

2023 Update of ε_K with lattice QCD inputs

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We report recent progress on ε_K evaluated directly from the standard model (SM) with lattice QCD inputs such as \hat{B}_K , $|V_{cb}|$, $|V_{us}|$, $|V_{ud}|$, ξ_0 , ξ_2 , ξ_{LD} , f_K , and m_c . We find that the standard model with exclusive $|V_{cb}|$ and lattice QCD inputs describes only 66% of the experimental value of $|\varepsilon_K|$ and does not explain its remaining 34%, which corresponds to a strong tension in $|\varepsilon_K|$ at the $4.9\sigma \sim 3.9\sigma$ level between the SM theory and experiment. We also find that this tension disappears when we use the inclusive value of $|V_{cb}|$ obtained using the heavy quark expansion based on the QCD sum rule approach.

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1. Introduction

This paper is an update of our previous reports [1–8]. We report recent progress in the determination of $|\varepsilon_K|$ with updated inputs from lattice QCD. Updated input parameters include λ , $\bar{\rho}$, $\bar{\eta}$, exclusive $|V_{cb}|$, $|V_{us}|$, $|V_{ud}|$, $|V_{us}|/|V_{ud}|$, M_W , and M_t .

We follow the color convention of our previous papers [1–8] in Tables 1–9. We use red for new input data used to evaluate ε_K . We use blue for new input data which is not used.

2. Input parameters: Wolfenstein parameters

In Table 1, we summarize results for $|V_{ud}|$, $|V_{us}|$ from lattice QCD and nuclear β decays, and $\frac{|V_{us}|}{|V_{ud}|}$ from lattice QCD.

| type | $ V_{us} $ | $ V_{ud} $ | Ref. | type | $ V_{us} / V_{ud} $ | Ref. |
|---------------------------|------------------|--------------------|-------------|---------|---------------------|-------------|
| Lattice $N_f = 2 + 1 + 1$ | 0.2248(6) | 0.97440(15) | FLAG-23 [9] | QCD | 0.2313(5) | FLAG-23 [9] |
| Lattice $N_f = 2 + 1$ | 0.2249(5) | 0.97438(12) | FLAG-23 [9] | QCD+QED | 0.2320(5) | FLAG-23 [9] |
| nuclear β decay | 0.2278(6) | 0.97370(14) | PDG-21 [10] | | | |

(a) $|V_{us}|$ and $|V_{ud}|$ from lattice QCD and nuclear β decays

(b) $|V_{us}|/|V_{ud}|$ from lattice QCD

Table 1: (a) $|V_{us}|$ and $|V_{ud}|$ (b) $|V_{us}|/|V_{ud}|$.

$$\lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} = \frac{r}{\sqrt{1+r^2}}, \quad r = \frac{|V_{us}|}{|V_{ud}|} \quad (1)$$

Using Eq. (1), we determine λ from $\frac{|V_{us}|}{|V_{ud}|}$ in Table 1 (b), because its error is less than that obtained by using the values for $|V_{us}|$ and $|V_{ud}|$ in Table 1 (a). Results for λ are presented in Table 2 (a), where we summarize the most updated Wolfenstein parameters (WP).

Recently the UTfit collaboration updated values for the WP in Ref. [11]. As explained in Ref. [4, 8], we use the results of the angle-only-fit (AOF) in Table 2 (a) in order to avoid unwanted correlations between $(\varepsilon_K, |V_{cb}|)$, and $(\bar{\rho}, \bar{\eta})$. We determine the parameter A from $|V_{cb}|$.

| WP | CKMfitter | UTfit | AOF | Input | Value | Ref. |
|--------------|------------------|-----------------------------|------------------------|-------------|------------|------|
| λ | 0.22500(24) [12] | 0.22519(83) [11, 13] | 0.22600(46) [9] | η_{cc} | 1.72(27) | [5] |
| $\bar{\rho}$ | 0.1566(85) [12] | 0.161(10) [11, 13] | 0.156(17) [11] | η_{tt} | 0.5765(65) | [14] |
| $\bar{\eta}$ | 0.3475(118) [12] | 0.347(10) [11, 13] | 0.334(12) [11] | η_{ct} | 0.496(47) | [15] |

(a) Wolfenstein parameters

(b) η_{ij}

Table 2: (a) Wolfenstein parameters and (b) QCD corrections: η_{ij} with $i, j = c, t$.

| channel | value | method | ref | source |
|------------------------------------|---------------|--------|---------------|--------------|
| $B \rightarrow D^* \ell \bar{\nu}$ | 38.40(78) | BGL | [17] p27e76 | FNAL/MILC-22 |
| ex-comb | 39.48(67) | comb | [9] p195e314 | FLAG-23 |
| ex-comb | 39.10(50) | comb | [18] p120e221 | HFLAV-23 |
| ex-comb | 39.31(54)(51) | comb | [19] p22e51 | HPQCD-23 |

(a) Exclusive $|V_{cb}|$ in units of 10^{-3} .

| scheme | value | ref | source |
|----------------|-----------|---------------|------------|
| kinetic scheme | 42.16(51) | [20] p1 | Gambino-21 |
| 1S scheme | 41.98(45) | [18] p108e200 | HFLAV-23 |

(b) Inclusive $|V_{cb}|$ in units of 10^{-3} .

Table 3: Results for (a) exclusive $|V_{cb}|$ and (b) inclusive $|V_{cb}|$. The abbreviation p27e76 means Eq. (76) on page 27.

3. Input parameters: $|V_{cb}|$

In Table 3, we summarize recent updated results for exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$. In Table 3 (a), we present updated results for exclusive $|V_{cb}|$ obtained by various groups: FNAL/MILC, FLAG, HFLAV, and HPQCD. They are consistent with one another within 1.0σ uncertainty.

In Table 3 (b), we present recent updated results for inclusive $|V_{cb}|$. There are a number of attempts to determine inclusive $|V_{cb}|$ from lattice QCD, but all of them at present belong to the category of exploratory study rather than that of precision calculation [16].

4. Input parameter ξ_0

The absorptive part of long distance effects on ε_K is parametrized by ξ_0 .

$$\xi_0 = \frac{\text{Im } A_0}{\text{Re } A_0}, \quad \xi_2 = \frac{\text{Im } A_2}{\text{Re } A_2}, \quad \text{Re} \left(\frac{\varepsilon'}{\varepsilon} \right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|} (\xi_2 - \xi_0). \quad (2)$$

There are two independent methods to determine ξ_0 in lattice QCD: the indirect and direct methods. The indirect method is to determine ξ_0 using Eq. (2) with lattice QCD results for ξ_2 combined with experimental results for ε'/ε , ε_K , and ω . The direct method is to determine ξ_0 using the lattice QCD results for $\text{Im } A_0$, combined with experimental results for $\text{Re } A_0$.

In Table 4 (a), we summarize experimental results for $\text{Re } A_0$ and $\text{Re } A_2$. In Table 4 (b), we summarize lattice results for $\text{Im } A_0$ and $\text{Im } A_2$ calculated by RBC-UKQCD. In Table 4 (c), we present results for ξ_0 obtained by using the results in Table 4 (a) and (b).

Here we use the results of the indirect method for ξ_0 to evaluate ε_K , since the systematic and statistical errors are much smaller than those of the direct method.

| parameter | method | value | Ref. | source |
|-----------------------------------|--------|---------------------------------|----------|----------|
| Re A_0 | exp | $3.3201(18) \times 10^{-7}$ GeV | [21, 22] | NA |
| Re A_2 | exp | $1.4787(31) \times 10^{-8}$ GeV | [21] | NA |
| ω | exp | 0.04454(12) | [21] | NA |
| $ \varepsilon_K $ | exp | $2.228(11) \times 10^{-3}$ | [10] | PDG-2021 |
| Re (ε'/ε) | exp | $1.66(23) \times 10^{-3}$ | [10] | PDG-2021 |

(a) Experimental results for ω , Re A_0 and Re A_2 .

| parameter | method | value (GeV) | Ref. | source |
|-----------|---------|----------------------------------|-------------|-------------|
| Im A_0 | lattice | $-6.98(62)(144) \times 10^{-11}$ | [23] p4t1 | RBC-UK-2020 |
| Im A_2 | lattice | $-8.34(103) \times 10^{-13}$ | [23] p31e90 | RBC-UK-2020 |

(b) Results for Im A_0 , and Im A_2 in lattice QCD.

| parameter | method | value | ref | source |
|-----------|----------|------------------------------|------|--------|
| ξ_0 | indirect | $-1.738(177) \times 10^{-4}$ | [23] | SWME |
| ξ_0 | direct | $-2.102(472) \times 10^{-4}$ | [23] | SWME |

(c) Results for ξ_0 obtained using the direct and indirect methods in lattice QCD.**Table 4:** Results for ξ_0 . Here, we use the same notation as in Table 3. The abbreviation p4t1 means Table 1 on page 4.

5. Input parameters: \hat{B}_K , ξ_{LD} , and others

The Flavour Lattice Averaging Group (FLAG) [9] reports results for \hat{B}_K in lattice QCD with $N_f = 2$, $N_f = 2 + 1$, and $N_f = 2 + 1 + 1$. Here we use the result for \hat{B}_K with $N_f = 2 + 1$, which is obtained by taking an average over the four data points from BMW 11, Laiho 11, RBC-UKQCD 14, and SWME 15 in Table 5 (a).

The dispersive long distance (LD) effect ξ_{LD} is

$$\xi_{LD} = \frac{m'_{LD}}{\sqrt{2}\Delta M_K}, \quad m'_{LD} = -\text{Im} \left[\mathcal{P} \sum_C \frac{\langle \bar{K}^0 | H_w | C \rangle \langle C | H_w | K^0 \rangle}{m_{K^0} - E_C} \right] \quad (3)$$

| Collaboration | Ref. | \hat{B}_K |
|---------------|------|-----------------|
| SWME 15 | [24] | 0.735(5)(36) |
| RBC/UKQCD 14 | [25] | 0.7499(24)(150) |
| Laiho 11 | [26] | 0.7628(38)(205) |
| BMW 11 | [27] | 0.7727(81)(84) |
| FLAG-23 | [9] | 0.7625(97) |

(a) \hat{B}_K

| Input | Value | Ref. |
|--------------|---|-------------|
| G_F | $1.1663788(6) \times 10^{-5}$ GeV ⁻² | PDG-23 [28] |
| θ | 43.52(5) $^\circ$ | PDG-23 [28] |
| m_{K^0} | 497.611(13) MeV | PDG-23 [28] |
| ΔM_K | $3.484(6) \times 10^{-12}$ MeV | PDG-23 [28] |
| F_K | 155.7(3) MeV | FLAG-23 [9] |

(b) Other parameters

Table 5: (a) Results for \hat{B}_K and (b) other input parameters.

| Collaboration | N_f | $m_c(m_c)$ | Ref. | Collaboration | M_t | $m_t(m_t)$ | Ref. |
|---------------|---------|------------|------|---------------|------------|----------------|------|
| FLAG 2023 | $2+1$ | 1.276(5) | [9] | PDG 2021 | 172.76(30) | 162.96(28)(17) | [10] |
| FLAG 2023 | $2+1+1$ | 1.280(13) | [9] | PDG 2022 | 172.69(30) | 162.90(28)(17) | [28] |
| FLAG 2023 | $2+1+1$ | 1.280(13) | [9] | PDG 2023 | 172.69(30) | 162.90(28)(17) | [28] |

(a) $m_c(m_c)$ [GeV] (b) $m_t(m_t)$ [GeV]

Table 6: Results for (a) charm quark mass and (b) top quark mass.

As explained in Ref. [4], there are two independent methods to estimate ξ_{LD} : one is the BGI estimate [29], and the other is the RBC-UKQCD estimate [30, 31]. The BGI method estimates ξ_{LD} using chiral perturbation theory, using Eq. (4).

$$\xi_{LD} = -0.4(3) \times \frac{\xi_0}{\sqrt{2}} \quad (4)$$

The RBC-UKQCD method estimates ξ_{LD} using Eq. (5).

$$\xi_{LD} = (0 \pm 1.6)\% \quad (5)$$

We use both methods to estimate the size of ξ_{LD} .

In Table 2 (b), we present higher order QCD corrections: η_{ij} with $i, j = t, c$. A new approach using $u-t$ unitarity instead of $c-t$ unitarity appeared in Ref. [32], which is supposed to have better convergence with respect to the charm quark mass. We are working to incorporate this into our analysis, which we will report soon.

In Table 5 (b), we present other input parameters needed to evaluate ε_K . Note that the Fermi coupling constant G_F has been updated in 2023.

6. Quark masses

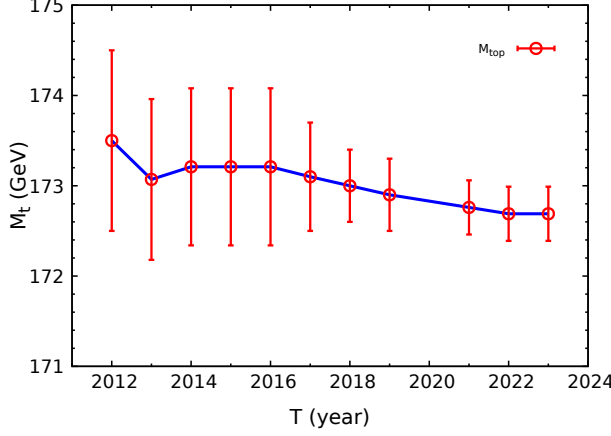
In Table 6 we present the charm quark mass $m_c(m_c)$ and top quark mass $m_t(m_t)$. From FLAG 2023 [9], we take the results for $m_c(m_c)$ with $N_f = 2+1$, since there is some inconsistency among the lattice results of various groups with $N_f = 2+1+1$. For the top quark mass, we use the PDG 2023 results for the pole mass M_t to obtain $m_t(m_t)$ ¹.

In Table 7 (a) we present the history of values for the top quark pole mass M_t . We find that the average value shifts downward by 0.47% over time. We also find that the error shrinks fast down to 30% of the original error (2012) thanks to accumulation of high statistics in the LHC experiments. The data for 2020 is dropped out intentionally in memory of the absence of Lattice 2020 due to COVID-19.

7. W boson mass

In Fig. 8 (a) we plot M_W (the W boson mass) as a function of time. The corresponding results for M_W are summarized in Table 8 (b). In Fig. 8 (a), the light-green band represents the standard model (SM) prediction, the red circles represent the PDG results from the experimental summary,

¹Here we use PDG results updated on 2023-5-31.

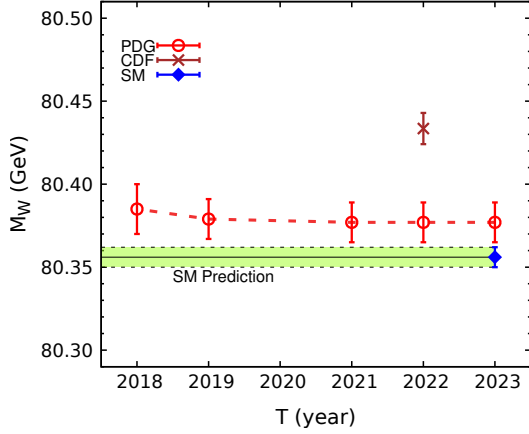


(a) History of M_t (top quark pole mass).

| source | error (%) | memo |
|--------------|-----------|-------------|
| $ V_{cb} $ | 49.6 | Exclusive |
| η_{ct} | 20.7 | $c - t$ Box |
| $\bar{\eta}$ | 13.3 | AOF |
| η_{cc} | 8.8 | $c - c$ Box |
| ξ_{LD} | 2.1 | RBC-UKQCD |
| $\bar{\rho}$ | 2.1 | AOF |
| \hat{B}_K | 1.7 | FLAG |
| \vdots | \vdots | \vdots |

(b) Error budget for $|\varepsilon_K|^{SM}$

Table 7: (a) M_t history (b) error budget for $|\varepsilon_K|^{SM}$.



(a) History of M_W .

| Source | M_W (GeV) | Ref. |
|----------|-------------|------|
| SM-2023 | 80.356(6) | [28] |
| CDF-2022 | 80.4335(94) | [33] |
| PDG-2023 | 80.377(12) | [28] |
| PDG-2022 | 80.377(12) | [28] |
| PDG-2021 | 80.377(12) | [10] |
| PDG-2019 | 80.379(12) | [34] |
| PDG-2018 | 80.385(15) | [35] |

(b) Table of M_W .

Table 8: W -boson mass: (a) M_W versus time, and (b) table of M_W .

and the brown cross represents the CDF-2022 result. The upside is that the CDF-2022 result is the most precise experimental result for M_W . The downside, however, is that it deviates by 6.9σ from the standard model prediction (SM-2023). Here we use the SM-2023 result for M_W to evaluate ε_K .

8. Results for ε_K

In Fig. 1 we show results for $|\varepsilon_K|$ evaluated directly from the standard model (SM) with the lattice QCD inputs given in the previous sections. In Fig. 1 (a), the blue curve represents the theoretical evaluation of $|\varepsilon_K|$ obtained using the FLAG-23 results for \hat{B}_K , AOF for Wolfenstein parameters, the FNAL/MILC-22 results for exclusive $|V_{cb}|$, results for ξ_0 with the indirect method, and the RBC-UKQCD estimate for ξ_{LD} . The red curve in Fig. 1 represents the experimental results for $|\varepsilon_K|$. In Fig. 1 (b), the blue curve represents the theoretical evaluation of $|\varepsilon_K|$ obtained using the same input parameters as in Fig. 1 (a) except for $|V_{cb}|$. Here we use the 1S-scheme results for inclusive $|V_{cb}|$ instead of those for exclusive $|V_{cb}|$.

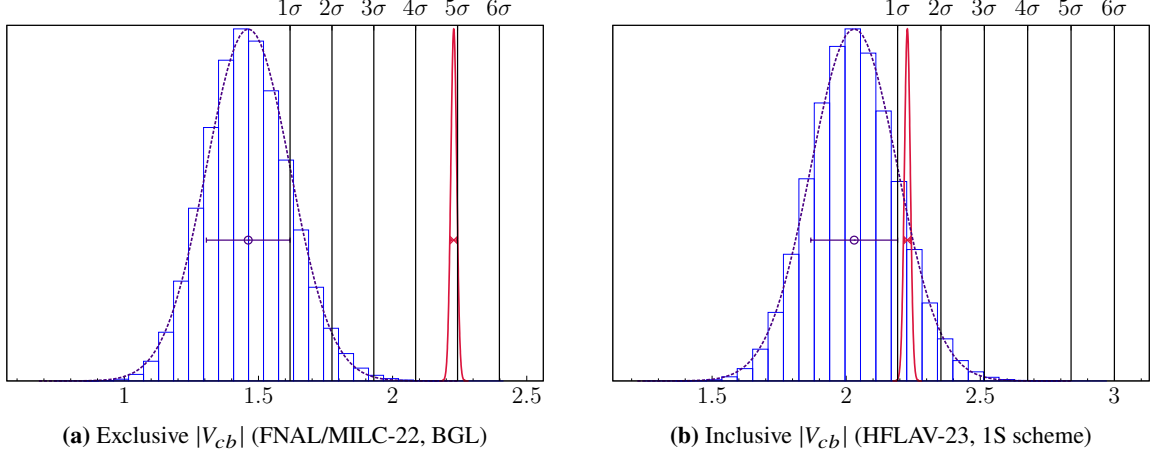


Figure 1: $|\varepsilon_K|$ with (a) exclusive $|V_{cb}|$ (left) and (b) inclusive $|V_{cb}|$ (right) in units of 1.0×10^{-3} .

Our results for $|\varepsilon_K|^{\text{SM}}$ and $\Delta\varepsilon_K = |\varepsilon_K^{\text{Exp}}| - |\varepsilon_K^{\text{SM}}|$ are summarized in Table 9. Here, the superscript SM represents the theoretical expectation value of $|\varepsilon_K|$ obtained directly from the SM. The superscript Exp represents the experimental value of $|\varepsilon_K| = 2.228(11) \times 10^{-3}$. Results in Table 9 (a) are obtained using the RBC-UKQCD estimate for ξ_{LD} , and those in Table 9 (b) are obtained using the BGI estimate for ξ_{LD} . In Table 9 (a), we find that the theoretical expectation values of $|\varepsilon_K|^{\text{SM}}$ with lattice QCD inputs (with exclusive $|V_{cb}|$) have $4.9\sigma \sim 3.9\sigma$ tension with the experimental value of $|\varepsilon_K|^{\text{Exp}}$, while there is no tension with inclusive $|V_{cb}|$ (obtained using the heavy quark expansion and QCD sum rules).

In Fig. 2 (a), we show the time evolution of $\Delta\varepsilon_K/\sigma$ starting from 2012 till 2023. In 2012, $\Delta\varepsilon_K$ was 2.5σ , but now it is 4.9σ with exclusive $|V_{cb}|$ (FNAL/MILC-22, BGL). Here we use the results

| $ V_{cb} $ | method | source | $ \varepsilon_K ^{\text{SM}}$ | $\Delta\varepsilon_K$ |
|------------|---------|--------------|-------------------------------|-----------------------|
| exclusive | BGL | FNAL/MILC-22 | 1.462 ± 0.156 | 4.90σ |
| exclusive | comb | HFLAV-23 | 1.561 ± 0.138 | 4.84σ |
| exclusive | comb | FLAG-23 | 1.619 ± 0.157 | 3.88σ |
| exclusive | comb | HPQCD-23 | 1.594 ± 0.162 | 3.90σ |
| inclusive | 1S | HFLAV-23 | 2.030 ± 0.162 | 1.22σ |
| inclusive | kinetic | Gambino-21 | 2.063 ± 0.169 | 0.98σ |

(a) $|\varepsilon_K|$ with RBC-UKQCD estimate for ξ_{LD}

| $ V_{cb} $ | method | reference | $ \varepsilon_K ^{\text{SM}}$ | $\Delta\varepsilon_K$ |
|------------|--------|--------------|-------------------------------|-----------------------|
| exclusive | BGL | FNAL/MILC-21 | 1.510 ± 0.159 | 4.52σ |
| exclusive | comb | HFLAV-23 | 1.609 ± 0.140 | 4.41σ |

(b) $|\varepsilon_K|$ with BGI estimate for ξ_{LD}

Table 9: $|\varepsilon_K|$ in units of 1.0×10^{-3} , and $\Delta\varepsilon_K = |\varepsilon_K|^{\text{Exp}} - |\varepsilon_K|^{\text{SM}}$ in units of σ .

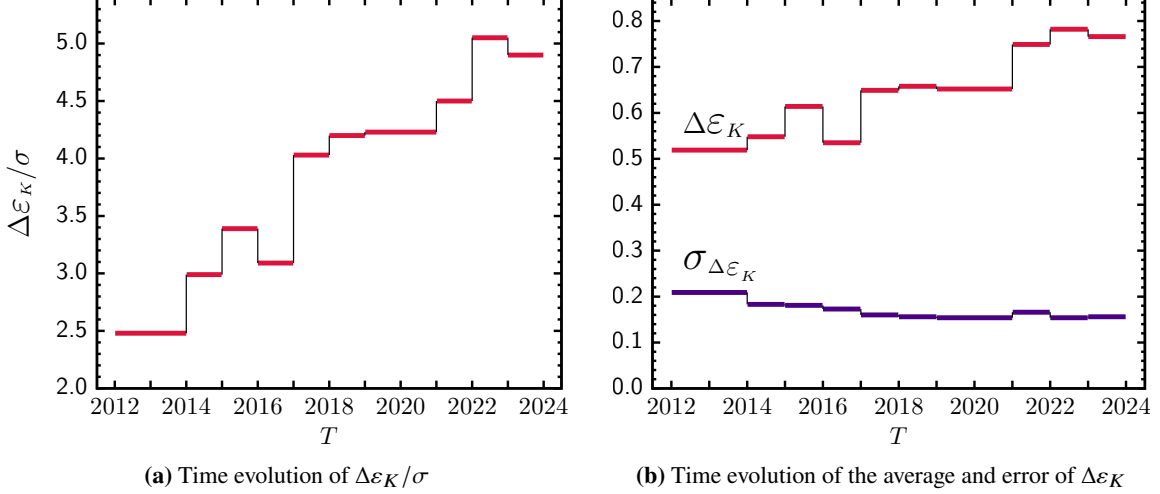


Figure 2: Time history of (a) $\Delta\varepsilon_K/\sigma$, and (b) $\Delta\varepsilon_K$ and $\sigma_{\Delta\varepsilon_K}$.

for exclusive $|V_{cb}|$ from FNAL/MILC-22, since it contains the most comprehensive analysis of the $\bar{B} \rightarrow D^* \ell \bar{\nu}$ decays at both zero recoil and non-zero recoil, while it incorporates both BELLE and BABAR experimental results. In Fig. 2 (b) we show the time evolution of the average $\Delta\varepsilon_K$ and the error $\sigma_{\Delta\varepsilon_K}$ during the period of 2012–2023.

At present we find $|V_{cb}|$ gives the largest error ($\approx 50\%$) in $|\varepsilon_K|^{\text{SM}}$. Refer to Table 7 (b) for more details. Hence, it is essential to reduce the errors in $|V_{cb}|$ as much as possible. Part of the errors come from experiments in BELLE, BELLE2, BABAR, and LHCb, which are beyond our control but will decrease thanks to on-going accumulation of higher statistics in BELLE2 and LHCb. Part of the errors come from the theory used to evaluate the semi-leptonic form factors for $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}$ decays, using the tools of lattice QCD. In order to reduce the errors on the theoretical side, there is an on-going project to determine exclusive $|V_{cb}|$ using the Oktay-Kronfeld (OK) action for the heavy quarks to calculate the form factors for $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}$ decays [36–42].

A large portion of interesting results for $|\varepsilon_K|^{\text{SM}}$ and $\Delta\varepsilon_K$ could not be presented in Table 9 and in Fig. 2 due to lack of space: For example, results for $|\varepsilon_K|^{\text{SM}}$ obtained using exclusive $|V_{cb}|$ (FLAG-23) with the BGI estimate for ξ_{LD} , results for $|\varepsilon_K|^{\text{SM}}$ obtained using ξ_0 determined by the direct method, and so on. We plan to report them collectively in Ref. [43]. We find that there was another analysis on ε_K in Ref. [44].

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