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ALPs, high-energy gamma-rays and magnetic fields

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We review the suggestion to study ALP-photon oscillations in the GeV–TeV energy range using the windowed discrete power spectrum, and discuss how the predicted signatures depend on the modelling of astrophysical magnetic fields.

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[*] Speaker

1. Introduction

Peccei and Quinn [1, 2] postulated in 1977 as a solution to the strong CP problem an additional U(1) symmetry that is spontaneously broken, thereby giving rise to a Nambu-Goldstone boson—the axion a. The two-gluon-axion vertex introduced to solve the strong CP problem induces a small axion mass through pion mixing, $m_a f_a \approx m_\pi f_\pi$, degrading the axion to a pseudo-Goldstone boson [3, 4]. Other pseudo-scalars bosons which have the same two-photon coupling as the axion, $\mathcal{L} = -\frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma} \boldsymbol{E} \cdot \boldsymbol{B}$, are collectively known as axion-like particles (ALPs). In the presence of an external magnetic field, such a coupling leads to a conversion between photons and ALPs, an effect on which most ALP searches are based on. In this contribution based on Refs. [5, 6], we review the methodology to study ALP-photon oscillations in the GeV–TeV energy range, and discuss how the predicted signatures depend on the modelling of astrophysical magnetic fields.

2. Remarks on the role of magnetic fields

Galactic and extragalactic magnetic fields The strength and structure of the Galactic and extragalactic magnetic fields are a crucial, but only poorly known ingredient in the description of ALP-photon oscillations. While the overall strength of the Galactic magnetic field (GMF) is rather well-constrained, the relative size of the regular, striated and turbulent components, the detailed structure of the regular field, and the nature of the magnetic turbulent cascade are rather uncertain. The origin and the strength of the extragalactic magnetic field (EGMF) are much more uncertain. Its seed fields may be generated in the primordial universe or by astrophysical processes like galactic plasma outflows. If the field strength of the EGMF is normalised such to reproduce observations in the cores of galaxy cluster, their filling factors differ in the two cases drastically. Observationally, the strength of the EGMF is limited independent of its creation mechanism by 2×10^{-9} G from rotation measures [7], while the present strength of fields with a primordial origin is restricted to 5×10^{-11} G from CMB anisotropies [8]. There exists also lower limits on the strength and the filling factor of the EGMF [9, 10]. It has been argued that they are invalidated by plasma instabilities [11]. However, the limits [12] derived from transients sources as GRB 221009A are immune against this criticism, since the growth of plasma instabilities is too slower than the life-time of this transients.

Modelling the turbulence Probably as a consequence of these uncertainties, the EGMF is often modelled in an extremely over-simplified way. Instead of using a turbulent random field with fluctuations on all scales $\ell = 2\pi/k$ weighted by its power spectrum $P(k)$, one often employs a "domain-like" field: Inside regions of size $L_{\rm max}^3$, a uniform magnetic field $\boldsymbol{B}$ is used which direction is randomly chosen. In contrast, the fluctuations of a physical random field are only suppressed above a maximal scale $L_{\rm max}$, and extend down to the dissipation scale $L_{\rm min}$. In the left panel of Fig. 1, we compare different choices of the power spectrum $P(k)$, which can be all represented by a broken power law. In the right panel, we compare the (volume) filling factor $f(B_i > B)$ for a Gaussian random field with a Kolmogorov spectrum and for a domain-like field. Clearly, the missing variance in $B_\perp$, the relevant component for ALP-photon oscillations, for a domain-like field will impact the signatures of these oscillations.

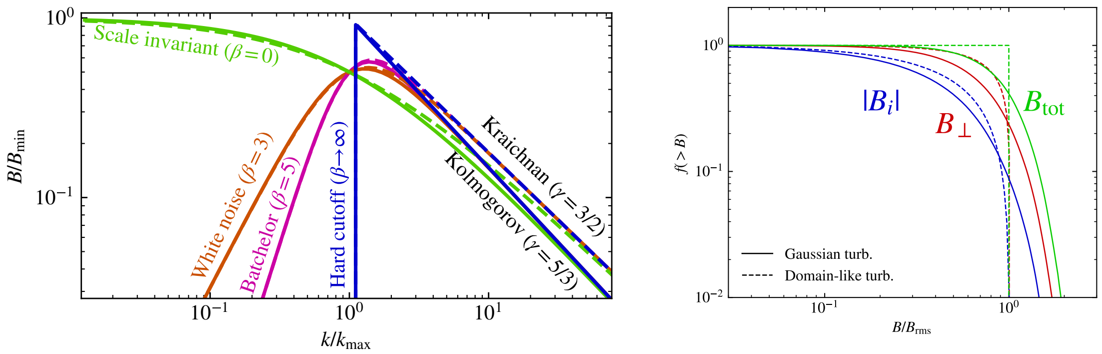

Figure 1: Left: Suggested power spectra for turbulent magnetic fields; Right: Filling factor for a Gaussian and domain-like turbulent field.

3. Parameter space of ALP-photon oscillations

After linearisation of the coupled ALP-photon Lagrangian (see the lecture [13] for details), the equation of motion for ALPs and photons with energy E propagating along the z-axis becomes

$$(E + \mathcal{M} - \mathrm{i}\partial_z)\,\phi(z) = 0, \tag{1}$$

where the wave function has the components $\phi = (A_\perp, A_\parallel, a)^T$ with $A_\perp$ and $A_\parallel$ as the two linear polarisation states of the photon perpendicular and parallel to the transverse magnetic field at a given position. The mixing matrix is given by

$$\mathcal{M} = \begin{pmatrix} \Delta_\perp & 0 & 0 \\ 0 & \Delta_\parallel & \Delta_{a\parallel} \\ 0 & \Delta_{a\parallel} & \Delta_a \end{pmatrix} \tag{2}$$

with $\Delta_a = -m_a{}^2/(2E)$, $\Delta_{\parallel,\perp} = (n_{\parallel,\perp} - 1)E$ and $n_{\parallel,\perp}$ as the refractive index of the photon. The off-diagonal terms $\Delta_{a\parallel} = g_{a\gamma}B_\perp/2$ lead to photon-ALP mixing in the presence of an external magnetic field.

Photon-ALP oscillations are essentially determined by the ALP parameters (m_a and $g_{a\gamma\gamma}$) and the refractive indices induced by the magnetic field, the medium and the EBL. In addition, the propagation distance and the photon energy enter the problem. The magnetic field leads via the QED vacuum polarisation to $n_\perp = 1 + 4\xi/2$ and $n_\parallel = 1 + 7\xi/2$ with $\xi \equiv (\alpha/45\pi)(B_\perp/B_{\rm cr})^2$. Among the medium effects, we neglect the Faraday contribution as the random direction of the turbulent magnetic field averages out its effect, as well as the Cotton-Mouton effect. Then the effective mass of the photon in a plasma given by the plasma frequency, $m_{\rm pl} \simeq \omega_{\rm pl} = \sqrt{4\pi\alpha n_e/m_e}$, leads to $\Delta^{\rm pl}_{\parallel,\perp} = -m^2_{\rm pl}/(2E)$ as the only additional contribution induced by the medium [14]. The isotropic EBL influences the two polarisation states equally, and its contribution is given below the pair creation threshold $E_{\rm th,CMB} \simeq 400\,\mathrm{TeV}$ by $\Delta_{\rm EBL} \simeq \Delta_{\rm CMB} \simeq 0.522 \cdot 10^{-42}E$ [15].

It is useful to consider the propagation through a homogeneous magnetic field to obtain an understanding of the problem. In this case, the photon conversion probability becomes

$$P_s(\gamma \to a) = \left|\left\langle A_\parallel(0)\middle|a(s)\right\rangle\right|^2 = \left(\Delta_{a\parallel}s\right)^2 \frac{\sin^2(\Delta_{\rm osc}s/2)}{(\Delta_{\rm osc}s/2)^2} \tag{3}$$

with

$$\Delta_{\rm osc}^2 = (\Delta_{\parallel} - \Delta_a)^2 + 4\Delta_{a\parallel}^2. \tag{4}$$

Similarly, the oscillation length in any sufficiently smooth environment is given by $L_{\rm osc} \simeq 2\pi/\Delta_{\rm osc}$. Thus the oscillation length, the correlation length $L_{\rm c}$ of the magnetic field and the mixing strength $2\pi/\Delta_{a\parallel}$ are the main parameters determining the effects of photon-ALP oscillations. For example, when $\Delta_{\rm osc} \sim 2\Delta_{a\parallel}$, we enter the strong mixing regime where Eq. (3) gives $P_s(\gamma \to a) = \sin^2(\Delta_{a\parallel}s)$.

From the solution (3) for a homogeneous magnetic field, one can conclude that the photon conversion probability will be governed by the relative ratios of $\Delta_{\rm osc}^{-1}$, $\Delta_{a\parallel}^{-1}$, $L_{\rm c}$ and the distance s travelled. That is, in order to have a significant conversion of photons, one must have a sizeable amount of oscillations ($s\Delta_{\rm osc} \gtrsim 1$) and a sizeable mixing ($\Delta_{a\parallel} \sim \Delta_{\rm osc}$). The coherence length, meanwhile, determines the intrinsic behaviour of the conversion probability: If $L_{\rm c} \gg 2\pi/\Delta_{\rm osc}$ the conversion probability "probes" the magnetic field with several oscillations per coherence length and the photon state parallel to the transverse magnetic field is completely mixed with the ALP for each coherence length. If $L_{\rm c} \ll 2\pi/\Delta_{\rm osc}$, on the other hand, the magnetic field changes quickly so that the mixing slowly converges. Observationally, one can measure the energy spectrum of single gamma-ray sources, which means that one can probe the energy dependence of the photon-ALP oscillation probability. The only energy dependence of the characteristic parameters lies in $\Delta_{\rm osc}$ and its generic behaviour is the same for all astrophysical environments, see also Ref. [16] for a similar discussion:

1. $\Delta_{\rm osc} \sim E^{-1}$ at low energies. Here, the oscillation length is determined by the effective photon mass or the ALP mass, depending on the magnetic field strength and the ALP parameters.

2. $\Delta_{\rm osc} \sim E^{0}$ at intermediate energies. This is the strong mixing regime where the oscillation length is determined by the mixing term, $\Delta_{\parallel} \sim 2\Delta_{a\parallel}$.

3. $\Delta_{\rm osc} \sim E^{1}$ at large energies. The oscillation length is here determined by either the CMB or the vacuum polarisation depending on the magnetic field strength.

The transitions between these regimes occur around the energies $E_{\rm min}$ and $E_{\rm max}$ defined by $4\Delta_{a\parallel} = (\Delta_{\parallel} - \Delta_a)^2$. Depending on the treatment of the magnetic fields, the oscillation probability in the transition region vary. For instance, the larger variance in $B_{\perp}$ of a turbulent field will lead to a larger variance in the threshold energies $E_{\rm min}$ and $E_{\rm max}$. This will effectively reduce or even cancel oscillations close to the thresholds upon averaging and shift the threshold energies.

The discussions above is summarised in Fig. 2, which presents $\Delta_{\rm osc}$ as function of energy and the transverse magnetic field strength. The three regions, $\Delta_{\rm osc} \sim E^{-1}$, $\Delta_{\rm osc} \sim E^{0}$ and $\Delta_{\rm osc} \sim E^{1}$, are indicated in the figure. Furthermore, the parameter space is divided into five regions depending on the dominant contributions to $\Delta_{\rm osc}$: ALP mass (upper left), plasma (lower left), mixing (middle), CMB (upper right) and QED (lower right). For very weak magnetic field strengths, there is no strong mixing regime. As an example of how this plot can be used, consider the EGMF: At low energies, the oscillations will be governed by the ALP mass term Δ_a. In the energy range $E = 10^9$–10^{12} eV, we are in the strong mixing regime where there are no energy-dependent oscillations. The exact energy of this transition will vary by around an order of magnitude for different realisations of the Gaussian turbulence. The energy oscillations will occur close to, but outside, the strong mixing regime, and their strength depends on $\Delta_{\rm osc}/\Delta_{a\parallel}$.

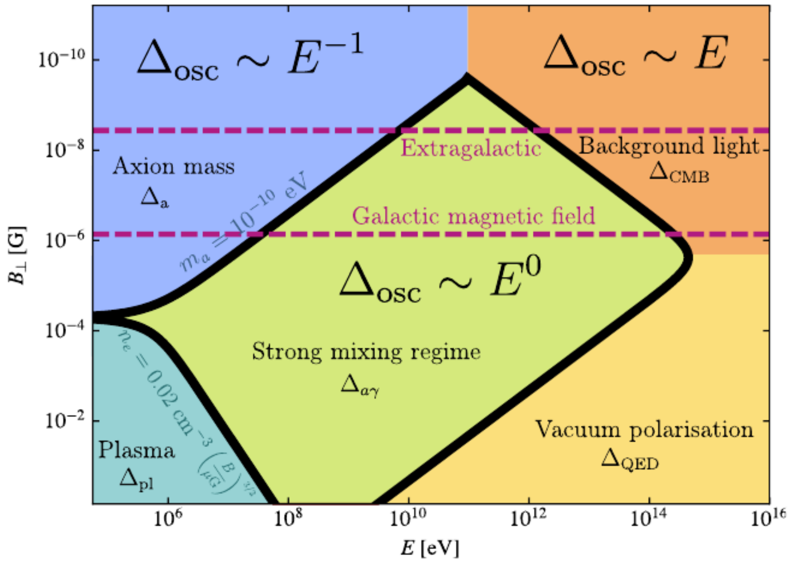

Figure 2: Photon-ALP parameter space divided into five main parts depending on the main contribution to the oscillation wave number $\Delta_{\rm osc}$, together with the border of the strong mixing regime.

4. ALP-photon oscillation at high energies

Photon opacity Since ALPs propagate practically without any interactions, there will be an increased photon survival probability at large energies, thus decreasing the opacity of the Universe. In the left panel of Fig. 3, we show the oscillation probability for a domain-like and Gaussian turbulence. It is clearly visible that the former case over-estimates the oscillation probability. As a result, the change of the opacity due to ALP-photon oscillations is over-estimated: In the right panel, we compare the normalised flux from a source with the injection spectrum $\mathrm{d}N/\mathrm{d}E \propto E^{-1.2}$ at a distance $z = 1$. The flux obtained averaged over many realisations of the magnetic field is shown as a solid (dashed) line for a Gaussian turbulent (domain-like) field, with the shaded regions corresponding to the 1σ variance between single realisations. For comparison, we plot also the spectrum obtained for a single realisation of the magnetic field using a Gaussian turbulent field with continuous attenuation (red squares) and the Monte Carlo treatment of the electromagnetic cascade (green circles) based on `ELMAG` [17, 18]. Furthermore, the spectrum obtained with the electromagnetic cascade without ALP, i.e. using `ELMAG` without ALP-photon oscillations, is shown (blue diamonds). It is clear from Fig. 3 that the two treatments of the magnetic field results in a significant difference in the predicted average flux which is increasing with energy: At $E \simeq 1$ TeV the difference is around a factor 20. Similarly, the variation between single realisations is noticeably larger for the Gaussian turbulence than for the domain-like approximation.

Search for wiggles in photon spectra Photon-ALP oscillations will perturb the photon spectrum with energy dependent oscillations, $k \sim \Delta_{\rm osc}$, even for a turbulent magnetic field [5]. At energies above the strong mixing regime, these oscillations with thus lead to wiggles in the energy spectrum of photons with $k \sim E$. Likewise, below the strong mixing regime, $k \sim E^{-1}$. In Ref. [5], we

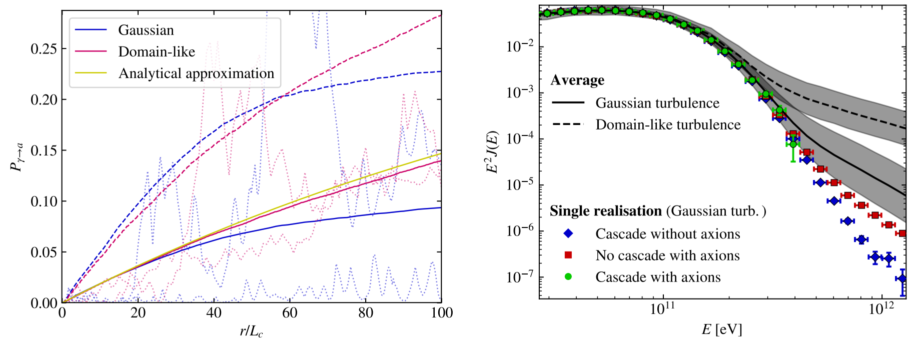

Figure 3: Left: The ALP-photon oscillation probability as function of distance. Right: The normalised diffuse photon flux from a single source as a function of energy.

suggested therefore to use, inspired by Refs. [19, 20], the windowed discrete power spectrum,

$$G_N(k) = \left| \frac{1}{N} \sum_{\text{events}} \mathrm{e}^{\mathrm{i}\eta k} \right|, \tag{5}$$

to extract information on the wiggles; for a similar approach see also Ref. [21]. For a turbulent magnetic field, the ALP signal is a broadened peak whose location and width is a priori unknown. While this makes a detection more challenging, it enables the extraction of information on the magnetic field. Importantly, one can use this model to search for ALPs without specifying the magnetic field.

In Refs. [5, 6], we suggested to use the test statistic given by the goodness-of-fit measure compared to an estimated background,

$$\mathrm{TS} = \frac{1}{\Delta k} \int_0^{\Delta k} \frac{[G_N(k) - G_N^B(k)]^2}{\sigma_N^B(k)^2}\, \mathrm{d}k \tag{6}$$

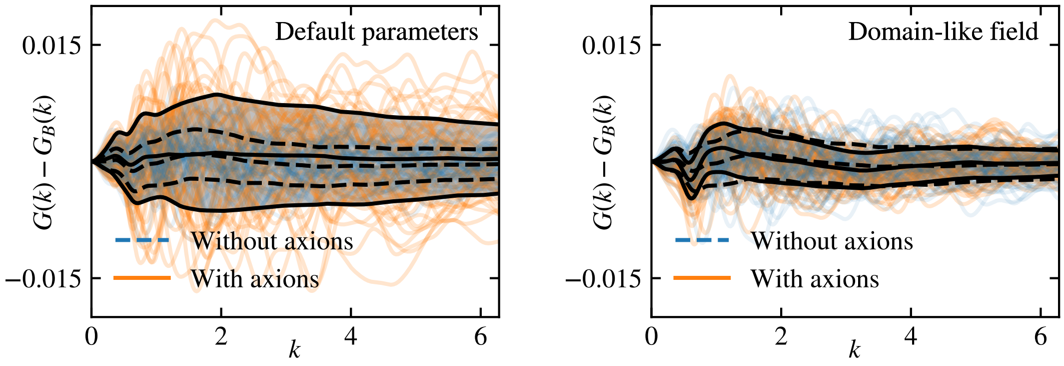

Figure 4: The power spectrum with the estimated background subtracted is plotted using a Gaussian turbulent field (left) and a domain-like field (right). The results for 50 realisation of the magnetic field with (orange) and without (blue) photon-ALP oscillations is shown, and the averages and the statistical standard deviation from a sample of 10^3 realisations are shown in black lines.

with $\Delta k = 6$. In the left pane of Fig. 4, this TS is shown for $N = 10^4$ detected photons in the energy range $E \in (10^{12}, 10^{14})$ eV with a spectrum $\mathrm{d}N/\mathrm{d}E \propto E^{-1.2}$, $g_{a\gamma} = 10^{-20}\,\mathrm{GeV}^{-2}$ and a Gaussian turbulent magnetic field with $B_{\mathrm{rms}} = 5\,\mathrm{nG}$. The results for 50 realisations with photon-ALP oscillations is shown in orange lines, while without photon-ALP oscillations in blue. The averages and the 1σ statistical variance (black lines) were computed using the full set of 10^3 realisations. For comparison, the results using a domain-like field is shown in the right panel of Fig. 4.

The results in Fig. 4 indicate the validity of the statistical procedure: There are clear peaks in the power spectrum including photon-ALP oscillations compared to the case without photon-ALP oscillations. Interestingly, due to the lack of cosmic variance in the simple domain-like field, there is a lack of variance in the photon spectra which represents itself as a clear signal in the discrete power spectrum, even after averaging over many realisations of the magnetic field. Hence, the use of simplified magnetic field models, such as the domain-like field, may lead to a bias in searches for ALP wiggles and impact the estimated limits on $g_{a\gamma}$. However, the larger variance in more realistic magnetic field models—in these examples represented by Gaussian turbulent fields—increases the rate of random encounters of regions of magnetic fields that may enhance the wiggles. Note also that magnetic fields found in MHD simulations are intermittent, i.e. their power spectrum has non-Gaussian tails, enhancing the variance even further [22]. Thus, a more realistic modelling of the magnetic fields may, in fact, improve the detection prospects, in particular if several sources with different line-of-sights are studied.

5. Summary

We have argued that the characteristic signatures expected in the photon spectra from distant gamma-ray sources depend strongly on the chosen magnetic field model. For instance, the change of the opacity of the Universe induced by ALP-photon oscillations is strongly reduced employing a more realistic description of the turbulent magnetic field. Thus the use of domain-like magnetic fields may be tenable for quantitative discussions, but should be abandoned in qualitative studies. The discrete power spectrum can be used to detect ALP-photon oscillations in upcoming gamma-ray experiments such as CTA. This method directly uses energy-dependent wiggles in the photon spectra as observable. In addition, these signatures can be used to infer information about the magnetic field environment.

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