PROCEEDINGS OF SCIENCE

# Precision muonium spectroscopy

R Iwai[1*], M Abe[1], S Fukumura[2], M Fushihara[2], Y Goto[2], M Hiraishi[3], S Kanda[1], S Nishimura[1], H Okabe[4], K Sasaki[1], P Strasser[1], K Shimomura[1], H Tada[2], N Teshima[1] and H A Torii[5]

[1]High Energy Accelerator Research Organization (KEK), Ibaraki, Japan
[2]Nagoya University, Nagoya, Japan
[3]Ibaraki University, Ibaraki, Japan
[4]Tohoku University, Miyagi, Japan
[5]The University of Tokyo, Tokyo, Japan

E-mail: riwai@post.kek.jp

**Abstract.**
In muon physics at low energies (keV-MeV), spectroscopy of muonium atoms takes a significant role in fundamental physics at precision and intensity frontiers. Efforts are invested nowadays in testing theoretical calculations of the Standard Model with a precision down to $\approx$ 1 ppb. New measurements for two energy intervals of the muonium, the 1s hyperfine structure and the 1s-2s interval, are designed utilizing new generations of muon sources and new experimental techniques for microwave and laser spectroscopy. These ongoing experiments will also determine the muon mass and magnetic moment with unprecedented precisions. This paper first reviews the current experimental activities on precision spectroscopy of the muonium atoms. The latter part focuses on new measurements of the 1s hyperfine splitting by the MuSEUM collaboration at J-PARC. Future measurement plans using a rectangular shaped microwave cavity are also described.

PoS(Muon4Future2023)007

## 1. Introduction

Muonium ($Mu = \mu^+ + e^-$) is a light hydrogen isotope with the proton replaced by a positive muon. In calculating its hydrogen-like energy levels, it is free from considerations of finite nuclear size effects, because it is simply made of two leptons. The accurate theoretical calculations are thus possible with the theoretical bound-state QED. While, using $Mu$ atoms for precision spectroscopy has several experimental advantages, owing to the muon's moderate mass ($m_\mu \approx$105 MeV). This production threshold allows us to produce muons in copious numbers with a high power ($\approx$1 MW) proton beam at moderate energies (typically up to a few-GeV). The muon lifetime $\tau_\mu \approx$2.2 µs allows us to efficiently transport a muon beam to the experimental site, providing a sufficient measurement time to perform various experiments. In spectroscopy of $Mu$ atoms, it provides a natural line width of 145 kHz (FWHM).

To attain higher precisions in spectroscopy, one needs to measure the energy intervals involved with the most populated 1s state. The following two energy intervals have been most precisely measured in spectroscopy of $Mu$ atoms. Upcoming new measurements at J-PARC and PSI will further improve the precisions with new a generation of muon sources:

*Muon4Future Workshop (Muon4Future2023)*
*29–31 May, 2023*
*Venezia, Italy*

*Speaker

*1s hyperfine structure*
The hyperfine structure (HFS) of $Mu$ ($\nu_{\mathrm{HFS}}$ ≈4.463 GHz) arises from an interaction between the spins of the muon and electron. This energy interval has been most precisely measured with a relative precision of 12 ppb at LAMPF in 1999 [1]. This value has been also providing the current knowledge of the muon mass and magnetic moment (120 ppb for each). In the previous measurement, the polarized $Mu$ atoms were formed by stopping surface muon beam ($p_\mu \approx 28\,\mathrm{MeV/c}$, spins fully polarized) in a krypton gas. A microwave field was subsequently applied to induce a transition between the spin states. The flipped muon spins were observed from decay positrons, preferably ejected along the muon spin direction. To accurately determine the resonance line center, a resonance narrowing technique was used: only the long lived $Mu$ atoms (several times longer than $\tau_\mu$) have been used for the analysis. At LAMPF providing continuous muon beams up to few $\times 10^7\,\mu^+/\mathrm{s}$, this technique was realized by chopping the muon beam, and only 29 % of muon flux were used for the measurement. Hence, for new measurements of the $\nu_{\mathrm{HFS}}$, a new high intensity pulsed muon beamline at J-PARC MUSE is an ideal muon source [2]. Utilizing the new beamline (H-line) at J-PARC, the MuSEUM collaboration aims to measure the $\nu_{\mathrm{HFS}}$ with a relative precision down to ≈1 ppb. More details are provided in the following sections.

*1s-2s interval*
The energy interval between the 1s-2s states ($\nu_{1s2s}$ ≈2.446 PHz) is measured with a doppler-free two-photon laser spectroscopy. Up to the present, this energy level has been measured with a relative precision of 4 ppb at RAL in 1999 [3]. In this measurement, a pulsed muon beam was impinged on a surface of a $SiO_2$ target, converting into a cloud of $Mu$ atoms in a vacuum. The 1s state $Mu$ atoms were excited to the 2s state by counter-propagating two laser pulse shots with a wavelength of 244 nm. The excited $Mu$ atoms were subsequently ionized with the same laser pulse, so that solely positive muons were electromagnetically transported to a detection system. The new measurements are planned at PSI [4] and J-PARC [5]. For example, the Mu-MASS collaboration at PSI utilizes a cold muon beam developed by [6]. In this beamline, low energy muons (1-30 keV with ≈1 kHz) can be focused on a sub-mm beam spot size to efficiently stop muons on the surface of the $SiO_2$ target. The Mu-MASS collaboration also develops a new CW-laser system to reduce the systematic uncertainty related to the chirp effect of the laser pulse. The collaboration aims to measure the $\nu_{1s2s}$ with a relative precision down to ≈1 ppt.

Combining results from the above two experiments planned in the near future, one can precisely test the theoretical calculation of the Standard Model. For example, the theoretical prediction of the HFS is $\nu_{\mathrm{HFS}} = 4\,463\,302\,872(515)\,\mathrm{Hz}$ (115 ppb) [7]. The contributions to the $\nu_{\mathrm{HFS}}$ are described as,

$$\nu_{\mathrm{HFS}} = \frac{16}{3} Z^4 \alpha^2 \frac{m_e}{m_\mu} \left(1 + \frac{m_e}{m_\mu}\right)^{-3} c R_\infty + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{QCD}} + \Delta_{\mathrm{EW}}, \tag{1}$$

where $Z$ is the charge ratio of the positive muon and positron charge, $\alpha$ is the fine structure constant, $m_e$ is the electron mass, $m_\mu$ is the muon mass, $c$ is the speed of light and $R_\infty$ is the Rydberg constant. In Equation 1, All contributions from QED calculations are included in the term $\Delta_{\mathrm{QED}}$. The contributions from electroweak interactions and strong interactions are included in $\Delta_{\mathrm{EW}}$ (237 Hz) and $\Delta_{\mathrm{QCD}}$ (-65 Hz), respectively. The current theoretical uncertainty of 515 Hz is dominated by the limited knowledge of the muon mass $m_\mu$, which originates an uncertainty of 512 Hz. Nevertheless, laser spectroscopy measuring the $\nu_{1s2s}$ will be able to determine the $m_\mu$ via,

$$\nu_{1s2s} \approx \frac{3}{4}\frac{R_\infty c}{1+m_e/m_\mu}. \tag{2}$$

With the new value of the $\nu_{1s2s}$, the uncertainty originating from the muon mass in Equation 1 will be reduced by two orders of magnitude (several-Hz), Hence, the combined new measurements of $\nu_{\mathrm{HFS}}$ and $\nu_{1s2s}$ will provide a test of the Standard Model calculations with a $\approx$1 ppb level, including contributions of the $\Delta_{\mathrm{EW}}$ and $\Delta_{\mathrm{QCD}}$.

## 2. New HFS measurements at J-PARC

The section below describes activities of the MuSEUM collaboration, which has been performing a series of new measurements of the HFS of $Mu$ atoms at J-PARC. The principle of the measurement is shown in Figure 1. Because of the Zeeman effect, the HFS of $Mu$ has four energy levels depending on the external magnetic field strength. Without having any external magnetic fields, the $\nu_{\mathrm{HFS}}$ corresponds to an energy interval between the spin singlet and degenerated spin triplet states (orange arrows in Figure 1). The measurements of this energy interval have been completed at J-PARC D-line in 2016-2018 [8, 9]. The most precise value of the $\nu_{\mathrm{HFS}}$ without external magnetic fields (159 ppb) has been attained by [9] with a newly developed technique to analyze Rabi oscillations.

When a strong magnetic field is applied, two energy intervals with opposite muon spin directions can be measured ($\nu_{12}$ and $\nu_{34}$ in Figure 1). The $\nu_{\mathrm{HFS}}$ can be determined via $\nu_{\mathrm{HFS}} = \nu_{12}+\nu_{34}$, freely for the accuracy of measuring external magnetic field strength. Whereas, by subtracting $\nu_{12}$ from $\nu_{34}$, we get a term with the external magnetic field strength and muon magnetic moment. Because the magnetic field strength can be precisely measured by observing the spin precision of protons (proton NMR), the subtraction provides the proton to muon magnetic moment ratio ($\mu_\mu/\mu_p \propto \nu_{12} - \nu_{34}$). This quantity is strongly correlated with ongoing experiments measuring the anomalous muon magnetic moment $a_\mu$, which is still puzzling with a $4.2\,\sigma$ discrepancy between the theoretical prediction and the measured value [10]. One can measure the $a_\mu$ via,

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}, \tag{3}$$

where the anomalous muon spin precession frequency $\omega_a$ and proton NMR frequency $\omega_p$ are measured in a storage ring. Currently, the MuSEUM collaboration prepares for the first measurement with an external magnetic field strength of 1.7 T (blue arrows in Figure 1, planned in 2023$\sim$). In this measurement, statistical uncertainties will be predominantly reduced with a new H-line at J-PARC, providing the highest muon rate ($\approx 1\times10^8\,\mu^+$/s at a proton beam power of 1 MW) [11]. Moreover, we can maintain the fully polarized muon spins of the surface muons in $Mu$ atoms with a strong magnetic field, enhancing an asymmetry of positron detector counts. The target precisions of the $\nu_{\mathrm{HFS}}$ and $\mu_\mu/\mu_p$ are $\approx$1 ppb and $\approx$10 ppb, respectively, improved by an order of magnitude from the previous work at LAMPF [1]. A cylindrical microwave cavity to sweep two mode frequencies of the $\nu_{12} = 1.896$ GHz and $\nu_{34} = 2.567$ GHz at the 1.7 T field has been developed by [12]. While, precise controlling and monitoring of the magnetic field in the spectroscopy region are keys to precisely determine the $\mu_\mu/\mu_p$. Hence, particularly for this measurement, we have developed a mapping system with multiple CW-NMR probes [13] and a passive shimming system with iron plates [14] to establish the homogeneity of 0.2 ppm (peak to peak).

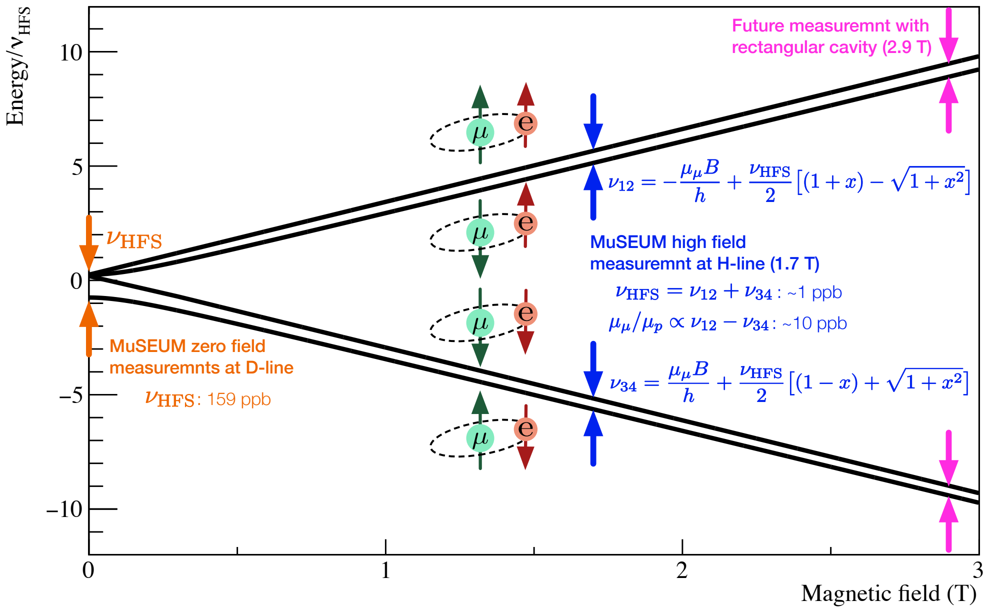


**Figure 1.** Breit-Rabi diagram of 1s state $Mu$ with a sketch showing the measurements by the MuSEUM collaboration. Black lines are normalized energy levels with different external magnetic field strengths. Muon and electron spin configurations are schematically illustrated for each level. Zeeman-split intervals with the presence of external magnetic fields ($\nu_{12}$ and $\nu_{34}$) are also expressed with the muon magnetic moment $\mu_\mu$, magnetic field strength $B$, proton magnetic moment $\mu_p$, Plank constant $h$ and dimensionless quantity $x$ ($x \propto B$).

## 3. Future measurement plans with a rectangular shaped microwave cavity

After completing the measurement at 1.7 T field, we have plans to perform measurements at different magnetic field strengths. This is mainly motivated by further improved determination of the $\mu_\mu/\mu_p$. For example, the uncertainty of measuring the magnetic field strength with the proton NMR is basically constant at different magnetic field strengths, so that it can be relatively reduced at higher magnetic fields. Hence, as a next step, we could perform the experiment at the maximum field strength produced by our magnet (2.9 T, magenta arrows in Figure 1). Nevertheless, two frequency modes of the cylindrical microwave cavity (TM110 for $\nu_{12}$ and TM210 for $\nu_{34}$) are suitable only for the measurement at 1.7 T field. This is due to the mode frequencies of the cylindrical microwave cavity, that only have dependencies on the cavity diameter, and the ratio of the frequencies $F_{\mathrm{TM110}}/F_{\mathrm{TM210}}$ yields a constant. Thus, the measurement can be performed only at 1.7 T field, where the relation $F_{\mathrm{TM110}}/F_{\mathrm{TM210}} \approx \nu_{12}/\nu_{34} = 0.739$ holds.

To allow the measurement at different field strengths (i.e. $\nu_{12}/\nu_{34} \neq 0.739$), a rectangular shaped microwave cavity needs to be used. The mode frequencies of the rectangular shaped microwave cavity have dependencies on three-dimensional parameters (i.e. cavity height, width and depth), providing more flexibility in choosing the mode frequencies. Such a rectangular shaped cavity has been designed for the measurement at 2.9 T field as shown in Figure 2. The TM210 and TM120 modes are used to sweep the frequencies over the $\nu_{12} = 1.778$ GHz and $\nu_{34} = 2.686$ GHz, respectively. These modes are favored because they have oscillating magnetic fields on the transverse plane ($xy$-plane, perpendicular to the muon spin direction), and they have maximum field strengths at the cavity center where most of $Mu$ atoms are formed. In addition, the oscillating fields have uniform distributions along the longitudinal direction, so that the measurement can be free from the muon stopping distribution along the $z$-axis. The oscillating magnetic field of each mode is coupled to a loop antenna of the wave port. Each mode frequency can be tuned by moving an alumina bar placed on the other side of the cavity wall. While, simulation studies have been performed to evaluate the precision in the spectroscopy at 2.9 T field. For example, the right panel of Figure 2 shows the simulation of muon beam injection. The muon beam from H-line is strongly focused by the 2.9 T field produced by the MRI magnet and it enters the cavity. About 32 % of muons form $Mu$ atoms inside the cavity by striping electrons from krypton gas atoms (at krypton gas pressure of 0.6 atm), while, rest of the muons are mostly stopped in the upstream window materials. Furthermore, to measure the muon spin flips with microwave fields, two layers of the positron detectors ($240 \times 240\,\mathrm{mm}^2$) will be placed to count decay positrons in the downstream of the cavity [15]. Resonance curves of the $\nu_{12}$ and $\nu_{34}$ have been calculated by combining time evolution of the muon spin states by microwave fields and detection efficiency of the decay positrons. These studies predict to have similar statistical uncertainty for $\nu_{12}$ and $\nu_{34}$ ($\approx$2 Hz for each), indicating the precision of measuring the $\mu_\mu/\mu_p$ will be improved with reduced systematic uncertainty of measuring the magnetic field. Currently, we are establishing mechanical production of the rectangular shaped cavity.

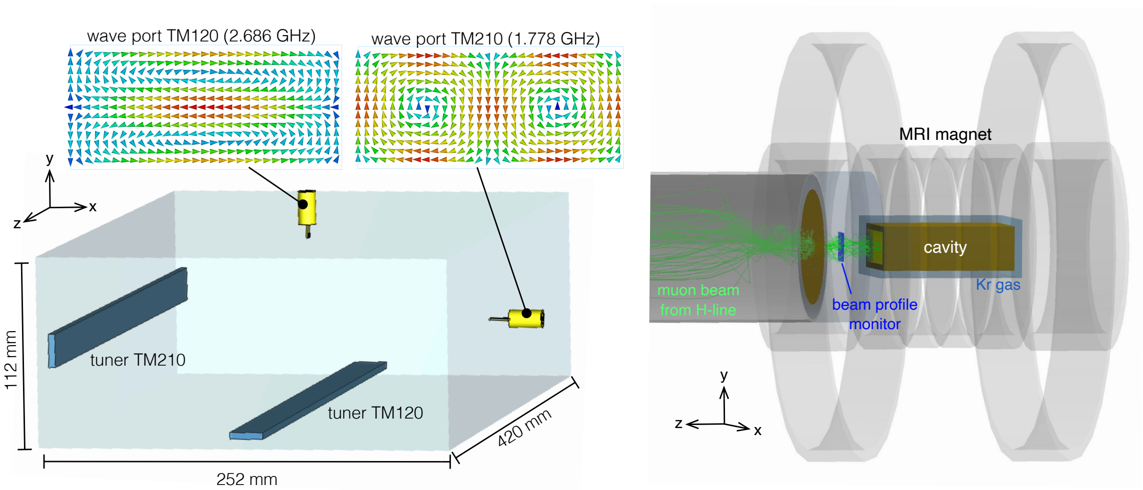


**Figure 2.** Design of the rectangular shaped microwave cavity for the future measurement at 2.9 T field. Left: Electromagnetic design of the cavity using CST Studio Suite [16]. Cavity walls are not shown. Two wave ports couple to the oscillating magnetic field of the TM210 and TM120 modes (yellow). Distributions of the oscillating magnetic field on the $xy$-plane are shown on top. A tuner (blue) is placed facing each wave port. Left: Simulation of muon beam injection into the cavity using Geant4 [17]. A muon beam from the H-line (green) enters the cavity by passing window materials and a beam profile monitor [18].

## 4. Conclusion

A new generation of precision muonium spectroscopy is being developed with a new generation of muon sources at PSI and J-PARC. The outcomes of microwave and laser spectroscopy is strongly connected towards testing the Standard Model with an unprecedented precision, also shedding light on the current puzzle of the muon anomalous magnetic moment. The energy interval of the 1s hyperfine structure will soon be the most precisely measured by the MuSEUM collaboration at 1.7 T magnetic field. To perform the measurement at different field strengths, the microwave cavity needs to be conceptually upgraded. Presently, we have designed a new rectangular shaped microwave cavity for the measurement at 2.9 T field.

## 5. Acknowledgment

The authors gratefully acknowledge Prof. Paolo Crivelli (ETH Zürich) for useful discussions. This work is supported by the KAKENHI grants No. 20H05646, 21H04463, 21H04481 and 21J00903.

## References

[1] Liu W *et al.* 1999 *Phys. Rev. Lett.* **82** 711–714
[2] Shimomura K *et al.* 2023 *Journal of Physics: Conference Series* **2462** 012033
[3] Meyer V *et al.* 2000 *Phys. Rev. Lett.* **84**(6) 1136–1139
[4] Cortinovis I *et al.* 2023 *Eur. Phys. J. D* **77** 66
[5] Uetake S 2019 New frontier with laser spectroscopy of muonium talk at the J-PARC Symposium
[6] Prokscha T *et al.* 2008 *Nucl. Instrum. Methods Phys. Res. A* **595** 317–331 ISSN 0168-9002
[7] Eides M I 2019 *Physics Letters B* **795** 113–116
[8] Kanda S *et al.* 2021 *Physics Letters B* **815** 136154 ISSN 0370-2693
[9] Nishimura S *et al.* 2021 *Phys. Rev. A* **104** L020801
[10] Abi B *et al.* 2021 *Phys. Rev. Lett.* **126**(14) 141801
[11] Yamazaki T *et al.* 2023 *EPJ Web Conf.* **282** 01016
[12] Tanaka K S *et al.* 2021 *Progress of Theoretical and Experimental Physics* **2021**
[13] Tada H *et al.* 2022 *IEEE Transactions on Applied Superconductivity* **32** 1–5
[14] Sasaki K *et al.* 2022 *IEEE Transactions on Applied Superconductivity* **32** 1–7
[15] Kanda S 2016 *PoS* **PhotoDet2015** 039
[16] CST Studio Suite `https://www.3ds.com/products-services/simulia/products/cst-studio-suite`
[17] G4Beamline `https://www.muonsinc.com/Website1/G4beamline`
[18] Kanda S 2014 *RIKEN APR* **48**