter capture

The ULDM around an (approximately point-like) astrophysical object of mass M, e.g. the Sun, admits bound state configurations corresponding to a gravitational atom, with ground state radius

$$R_\star = \frac{1}{m\alpha} = 1\,{\rm AU}\left[\frac{1.3\cdot 10^{-14}\,{\rm eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right], \tag{1}$$

where $\alpha \equiv GMm$ is the gravitational coupling of the DM to the body and $M_\odot$ the solar mass. On the other hand, the DM in the galactic halo has a mean velocity $\mathbf{v}_{\rm dm}$ in the rest frame of the Sun and variance $\sigma^2 \simeq v_{\rm dm}^2/2$. This can be thought of as all of the ULDM being the continuum (unbound states of the atom). However, bound states do get populated from the DM in the continuum via processes mediated by the quartic self-interactions, which I write as $V \supset \lambda\phi^4/4!$, where ϕ is the ULDM field and λ the quartic coupling. Indeed the mass $M_\star$ of the ULDM bound to the Sun changes as [33]

$$\dot{M}_\star = C + (\Gamma_1 - \Gamma_2)M_\star\,, \tag{2}$$

where $C, \Gamma_1, \Gamma_2 \propto \lambda^2$ are positive coefficients and arise from the self-interactions.

The first contribution to $\dot{M}_\star$, which I call 'capture', is interpreted as arising from a $2 \to 2$ process where two unbound particles scatter into a bound-state particle and an unbound one. The second, 'stimulated capture', stands for the same process, but is proportional to $M_\star$ as a consequence of the Bose enhancement of the indistinguishable bosons. The last represents the reduction of the bound state population via the inverse 'stripping' process.

Significant DM capture happens in the regime where the stimulated capture rate Γ_1 exceeds the stripping rate Γ_2. The main point is that this occurs when the DM waves in the galactic halo are

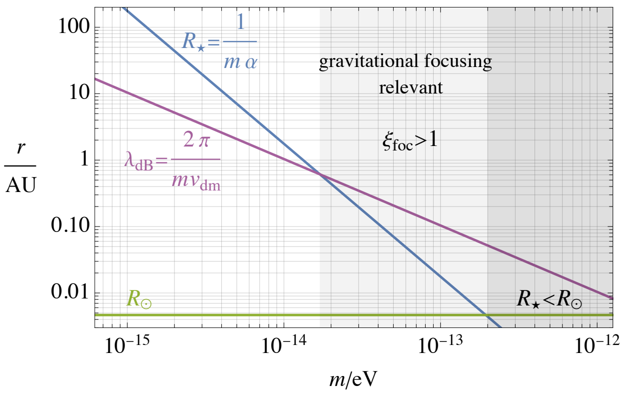

Figure 1: The typical de Broglie wavelength of the DM waves in the galactic halo, $\lambda_{\rm dB} = 2\pi/mv_{\rm dm}$ for $v_{\rm dm} \simeq 240$km/s, and the radius $R_\star$ of the ground state, in our Solar System for varying m. In the regime $\lambda_{\rm dB} > R_\star$, occurring for $m \gtrsim 1.7 \cdot 10^{-14}$ eV, gravitational focusing is relevant. Picture from Ref. [33].

gravitationally focused by the external body, i.e. if

$$\xi_{\rm foc} \equiv \frac{\lambda_{\rm dB}}{R_\star} = \frac{2\pi\alpha}{v_{\rm dm}} \simeq \left[\frac{m}{1.7\times 10^{-14}\,{\rm eV}}\right]\left[\frac{M}{M_\odot}\right]\left[\frac{240\,{\rm km/s}}{v_{\rm dm}}\right] \tag{3}$$

is larger than unity. In particular, if $\lambda_{\rm dB} < R_\star$, corresponding to $v_{\rm dm} > 2\pi\alpha$, the incoming particles are fast with respect to $2\pi\alpha$ and the gravitational potential is negligible; in this regime, their kinetic energy $mv_{\rm dm}^2/2$ exceeds the binding energy of the ground state, $-m\alpha^2/2$, so capture is inefficient ($\Gamma_1 < \Gamma_2$). Instead, if $\lambda_{\rm dB} > R_\star$, the waves are gravitationally focused as their dynamics close to the Sun is dominated by the Sun itself. In this regime, the kinetic energy of the corresponding particles is small enough that order-one energy changes – from particles scattering via the self-interactions – have a chance of trapping them in the gravitational well. Additionally, the particles in the galactic halo are not energetic enough to strip out a particle in the ground state, without getting themselves captured. Stripping is thus suppressed compared to stimulated capture ($\Gamma_1 > \Gamma_2$). Figure 1 compares $\lambda_{\rm dB}$ and $R_\star$ for the Sun a function of m. From Eq. (3), $\xi_{\rm foc} \gtrsim 1$ for $m \gtrsim 10^{-14}$ eV, for which, importantly for observations, the ground state radius is of order AU or smaller; see Eq. (1).

3. Formation stages and dark matter overdensity

Depending on $\Gamma_1 > \Gamma_2$ or $\Gamma_2 < \Gamma_1$ the atom's evolution undergoes different phases, according to Eq. (2). Their dynamical time-scale is similar to the *relaxation time* via self-interactions

$$\tau_{\rm rel} \equiv \frac{64 m^7 v_{\rm dm}^2}{\lambda^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr}\left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4\left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3\left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2\left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2, \tag{4}$$

where I wrote $\lambda \equiv -m^2/f_a^2$ valid for an axion with decay constant f_a. $\tau_{\rm rel}$ is the time a particle in a gas takes to change its velocity by order one via the self-interactions, in the absence of external potentials [42, 43].

The bound mass $M_\star$ initially increases linearly as a result of direct capture only, see Eq. (2), until $t \simeq 1/|\Gamma_1 - \Gamma_2|$, at which point the stimulated capture/stripping terms become relevant. After this time, for $\xi_{\rm foc} \gtrsim 1$ the bound mass increases exponentially because of the dominance of stimulated

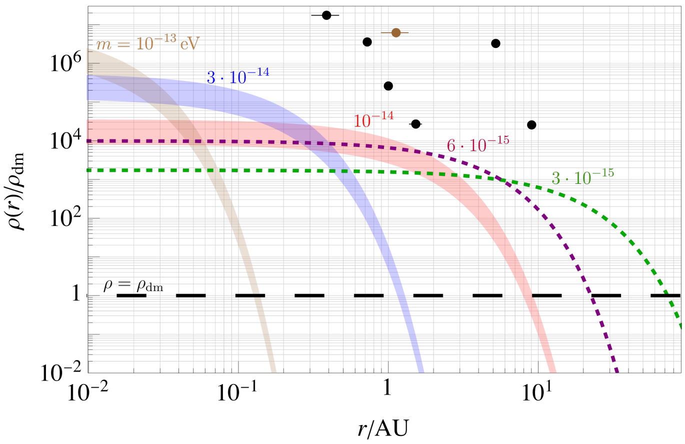

Figure 2: The overdensity profile of the dense solar halo $\rho(r)/\rho_{\rm dm}$, for different values of m. In the shaded regions the values of the DM velocity and dispersion are varied from $v_{\rm dm} = \sqrt{2}\sigma = 240$ km/s to 50 km/s, with the lower edge corresponding to the largest velocity in this range. Dashed lines correspond to the profile for $m < 10^{-14}$ eV, for which the exponential growth of the bound state occurs only when $v_{\rm dm} \ll 240$ km/s (and results for $v_{\rm dm} = 50$ km/s are shown). A large overdensity at the position of the Earth, as well as within the Earth's orbit, is predicted. Picture from Ref. [33].

capture over stripping, with an exponential rate $1/\Gamma_1 \simeq 0.3\tau_{\rm rel}$ similar to the relaxation time. This leads to the formation of a 'dense' gravitational atom. Instead, for $\xi_{\rm foc} \lesssim 1$ the capture and stripping processes reach equilibrium (much after $\tau_{\rm rel}$) resulting in a constant bound mass, leading to a 'dilute' atom with density at most a few percent of the average DM density $\rho_{\rm dm}$. In the Solar System, for $v_{\rm dm} \simeq 240$ km/s the exponential increase happens for 10^{-14} eV $\lesssim m \lesssim 2 \cdot 10^{-13}$ eV. The upper limit correspond to the smallest possible atom, with $R_\star = R_\odot$, see Figure 1.

The exponential growth of the dense atoms terminates when the typical bound state density approaches the critical density

$$\rho_{\rm crit} \simeq 16\frac{\alpha^2 m^4}{|\lambda|} \simeq 7 \cdot 10^3 \rho_{\rm dm} \left[\frac{f_a}{5\cdot 10^7\,{\rm GeV}}\right]^2 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^4 \left[\frac{M}{M_\odot}\right]^2 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right] . \tag{5}$$

At this point the self-interaction energy is similar to the gravitational potential energy. $\rho_{\rm crit}$ can be much larger than the background density. For attractive self-interactions, $\lambda < 0$, the atom is unstable and collapses. After the collapse starts, higher-order self-interaction terms become important and for an axion-like potential the bound state should experience a *Bosenova* explosion, radiating an order-one fraction of its mass into relativistic particles via $3 \to 1$ processes (similarly to a boson star [44–46]). For $\lambda > 0$, the density saturates to at least $\rho_{\rm crit}$ and there is no collapse. Note that $|\lambda|$ only sets the capture timescale via the relaxation time and $\rho_{\rm crit}$.

In Figure 2, I show the density profile of the dense solar halo, $\rho(r) \propto e^{-2r/R_\star}$, for different m in the range 10^{-13} eV to 10^{-15} eV. This is shown at the final stages of the exponential increase, when $\rho_{\rm crit}$ is reached, choosing λ for each line such that $\rho_{\rm crit}$ is reached at 5 Gyr. The values of λ used, of order $10^{-57} \div 10^{-61}$, for an axion correspond to f_a within the range 10^7 GeV to 10^8 GeV. Over the bands in Figure 2 the velocity is varied from $v_{\rm dm} = \sqrt{2}\sigma = 240$ km/s to 50 km/s; the lower value could arise if the DM is in a dark disk, with smaller average velocity and dispersion [41, 47, 48]. For comparison, black dots show the constraints on the maximum mass that can be bound to the

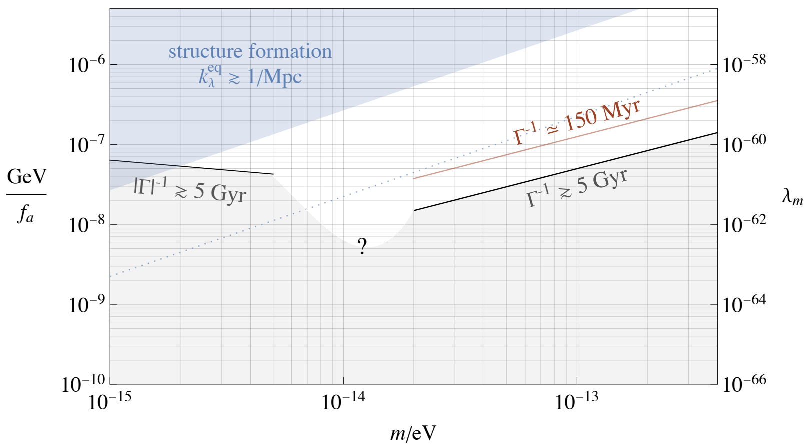

Figure 3: In blue, an approximate constraint on the size of the self-interaction coupling λ (written in terms of f_a) from the matter power spectrum at largest measured momenta, $k \simeq 1\,\mathrm{Mpc}^{-1}$. This is valid both for $\lambda > 0$ and $\lambda < 0$. The dotted line is an indication of an upper bound that could be set on $\lambda < 0$ from the matter power spectrum due to the exponential enhancement of the perturbations during radiation domination. Black lines show the values of m and λ for which the typical formation time $1/|\Gamma_1 - \Gamma_2|$ in the Solar System is 5 Gyr or 150 Myr. I also show the corresponding value of λ parameterized by $\lambda_m \equiv |\lambda|(10^{-14}\,\mathrm{eV}/m)^2$.

Sun from Solar System ephemerides [31, 49]. Clearly, the density in the solar halo is orders of magnitude larger than $\rho_{\rm dm}$, but it does not violate these direct constraints.

4. Bounds and impact on structure formation

As illustrated by Eq. (4), sufficiently large ULDM self-interactions are needed for the halo to form over Gyr time scales, but they also modify the cosmological evolution of DM perturbations. Attractive/repulsive self-interactions enhance/suppress the perturbation growth and can be thus constrained by measurements of the matter power spectrum [50–52]. As shown in Figure 3, taken from [33], the values of λ required for the dense halos to form within 5 Gyr (those above the black lines) are consistent with these constraints (shown in blue) over a few orders of magnitude. The constraints from structure formation may become considerably stronger for $\lambda < 0$ if one considers the evolution of the field before matter-radiation equality, due to the exponential enhancement of perturbations during radiation domination [33]. Note that the misalignment mechanism for a conventional axion potential underproduces the DM abundance for the considered values of m and f_a, but different production mechanisms or scalar potentials may provide the observed abundance.

5. Summary and Outlook

In this contribution I presented a generic mechanism of formation of ULDM halos around massive astrophysical objects, arising from the effect of the weak quartic self-interactions of the ULDM. This mechanism occurs also around our Sun, leading to a solar halo with a density orders of magnitude larger than the background DM density at the position of the Earth, see Figure 2. More generally, the formation of halos around astrophysical objects in our Galaxy could provide novel ULDM signatures.

The most striking effect happens in the exponential-growth regime, when gravitational focusing is relevant ($\xi_{\rm foc} \gtrsim 1$), corresponding to $m \gtrsim 10^{-14}$ eV, which predicts a large overdensity at $r \lesssim$ AU, see Figure 2. This could enhance the ULDM detection prospects on Earth and is also a clear target for the proposed searches in space [53]. For attractive self-interactions, the halo is expected to collapse, triggering a Bosenova process and the corresponding emission of relativistic particles. This burst of light bosons can be detectable on Earth. Similarly, Bosenovas from nearby stars can provide novel signals. Finally, the effective coherence time of the DM in the solar halo could be larger than that of virialized DM even for $\xi_{\rm foc} \lesssim 1$ when the solar halo is underdense [33], possibly improving the detection prospects in resonance experiments looking for direct ULDM couplings to the SM.

Acknowledgements. I am grateful the organizers of the 1st General Meeting and 1st Training School of the COST Action COSMIC WSIPers for the nice invitation to this plenary talk. I also thank Dima Budker, Josh Eby, Minyuan Jiang and Gilad Perez for the fruitful collaboration on the topics I presented. This article is based upon work from the COST Action COSMIC WISPers CA21106, supported by COST (European Cooperation in Science and Technology). This work is supported by the Alexander von Humboldt foundation and has been partially funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - 491245950.

References

[1] R. Peccei and H. R. Quinn, *CP Conservation in the Presence of Instantons*, *Phys. Rev. Lett.* **38** (1977) 1440–1443.

[2] S. Weinberg, *A New Light Boson?*, *Phys. Rev. Lett.* **40** (1978) 223–226.

[3] F. Wilczek, *Problem of Strong P and T Invariance in the Presence of Instantons*, *Phys. Rev. Lett.* **40** (1978) 279–282.

[4] J. E. Kim, *Weak Interaction Singlet and Strong CP Invariance*, *Phys. Rev. Lett.* **43** (1979) 103.

[5] M. A. Shifman, A. Vainshtein and V. I. Zakharov, *Can Confinement Ensure Natural CP Invariance of Strong Interactions?*, *Nucl. Phys. B* **166** (1980) 493–506.

[6] M. Dine, W. Fischler and M. Srednicki, *A Simple Solution to the Strong CP Problem with a Harmless Axion*, *Phys. Lett. B* **104** (1981) 199–202.

[7] P. W. Graham, D. E. Kaplan and S. Rajendran, *Cosmological Relaxation of the Electroweak Scale*, *Phys. Rev. Lett.* **115** (2015) 221801, [1504.07551].

[8] P. Svrcek and E. Witten, *Axions In String Theory*, *JHEP* **06** (2006) 051, [hep-th/0605206].

[9] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, *String Axiverse*, *Phys. Rev. D* **81** (2010) 123530, [0905.4720].

[10] J. Preskill, M. B. Wise and F. Wilczek, *Cosmology of the Invisible Axion*, *Phys. Lett. B* **120** (1983) 127–132.

[11] L. F. Abbott and P. Sikivie, *A Cosmological Bound on the Invisible Axion*, *Phys. Lett. B* **120** (1983) 133–136.

[12] M. Dine and W. Fischler, *The Not So Harmless Axion*, *Phys. Lett. B* **120** (1983) 137–141.

[13] M. Weber and W. de Boer, *Determination of the local dark matter density in our galaxy*, *Astronomy and Astrophysics* **509** (jan, 2010) A25.

[14] F. Nesti and P. Salucci, *The Local Dark Matter Density*, *PoS* **DSU2012** (2012) 041, [1212.3670].

[15] J. Bovy and S. Tremaine, *On the local dark matter density*, *Astrophys. J.* **756** (2012) 89, [1205.4033].

[16] J. I. Read, *The Local Dark Matter Density*, *J. Phys. G* **41** (2014) 063101, [1404.1938].

[17] N. W. Evans, C. A. J. O'Hare and C. McCabe, *Refinement of the standard halo model for dark matter searches in light of the Gaia Sausage*, *Phys. Rev. D* **99** (2019) 023012, [1810.11468].

[18] L. Necib, M. Lisanti, S. Garrison-Kimmel, A. Wetzel, R. Sanderson, P. F. Hopkins et al., *Under the FIRElight: Stellar Tracers of the Local Dark Matter Velocity Distribution in the Milky Way*, *The Astrophysical Journal* **883** (10, 2019) , [1810.12301].

[19] J. W. Foster, N. L. Rodd and B. R. Safdi, *Revealing the Dark Matter Halo with Axion Direct Detection*, *Phys. Rev. D* **97** (2018) 123006, [1711.10489].

[20] G. P. Centers et al., *Stochastic fluctuations of bosonic dark matter*, *Nature Commun.* **12** (2021) 7321, [1905.13650].

[21] M. Lisanti, M. Moschella and W. Terrano, *Stochastic properties of ultralight scalar field gradients*, *Phys. Rev. D* **104** (2021) 055037, [2107.10260].

[22] D. J. Kaup, *Klein-Gordon Geon*, *Phys. Rev.* **172** (1968) 1331–1342.

[23] R. Ruffini and S. Bonazzola, *Systems of selfgravitating particles in general relativity and the concept of an equation of state*, *Phys. Rev.* **187** (1969) 1767–1783.

[24] E. W. Kolb and I. I. Tkachev, *Axion miniclusters and Bose stars*, *Phys. Rev. Lett.* **71** (1993) 3051–3054, [hep-ph/9303313].

[25] D. G. Levkov, A. G. Panin and I. I. Tkachev, *Gravitational Bose-Einstein condensation in the kinetic regime*, *Phys. Rev. Lett.* **121** (2018) 151301, [1804.05857].

[26] B. Eggemeier and J. C. Niemeyer, *Formation and mass growth of axion stars in axion miniclusters*, *Phys. Rev. D* **100** (2019) 063528, [1906.01348].

[27] J. Veltmaat, B. Schwabe and J. C. Niemeyer, *Baryon-driven growth of solitonic cores in fuzzy dark matter halos*, *Phys. Rev. D* **101** (2020) 083518, [1911.09614].

[28] R. Brito, V. Cardoso, C. A. R. Herdeiro and E. Radu, *Proca stars: Gravitating Bose–Einstein condensates of massive spin 1 particles*, *Phys. Lett. B* **752** (2016) 291–295, [1508.05395].

[29] M. A. Amin and P. Mocz, *Formation, gravitational clustering, and interactions of nonrelativistic solitons in an expanding universe*, *Phys. Rev. D* **100** (2019) 063507, [1902.07261].

[30] M. Gorghetto, E. Hardy, J. March-Russell, N. Song and S. M. West, *Dark photon stars: formation and role as dark matter substructure*, *JCAP* **08** (2022) 018, [2203.10100].

[31] A. Banerjee, D. Budker, J. Eby, H. Kim and G. Perez, *Relaxion Stars and their detection via Atomic Physics*, *Commun. Phys.* **3** (2020) 1, [1902.08212].

[32] A. Banerjee, D. Budker, J. Eby, V. V. Flambaum, H. Kim, O. Matsedonskyi et al., *Searching for Earth/Solar Axion Halos*, *JHEP* **09** (2020) 004, [1912.04295].

[33] D. Budker, J. Eby, M. Gorghetto, M. Jiang and G. Perez, *A Generic Formation Mechanism of Ultralight Dark Matter Solar Halos*, 2306.12477.

[34] A. Kryemadhi, M. Maroudas, A. Mastronikolis and K. Zioutas, *Gravitational focusing effects on streaming dark matter as a new detection concept*, 2210.07367.

[35] D. H. H. Hoffmann, J. Jacoby and K. Zioutas, *Gravitational lensing by the Sun of non-relativistic penetrating particles*, *Astropart. Phys.* **20** (2003) 73–78.

[36] M. S. Alenazi and P. Gondolo, *Phase-space distribution of unbound dark matter near the Sun*, *Phys. Rev. D* **74** (2006) 083518, [astro-ph/0608390].

[37] B. R. Patla, R. J. Nemiroff, D. H. H. Hoffmann and K. Zioutas, *Flux Enhancement of Slow-moving Particles by Sun or Jupiter: Can they be Detected on Earth?*, *Astrophys. J.* **780** (2014) 158, [1305.2454].

[38] G. Prézeau, *Dense Dark Matter Hairs Spreading Out from Earth, Jupiter and Other Compact Bodies*, *Astrophys. J.* **814** (2015) 122, [1507.07009].

[39] Y. Sofue, *Gravitational Focusing of Low-Velocity Dark Matter on the Earth's Surface*, *Galaxies* **8** (2020) 42, [2005.08252].

[40] S. K. Lee, M. Lisanti, A. H. G. Peter and B. R. Safdi, *Effect of Gravitational Focusing on Annual Modulation in Dark-Matter Direct-Detection Experiments*, *Phys. Rev. Lett.* **112** (2014) 011301, [1308.1953].

[41] H. Kim and A. Lenoci, *Gravitational focusing of wave dark matter*, *Phys. Rev. D* **105** (2022) 063032, [2112.05718].

[42] J. Chen, X. Du, E. W. Lentz and D. J. E. Marsh, *Relaxation times for Bose-Einstein condensation by self-interaction and gravity*, *Phys. Rev. D* **106** (2022) 023009, [2109.11474].

[43] K. Kirkpatrick, A. E. Mirasola and C. Prescod-Weinstein, *Analysis of Bose-Einstein condensation times for self-interacting scalar dark matter*, *Phys. Rev. D* **106** (2022) 043512, [2110.08921].

[44] J. Eby, M. Leembruggen, P. Suranyi and L. C. R. Wijewardhana, *Collapse of Axion Stars*, *JHEP* **12** (2016) 066, [1608.06911].

[45] D. G. Levkov, A. G. Panin and I. I. Tkachev, *Relativistic axions from collapsing Bose stars*, *Phys. Rev. Lett.* **118** (2017) 011301, [1609.03611].

[46] T. Helfer, D. J. E. Marsh, K. Clough, M. Fairbairn, E. A. Lim and R. Becerril, *Black hole formation from axion stars*, *JCAP* **03** (2017) 055, [1609.04724].

[47] J. I. Read, G. Lake, O. Agertz and V. P. Debattista, *Thin, thick and dark discs in LCDM*, *Mon. Not. Roy. Astron. Soc.* **389** (2008) 1041–1057, [0803.2714].

[48] C. W. Purcell, J. S. Bullock and M. Kaplinghat, *The Dark Disk of the Milky Way*, *apj* **703** (Oct., 2009) 2275–2284, [0906.5348].

[49] N. P. Pitjev and E. V. Pitjeva, *Constraints on dark matter in the solar system*, *Astron. Lett.* **39** (2013) 141–149, [1306.5534].

[50] A. Arvanitaki, J. Huang and K. Van Tilburg, *Searching for dilaton dark matter with atomic clocks*, *Phys. Rev. D* **91** (2015) 015015, [1405.2925].

[51] J. Fan, *Ultralight Repulsive Dark Matter and BEC*, *Phys. Dark Univ.* **14** (2016) 84–94, [1603.06580].

[52] J. A. R. Cembranos, A. L. Maroto, S. J. Núñez Jareño and H. Villarrubia-Rojo, *Constraints on anharmonic corrections of Fuzzy Dark Matter*, *JHEP* **08** (2018) 073, [1805.08112].

[53] Y.-D. Tsai, J. Eby and M. S. Safronova, *Direct detection of ultralight dark matter bound to the Sun with space quantum sensors*, *Nature Astron.* **7** (2023) 113–121, [2112.07674].

PoS(COSMICWISPers)024