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Lattice QCD predictions of pion and kaon electromagnetic form factors at large momentum transfer

Heng-Tong Ding,[a] **Xiang Gao,**[b] **Andrew D. Hanlon,**[c] **Swagato Mukherjee,**[c] **Peter Petreczky,**[c] **Qi Shi,**[a,c,*] **Sergey Syritsyn**[d,e] **and Yong Zhao**[b]

[a] *Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

[b] *Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA*

[c] *Physics Department, Brookhaven National Laboratory, Bldg. 510A, Upton, New York 11973, USA*

[d] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

[e] *Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11790, USA*

E-mail: gaox@anl.gov, qi-shi@mails.ccnu.edu.cn

We present a first Lattice Quantum Chromodynamics (QCD) prediction for the electromagnetic form factors (EMFFs) of the pion and kaon up to very large momentum transfers, i.e., $Q^2 \sim 10$ and 28 GeV^2, respectively. Our calculations utilize Wilson-clover fermions within the $N_f = 2 + 1$ Highly Improved Staggered Quark (HISQ) ensemble at the physical point. For pion, we employ a lattice spacing of $a = 0.076$ fm, and for kaon, we use two spacings, $a = 0.04$ and 0.076 fm, to mitigate discretization effects. The renormalized results indicate a rapid increase in $Q^2 F(Q^2)$ at lower Q^2 values, which then stabilize for both pion and kaon. Notably, our findings align well with existing experimental data, establishing a reliable QCD benchmark for future experimental analyses from JLab12, EicC, and EIC.

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PoS(LATTICE2023)320

1. Introduction

Understanding the fundamental properties of hadrons is crucial for a deeper insight into the fabric of the universe. Among these properties, EMFFs play a pivotal role. They provide essential information about the distribution of charge and magnetization within these particles, offering a window into their internal structure and dynamics. Particularly, as pion and kaon are pseudo-Goldstone bosons, their EMFFs are of paramount importance and can provide crucial insights into the non-perturbative aspects of QCD, particularly in understanding chiral symmetry breaking and the strong interaction at low energies. Additionally, with its strange quark content, kaon is crucial to exploring flavor physics and CP violation. Thus, investigating pion and kaon not only enhances our understanding of fundamental particles and forces but also tests and refines theoretical frameworks like chiral perturbation theory, making it a cornerstone in the study of particle physics.

The traditional approach to obtaining the EMFFs primarily involved experimental extractions and theoretical model predictions. Experimental efforts, though insightful, often grapple with the challenges of high-precision measurements, especially at large momentum transfers. Similarly, theoretical predictions, while valuable, are sometimes constrained by approximations and assumptions that may not fully encapsulate the complex dynamics of strong interactions. As a non-perturbative approach grounded in the first principles of QCD, Lattice QCD has emerged as a formidable tool in the study of hadronic physics. It enables the calculation without any model dependence, bypassing the limitations of perturbative techniques. Recent advancements in computational power and algorithms have allowed Lattice QCD to perform more precise and comprehensive simulations.

Despite the long-standing discovery of pion and kaon, which dates back several decades, the comprehensive study of their EMFFs has faced considerable limitations. The largest Q^2 in existing experimental and lattice results are smaller than 3 GeV^2 [1, 2] and 6 GeV^2 [3–6], respectively. In this work, we explore a wide range of momentum transfers. By pushing Q^2 up to 10 and 28 GeV^2 for pion and kaon, respectively, our study not only contributes to a more comprehensive understanding of the pion and kaon EMFFs but also offers invaluable insights for the future JLab12 [7], EicC [8], and EIC [9] experimental programs to refine the designs and interpretative frameworks.

2. Lattice calculation

Our research utilizes two distinct gauge ensembles from the HotQCD collaboration [10], both featuring a 2+1 flavor HISQ [11] setup. These ensembles share the same lattice dimensions of $N_s \times N_t = 64^3 \times 64$ but differ in their lattice spacings, which are 0.076 fm and 0.04 fm. In these setups, we use light quark masses in the sea that correspond to the pion mass of 140 MeV for the coarser lattice and 160 MeV for the finer lattice. The strange quark masses are aligned with the physical value for both lattices. In the valence quark sector, we employ the Wilson-Clover action, setting the clover coefficients at $c_{sw} = 1.0372$ [12] for the 0.076 fm lattice and 1.02868 [13] for the 0.04 fm lattice. These coefficients are determined using the average plaquette value post-1-step HYP smearing [14]. Furthermore, the Wilson-Clover valence quark masses are tuned to the physical point for both lattices. Our computational methods include the use of the QUDA multigrid algorithm [15–18] for Wilson-Dirac operator inversions, crucial for obtaining quark propagators.

Additionally, we implement the all-mode averaging (AMA) technique [19] to enhance the statistical reliability of our results.

To accomplish the primary goal of this study, which is to achieve a high momentum transfer $Q^2 = -(p^f - p^i)^2$, our calculations predominantly focus on the three-point function within the Breit frame, where the initial momentum, final momentum, and momentum transfer are $\mathbf{P}^i = (0, 0, P_z)$, $\mathbf{P}^f = (0, 0, -P_z)$, and $\mathbf{q} = (0, 0, -2P_z)$, respectively. Additionally, we maintain the quark boost parameter ζ [12, 13] of the initial and final states consistent, which means $\zeta = \mathbf{k}^i/\mathbf{P}^i = \mathbf{k}^f/\mathbf{P}^f$ with the quark boost momentum $\mathbf{k}^f = -\mathbf{k}^i$. Given that the hadron states with slight momentum variation share the same propagator [12], it is possible to calculate multiple reliable data sets showing slight deviations from the Breit frame while saving computational time. In this calculation, we only consider quark line-connected diagrams.

The correlation functions are the main objects of lattice calculation. To get the bare matrix elements, we need to calculate the two-point and three-point correlation functions. For the analysis, we need to first extract the energy and amplitude from the two-point correlation function, through which we can also check the quality of our signal. The spectral decomposition of the two-point correlation function can be expressed as

$$C_{2\mathrm{pt}}(\mathbf{P}, t_s) = \left\langle H(\mathbf{P}, t_s) H^\dagger(\mathbf{P}, 0)\right\rangle = \sum_{n=0}^{N-1} A_n A_n^* \left[e^{-E_n(\mathbf{P}) t_s} + e^{-E_n(\mathbf{P})(aN_t - t_s)}\right], \tag{1}$$

where N is the number of energy levels we consider, E_n and $A_n = \langle \Omega|\hat{H}|n\rangle$ are the energy and amplitude of the n^{th} state, respectively, and $|\Omega\rangle$ denotes the vacuum. Due to the periodic boundary condition, the spatial momentum $\mathbf{P} = 2\pi\mathbf{n}/(aN_s)$ with $\mathbf{n}$ being the lattice unit vector. The specific fit results of the ground state for kaon are shown in Fig. 1, which pretty much agree with the dispersion relation $E = \sqrt{m_K^2 + \mathbf{P}^2}$.

The spectral decomposition of the three-point correlation function is

$$\begin{aligned} C_{3\mathrm{pt}}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) &= \left\langle H(\mathbf{P}^f, t_s) O_\Gamma(\mathbf{q}, \tau) H^\dagger(\mathbf{P}^i, 0)\right\rangle \\ &= \sum_{n,m=0}^{N-1} A_n^f (A_m^i)^* e^{-E_n(\mathbf{P}^f)(t_s-\tau)} e^{-E_m(\mathbf{P}^i)\tau} \left\langle n; \mathbf{P}^f|\hat{O}_\Gamma|m; \mathbf{P}^i\right\rangle, \end{aligned} \tag{2}$$

where $\mathbf{q} = \mathbf{P}^f - \mathbf{P}^i$, and the vector currents O_Γ are $\frac{2}{3}\bar{u}\gamma_t u - \frac{1}{3}\bar{d}\gamma_t d$ and $\frac{2}{3}\bar{u}\gamma_t u - \frac{1}{3}\bar{s}\gamma_t s$ for pion and kaon, respectively. To take advantage of the correlation between the two-point and three-point correlation functions, we construct the ratio

$$R^{fi}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) \equiv \frac{2\sqrt{E_0^f E_0^i}}{E_0^f + E_0^i} \frac{C_{3\mathrm{pt}}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s)}{C_{2\mathrm{pt}}(t_s, \mathbf{P}^f)} \times \left[\frac{C_{2\mathrm{pt}}(t_s-\tau, \mathbf{P}^i) C_{2\mathrm{pt}}(\tau, \mathbf{P}^f) C_{2\mathrm{pt}}(t_s, \mathbf{P}^f)}{C_{2\mathrm{pt}}(t_s-\tau, \mathbf{P}^f) C_{2\mathrm{pt}}(\tau, \mathbf{P}^i) C_{2\mathrm{pt}}(t_s, \mathbf{P}^i)}\right]^{1/2}, \tag{3}$$

which approaches the bare matrix elements of the ground state $F_M^B = \langle 0; \mathbf{P}^f|O_\Gamma|0; \mathbf{P}^i\rangle$ with $M = \pi^+, K^+$ in the limit $t_s \to \infty$. Thus, by combining the Eqs. 1 - 3 and using the extracted values of energy and amplitude, we can perform a N-state fit on the lattice data. In Fig. 2, we show the 2-state fit results of R^{fi} at $Q^2 = 9.4$ GeV^2 for pion and $Q^2 = 23.4$ GeV^2 for kaon which are both from the Breit-frame. The grey bands denote the extrapolated bare form factor F_M^B, while the bands

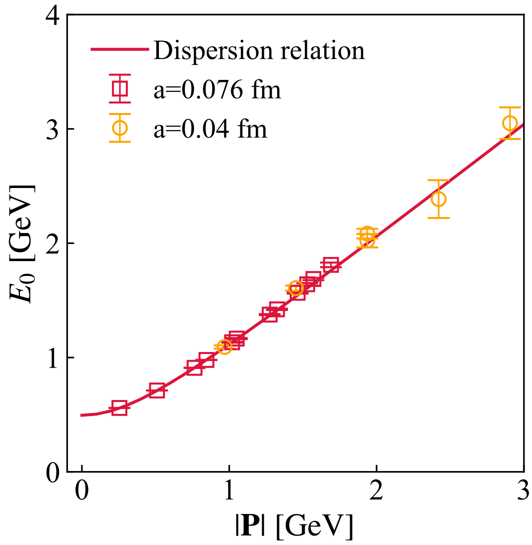

Figure 1: The extracted ground state energy E_0 for kaon are shown. The squared and circled symbols denote the results of a = 0.076 and 0.04 fm lattices, respectively. The red line shows the dispersion relation.

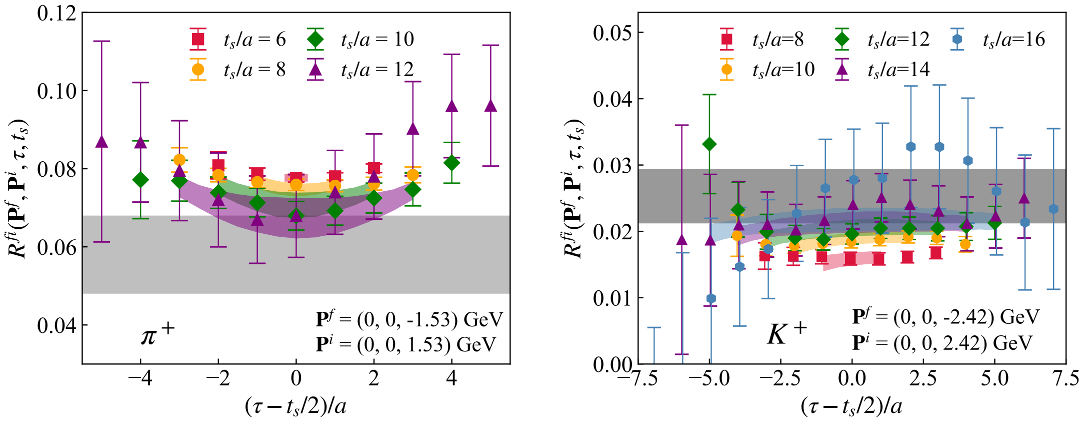

Figure 2: The 2-state fit results and lattice data of R^{fi} are shown. Left panel: pion results of $Q^2 = 9.4$ GeV^2 from the $a = 0.076$ fm lattice. Right panel: kaon results of $Q^2 = 23.4$ GeV^2 from the $a = 0.04$ fm lattice.

in other colors correspond to the fit results of the lattice data, matching in color. Even for the largest momentum transfer we have a reasonably good signal for $R^{fi}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s)$ for all t_s values and the fits work well.

3. Results and conclusion

To obtain the final EMFFs, the bare form factors need to be renormalized by the vector current renormalization factor $Z_V = \langle 0; \mathbf{P}|O_\Gamma|0; \mathbf{P}\rangle$. Here we use the values $Z_V = 1.048$ [5] and 1.024 [13], extracted previously, for $a = 0.076$ fm and 0.04 fm lattices, respectively. The renormalized results $F_M = F_M^B/Z_V$ are shown in Fig. 3 as $Q^2 F_M(Q^2)/f_M^2$. We use the following values of the pion and

PoS(LATTICE2023)320

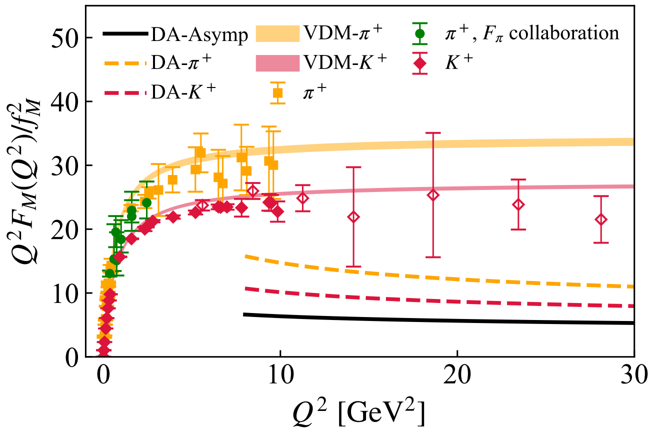

Figure 3: The $Q^2F_M(Q^2)/f_M^2$ are shown as a function of Q^2. The squared symbols denote lattice results for pion. The filled and open diamond symbols are lattice results for kaon obtained from lattices with $a = 0.076$ and 0.04 fm, respectively. The circle symbols represent the existing experimental extractions of pion from F_π collaboration. For comparison, the predictions from the VMD model (bands) and the pQCD contribution using the asymptotic DAs (solid line) and lattice calculated DAs (dash lines) are also shown.

kaon decay constants $f_\pi = 130.2$ MeV and $f_K = 155.7$ [21–23]. For the pion case, we show the previous lattice results [5] at lower Q^2, experimental extractions [1] at middle Q^2, and lattice results in this work at higher Q^2. Notably, our lattice results overlap with the experimental extractions on both sides with small errors and show excellent agreement. This observation verifies the model-based determination from experimental measurements and encourages future experimental extractions. For the kaon case, the outcomes from two lattice spacings seem to align well, indicating that the discretization errors are relatively minor compared to the current statistical uncertainties. For both pion and kaon, we also show the predictions from the Vector Meson Dominance (VMD) model [28] fitted from data at low Q^2 ($\lesssim 0.4$ GeV^2) and the LO perturbative QCD (pQCD) factorization [29] at high Q^2 using Distribution Amplitudes (DAs) [27]. Surprisingly, the VDM predictions can roughly describe the data at high Q^2 within a 2-σ range, while the pQCD predictions remain below.

In summary, we present the Lattice QCD predictions for the EMFFs of pion and kaon, which are extended to a significantly high Q^2 for the first time. Our findings are consistent with existing experimental data and VMD model predictions. However, the observed discrepancies with predictions from pQCD employing DAs warrant further exploration. Importantly, our results set a valuable benchmark for future model-based QCD studies and upcoming experimental investigations in this field.

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