PROCEEDINGS OF SCIENCE

The $B \to \pi\pi\ell\bar{\nu}$ transition

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V_{ub} is the smallest and least known of all CKM matrix elements; it is currently determined primarily through the exclusive process $B \to \pi\ell\bar{\nu}$, and additional channels to determine it are welcomed by the community. We will present progress toward a lattice QCD determination of $|V_{ub}|$ from the $B \to \pi\pi\ell\bar{\nu}$ process, where the $\pi\pi$ system is in a P-wave and features the $\rho(770)$ resonance as an enhancement. After an overview of the theoretical framework, we will discuss some preliminary results.

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1. Introduction

One of the remaining puzzles of the Standard Model concerns the smallest and least well-known element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2], V_{ub}. The puzzle stems from the discrepancy between the extractions of $|V_{ub}|$ from inclusive analyses, i.e., B mesons decaying to all possible light hadrons with a u quark and a lepton-anti-neutrino pair, and exclusive analyses, i.e., a B meson decaying to a specific final state [3]. Currently, the PDG [4] quotes the inclusive value $|V_{ub}| = 4.13(26) \times 10^{-3}$ and the exclusive value $|V_{ub}| = 3.70(16) \times 10^{-3}$, which are still in tension even after a significant increase in the estimate of systematic uncertainties of the former. Similarly, HFLAV [5] reports $|V_{ub}|^{\text{excl}} = 3.51(12) \times 10^{-3}$ and $|V_{ub}|^{\text{incl}} = 4.19(17) \times 10^{-3}$, which are in larger tension than the PDG values. FLAG [6] also performed an exclusive determination, which agrees with the exclusive analyses from PDG and HFLAV. The five different values are shown in Fig. 1; the fairly large discrepancy between the inclusive and exclusive determinations indicates that something is unclear about this Standard Model parameter, be it New Physics contributions or uncontrolled systematic errors. There are multiple paths forward, but the clearest is to start

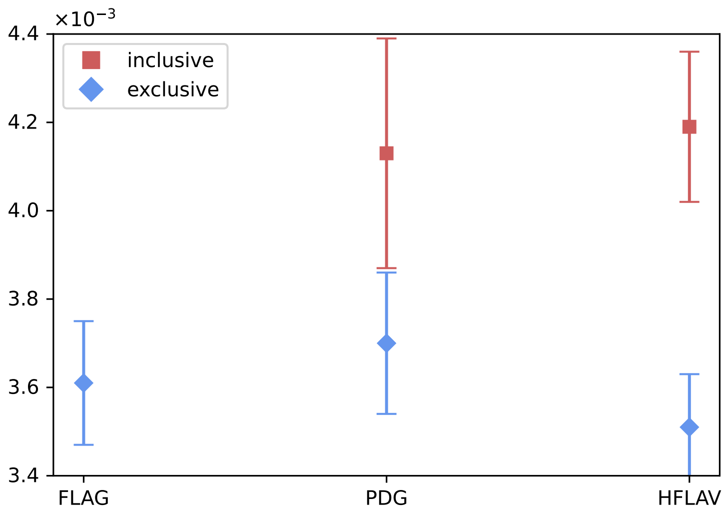

Figure 1: Comparing the values of $|V_{ub}|$ determined by PDG [4], FLAG [6] and [5]. Diamonds correspond to exclusive determinations while squares belong to inclusive determinations.

systematically adding further exclusive semileptonic processes and exploring alternative methods for the inclusive determination [7]. A good candidate for a new exclusive process is $B \to \rho\ell\bar{\nu}$. Because the ρ is a vector resonance, which decays strongly to a pair of pions, this process allows for more angular observables than $B \to \pi\ell\bar{\nu}$. Building a path toward resolving the V_{ub} puzzle by adding $B \to \rho\ell\bar{\nu}$ requires a reliable method for the underlying theory of hadrons. We use lattice QCD, the first-principles, non-perturbative approach to Quantum Chromodynamics (QCD). Previous lattice calculations of $B \to \rho\ell\bar{\nu}$ were performed in the narrow-width approximation, where the ρ is assumed to behave like a QCD-stable hadron [8–11]. This incurs uncontrolled systematic effects of $\mathcal{O}(\frac{\Gamma}{m_\rho})$ [12], where Γ is the ρ strong decay width and m_ρ is the ρ mass. Here, we do not use the narrow-width approximation and instead use the proper finite-volume formalism to treat the

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ρ as a resonance.

These proceedings are organized as follows. In Section 2 we describe our lattice setup and in Section 3 we overview the finite-volume formalism employed in this work. Section 4 shows an example of our data and discusses how we obtain the matrix elements, and we show our vector-current form-factor results in Section 5. Section 6 summarizes.

2. Lattice Setup

In order to obtain a physical result, the appropriate limits in $a \to 0$, $m_\pi \to m_\pi^{\rm phys}$ and $L \to \infty$ need to be taken, but here we present preliminary results only on a single gauge field ensemble with $N_f = 2+1$ clover fermions, where the light-quark mass corresponds to $m_\pi \approx 320$ MeV. The lattice is $L^3 \times N_t = 32^3 \times 96$ with a lattice spacing of $a \approx 0.114$ fm. The dispersion relations for the pion and the B meson can be found in Refs. [13], and [14] respectively, and are consistent with the relativistic continuum dispersion relations. This is achieved by using an anisotropic action for the b quark [15, 16].

3. The Finite-Volume formalism

One limit that differs significantly between what kind of hadrons are present in the process is the $L \to \infty$ limit. For processes such as $B \to \pi\ell\bar{\nu}$, where both the hadrons in the process are stable under QCD, the $L \to \infty$ limit is straightforward - determine the matrix elements related to the form factors at multiple volumes L and then extrapolate to $L = \infty$. This approach fails for the ρ resonance, and needs to be replaced by an analysis of $\pi\pi$ interactions in the finite volume. In their seminal work, Lellouch and Lüscher [17] introduced a normalization correction factor for $K \to \pi\pi$ decays. Their work was generalized by many [18–21] and finally made suitable for $B \to \pi\pi\ell\bar{\nu}$ by Briceño, Hansen, and Walker-Loud in Ref. [22].

When a two-hadron state that interacts strongly is placed in a finite volume, the finite-volume state is not the same as the infinite-volume state. Two main (finite-volume) effects of the strong interactions between them are: i) the finite-volume energy shifts with respect to the infinite volume [23], and ii) the normalization of the finite-volume state changes with respect to the infinite volume [24]. Effect i) leads to the well-known Lüscher method in spectroscopy, where the finite-volume spectrum can be related to the infinite-volume scattering amplitude via the quantization condition [25, 26]

$$\det\left[F^{-1}(E^\star) + T(E^\star)\right]|_{E^\star = E_n^\star} = 0. \tag{1}$$

Here, F is a linear combination of the Lüscher Zeta functions [27], where the coefficients are determined by the symmetry of the finite volume for the given momentum, T is the infinite-volume $2 \to 2$ scattering matrix, and the quantization condition is zero when the square root of the two-particle invariant mass, $\sqrt{s} = E^\star$, is equal to one of the finite-volume energies $E_n^\star$. Effect ii) leads to a change in the relation between the infinite-volume and finite-volume states through [24]

$$|\pi\pi; E_n^\star)\rangle_L \sim \sqrt{R}|\pi\pi(E^\star = E_n^\star)\rangle_\infty, \tag{2}$$

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where $|\pi\pi; E_n^\star\rangle_L$ is the finite-volume state with the finite-volume energy $E_n^\star$, and $|\pi\pi(E^\star = E_n^\star)\rangle_\infty$ is the infinite-volume two-hadron state at the root of invariant mass $E^\star$ equal to the finite-volume energy. Here, R is the residue of the pole associated with the finite-volume state, which is defined as

$$R = \lim_{E\to E_n} \frac{E - E_n}{F^{-1} + T}. \tag{3}$$

The transition amplitude describing the $B \to \rho\ell\bar{\nu}$ can be obtained from the transition amplitude of the $B \to \pi\pi\ell\bar{\nu}$ process when the $\pi\pi$ are in $I = 1$ and P-wave so that they couple to the ρ resonance. The infinite volume $I = 1$, P-wave $B \to \pi\pi\ell\bar{\nu}$ matrix element can be written as

$$\langle\pi\pi; P, \epsilon(P,m)|J^\mu|B; P_B\rangle_\infty, \tag{4}$$

where the initial state is a B meson with four-momentum $P_B = (E_B, \vec{p}_B)$, and the final state $\langle\pi\pi; P, \epsilon(P,m)$ is a two-hadron state with total four-momentum $P = (E_{\pi\pi}, \vec{P})$ and polarization vector $\epsilon(P,m)$. In the following, we limit the discussion to the vector current, $J^\mu = V^\mu = \bar{u}\gamma^\mu b$, whose matrix element can be decomposed under Lorentz symmetry to a single form-factor, $V(q^2, s)$, which is a function of the momentum transfer $q^2 = (P_B - P)^2$ and the two-pion invariant mass $s = (E_{\pi\pi}^2 - \vec{P}^2) = E^{\star 2}$. The Lorentz decomposition takes the form

$$\langle\pi\pi; P, \epsilon(P,m)|J^\mu|B; P_B\rangle_\infty = \frac{T(s)}{k}\frac{iV(q^2,s)}{m_B + 2m_\pi}\varepsilon^{\mu\nu\alpha\beta}\epsilon^{\nu*}(P,m)P_\alpha(P_B)_\beta, \tag{5}$$

where k is the two-pion scattering momentum, $k = \frac{1}{2}\sqrt{s - 4m_\pi^2}$, and $\varepsilon^{\mu\nu\alpha\beta}$ the four-dimensional Levi-Civita symbol. Here, we included a factor of the scattering amplitude T, which fully describes the pole structure and singularities in s. Thus, $V(q^2, s)$ is a smooth function of s, with singularities only in q^2.

To relate the infinite-volume matrix element, $\langle\pi\pi; P, \epsilon(P,m)|J^\mu|B; P_B\rangle_\infty$, with the finite-volume matrix element, $\langle\pi\pi; n, \vec{P}, \Lambda, r|J^\mu|B; P_B\rangle_L$, we project the infinite-volume matrix elements to definite irreducible representations of the relevant finite-volume groups [13], obtaining $\langle\pi\pi; n, \vec{P}, \Lambda, r|J^\mu|B; P_B\rangle_\infty$. Here, n stands for the n-th state of the finite-volume spectrum for total momentum $\vec{P}$, and Λ, r are the irreducible representation and the row in which the $\pi\pi$ state is, respectively. The relation between the finite and infinite-volume matrix elements is then given by

$$\langle\pi\pi; n, \vec{P}, \Lambda, r|J^\mu|B; P_B\rangle_L = \sqrt{R_n^\Lambda}\langle\pi\pi; n, \vec{P}, \Lambda, r|J^\mu|B; P_B\rangle_\infty, \tag{6}$$

where we added indices Λ and n to the residue matrix, R, to emphasize that it needs to be in the same irreducible representation and evaluated at the n-th energy level.

4. Lattice QCD calculation of the 3-point functions

The 3-point correlation functions relevant for the $B \to \pi\pi\ell\bar{\nu}$ are

$$C_3^i = \langle\Omega|O_i(t_\rho; \vec{P}, \Lambda, r)V^\mu(t_J; \vec{q})O_B^\dagger(0; \vec{p}_B)|\Omega\rangle, \tag{7}$$

where the B-meson is created at timeslice 0, the current $V^\mu(t_J; \vec{q})$ with momentum $\vec{q}$ at timeslice t_J changes the b-quark to a light quark, which propagates to timeslice t_ρ, where it is annihilated within

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a two-quark or four-quark operator labeled $O_i(t_\rho;\vec{P},\Lambda,r)$. Depending on the irrep Λ, we use three or four interpolators $O_i(t_\rho;\vec{P},\Lambda,r)$ built from either one- or two-hadron operators and projected to the irreducible representation Λ and row r. We use an $O(a)$ improved current of the form

$$V^\mu(t_J;\vec{q}) = \sqrt{Z^u Z^b}(\bar{u}\gamma^\mu b + d^{(b)}\bar{u}\gamma^\mu\gamma^i\nabla_i b), \tag{8}$$

where $d^{(b)}$ is the improvement coefficient and Z^f are the renormalization coefficients of the flavor-conserving temporal vector currents for quark flavors $f = u, b$. The 3-point correlation functions

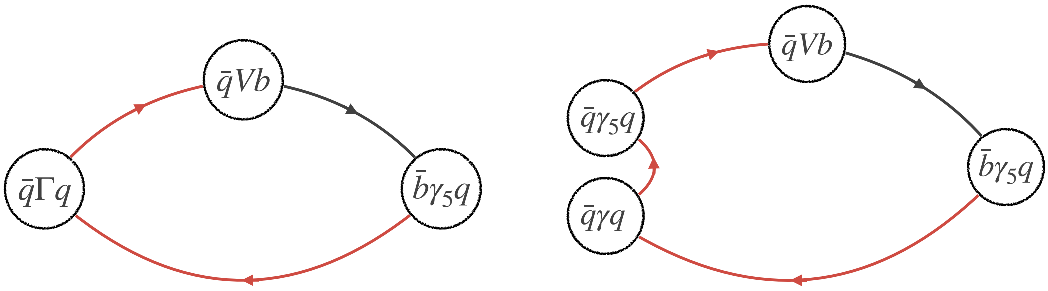

Figure 2: The Wick contractions of the $B \to \pi\pi\ell\bar{\nu}$ transition. The left diagram shows the Wick contraction for the single-hadron sink operator O_i, while the right diagram shows the Wick contraction for the two-hadron sink operator O_i.

can be diagrammatically represented as the Wick contractions shown in Fig. 2, while their numerical value is shown in Fig. 3 for the example B_3 irrep of $\vec{P} = \frac{2\pi}{L}[0,1,1]$. The left two panels show the

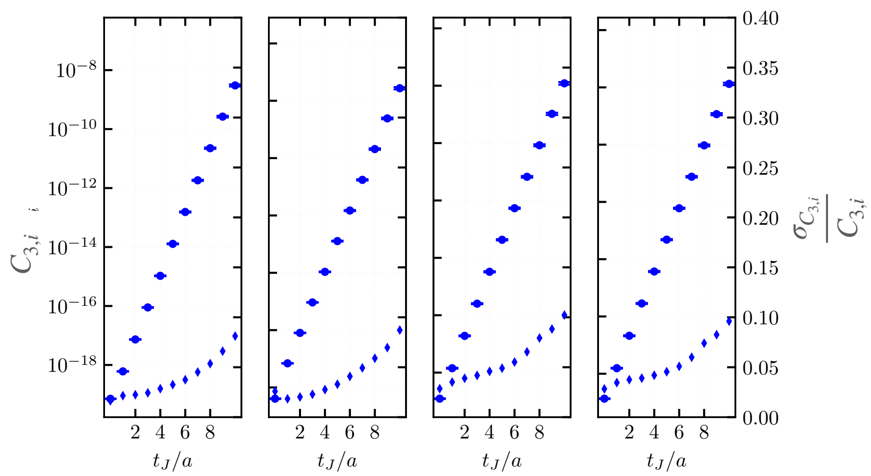

Figure 3: Examples of 3-point correlation functions are shown with filled circles; the left-most panel shows the $O_{i=1} = \bar{q}\gamma_i q$ operator, the second-to-left-most shows the $O_{i=2} = \bar{q}\gamma_t\gamma_i q$ operator, the second-to-right-most shows $O_{i=3} = \pi(\vec{p}_1)\pi(\vec{p}_2)$ with $|\vec{p}_1| = 0$, $|\vec{p}_2| = \frac{2\pi}{L}\sqrt{2}$, and the right-most shows $O_{i=4} = \pi(\vec{p}_1)\pi(\vec{p}_2)$ with $|\vec{p}_1| = \frac{2\pi}{L}$, $|\vec{p}_2| = \frac{2\pi}{L}\sqrt{3}$. The relative uncertainties, $\frac{\sigma_{C_{3,i}}}{C_{3,i}}$, are shown as filled diamonds.

one-hadron, $\bar{q}\gamma_i q$ and $\bar{q}\gamma_t\gamma_i q$ sink operators, while the right two panels show 3-point correlation functions with two-hadron operators. To project the 3-point functions to a single finite-volume state with definite energy, we use the generalized eigenvectors, u_i^n, of the variational analysis [13]:

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$C_3^n = u_i^n C_3^i$. The linear combination yields an optimized correlation function that dominantly overlaps with a single finite-volume state:

$$C_3^n = \langle \pi\pi; n, \vec{P}, \Lambda, r | V^\mu | B; P_B \rangle_L Z_B \frac{e^{-E_n(t_\rho - t_J)} e^{-E_B t_J}}{2E_n 2E_B} + \text{excited state cont.,} \tag{9}$$

where $\langle \pi\pi; n, \vec{P}, \Lambda, r | V^\mu | B; P_B \rangle_L$ is the desired finite-volume matrix element. The "excited state cont." are similar products with matrix elements involving higher-energy states. By fitting the optimized 3-point functions with models that can consider the excited state contamination, we can determine the values of the matrix elements. In Fig. 4, we show such an example. We vary the fit models and fit windows, yielding a plethora of fits for each of the 64 matrix elements spread across different q^2 and $\sqrt{s}$. To determine a single matrix element, we use the AIC approach [28] to average over the models and reduce model dependence.

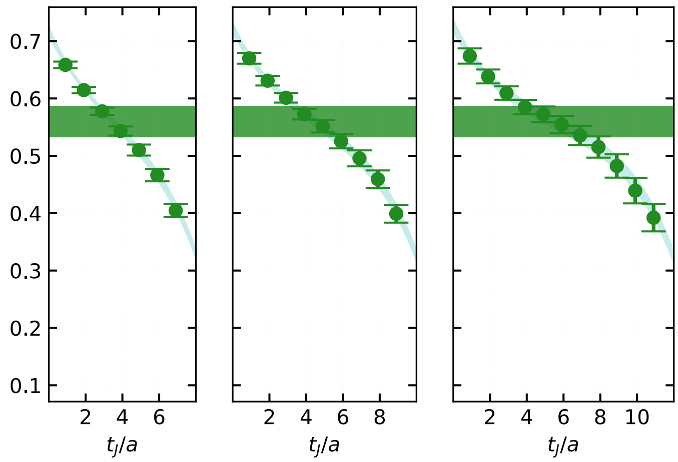

Figure 4: An example of the state-projected 3-point correlation function without the Lorentz symmetry factor. We factor out the leading-order temporal dependence to demonstrate the matrix element and excited state contributions. Shown is the ground state of the irreducible representation B_3 of $\vec{P} = \frac{2\pi}{L}[0, 1, 1]$ with $\vec{p}_B = \frac{2\pi}{L}[0, 1, 1]$. The discrete data points are the lattice-determined matrix element, the light-shaded region is the full model, which includes source and sink excited-state contamination, and the dark-shaded region is the determined matrix element.

5. The fits to the finite-volume matrix elements

The fits to the finite-volume matrix elements are performed using parametrizations of the infinite-volume matrix elements. We consider two types of scattering amplitudes T, BWI and BWII of Ref. [13], and parametrize $V(q^2, s)$ using a generalization of the z-expansion [29–31]

$$V(q^2, s) = \frac{1}{1 - \frac{q^2}{m_{B^\star}}} \sum_{n=0,m=0}^{n_{\max}, m_{\max}} a_{n,m} z(q^2)^n \mathcal{S}^m, \tag{10}$$

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where $\mathcal{S} = \frac{s-(2m_\pi)^2}{(2m_\pi)^2}$ and

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}. \tag{11}$$

Above, t_+ corresponds to the $B\pi$ threshold, and we use $t_0 = 6.0$ in lattice units. The B^*-meson pole is included explicitly as a prefactor in Eq. (10) and is located at our lattice energy $am_{B^*} \approx 3.09556$. We use four different parameterizations of V, which fall into two families of parametrizations:

F1) Combined order K:

$$V(q^2, s) = \frac{1}{1 - \frac{q^2}{m_P^2}} \sum_{n+m \leq K} A_{nm} z^n \mathcal{S}^m, \tag{12}$$

F3) Order N in z, order M is $\mathcal{S}$:

$$V(q^2, s) = \frac{1}{1 - \frac{q^2}{m_P^2}} \sum_{n=0}^{N} \sum_{m=0}^{M} A_{nm} z^n \mathcal{S}^m. \tag{13}$$

The fits of the finite-volume $B \to \pi\pi\ell\bar{\nu}$ matrix elements are summarized in Tab. 1. The average of all the V's, evaluated at $s = m_R^2$, is shown in Fig. 5. The three-dimensional representation is

model	N	M	χ^2/dof
BWI + N0M0	0	0	1.18
BWI + N1M0	1	0	1.07
BWI + N0M1	0	1	0.63
BWI + N0M0	1	1	0.50
BWII + N0M0	0	0	0.55
BWII + N1M0	1	0	0.55
BWII + N0M1	0	1	0.52
BWII + N0M0	1	1	0.51

Table 1: Results of fits to 8 different models of the $B \to \pi\pi\ell\bar{\nu}$ transition amplitude. The left column lists the parametrization (scattering amplitude + V). In the middle two columns, the truncation limits of the sum in Eqs. (12) and (13) are listed, and the right-most column shows the χ^2/dof.

shown in Fig. 6 for the combination of the scattering amplitude BWII and the form-factor model N1M1.

6. Summary

We have outlined the methodology used in determining the vector form factor of the process $B \to \pi\pi\ell\bar{\nu}$ with the $I = 1$, $\ell = 1$ $\pi\pi$ final state and presented preliminary results. For the ensemble shown, with a pion mass of 320 MeV and $a \approx 0.114$ fm, we have achieved an approximately 6% statistical and parametrization uncertainty in the high-q^2 region. These results demonstrate that analyses of heavy-light $1 \to 2$ transition form factors are feasible and reasonable precision can be obtained, encouraging further investigation in this process as well as other processes such as $B \to K\pi\ell\ell$ as well as $B \to D\pi\ell\bar{\nu}$ [32].

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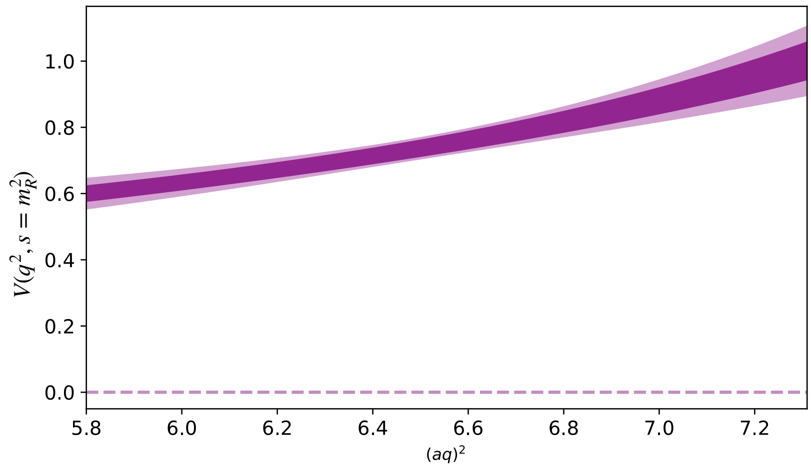

Figure 5: Preliminary results for $V(q^2, s = m_R^2)$. The curve shows the average over all the parametrizations; dark-shaded regions represent statistical uncertainties, while light-shaded regions are the combined statistical and systematical uncertainties from the variation of parametrizations.

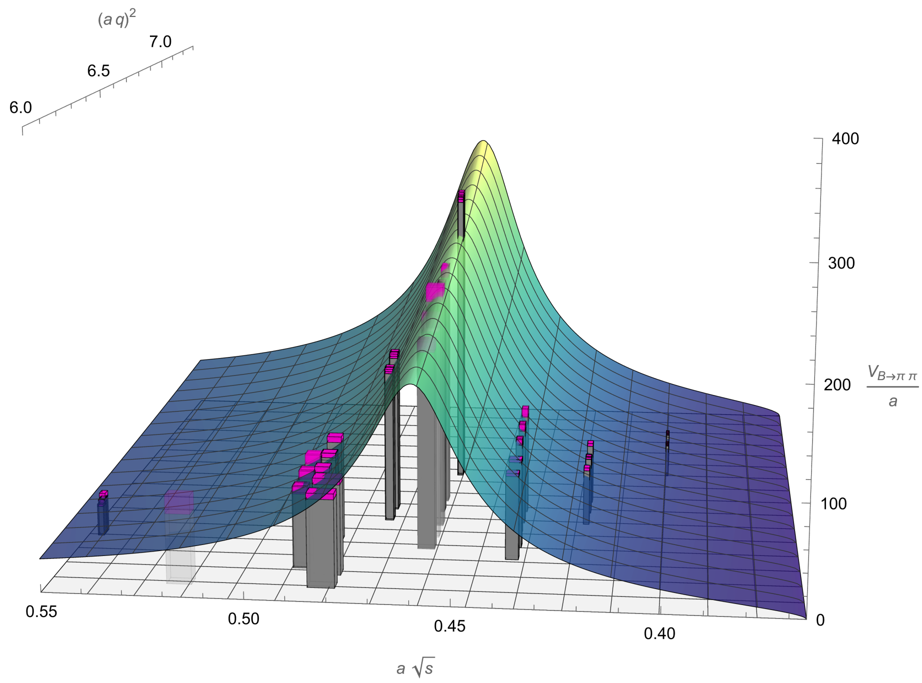

Figure 6: The transition amplitude $V_{B\to\pi\pi} = V_{N1M1}(q^2, s)\frac{T_{BWII}}{k}$ as a function of both $\sqrt{s}$ and q^2. The curve shows the fitted transition amplitude, the bars are the lattice data mapped into infinite volume, and the pink caps represent their statistical uncertainties.

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References

[1] N. Cabibbo, "Unitary Symmetry and Leptonic Decays," *Phys. Rev. Lett.* **10** (1963) 531–533.

[2] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction," *Prog. Theor. Phys.* **49** (1973) 652–657.

[3] **Particle Data Group** Collaboration, M. Tanabashi *et al.*, "Review of Particle Physics," *Phys. Rev. D* **98** no. 3, (2018) 030001.

[4] **Particle Data Group** Collaboration, R. L. Workman *et al.*, "Review of Particle Physics," *PTEP* **2022** (2022) 083C01.

[5] **HFLAV** Collaboration, Y. S. Amhis *et al.*, "Averages of b-hadron, c-hadron, and τ-lepton properties as of 2021," *Phys. Rev. D* **107** no. 5, (2023) 052008, `arXiv:2206.07501 [hep-ex]`.

[6] **Flavour Lattice Averaging Group (FLAG)** Collaboration, Y. Aoki *et al.*, "FLAG Review 2021," *Eur. Phys. J. C* **82** no. 10, (2022) 869, `arXiv:2111.09849 [hep-lat]`.

[7] A. Barone, S. Hashimoto, A. Jüttner, T. Kaneko, and R. Kellermann, "Approaches to inclusive semileptonic $B_{(s)}$-meson decays from Lattice QCD," *JHEP* **07** (2023) 145, `arXiv:2305.14092 [hep-lat]`.

[8] **UKQCD** Collaboration, J. M. Flynn *et al.*, "Lattice study of the decay $\bar{B}_0 \to \rho^+\ell\bar{\nu}_\ell$ Model independent determination of $|V(ub)|$," *Nucl. Phys. B* **461** (1996) 327–349, `arXiv:hep-ph/9506398`.

PoS(EuroPLEx2023)018

[9] J. M. Flynn, Y. Nakagawa, J. Nieves, and H. Toki, "$|V(ub)|$ from Exclusive Semileptonic $B \to \rho$ Decays," *Phys. Lett. B* **675** (2009) 326–331, arXiv:0812.2795 [hep-ph].

[10] **UKQCD** Collaboration, K. C. Bowler, J. F. Gill, C. M. Maynard, and J. M. Flynn, "$B \to \rho\ell\nu$ form-factors in lattice QCD," *JHEP* **05** (2004) 035, arXiv:hep-lat/0402023.

[11] **UKQCD** Collaboration, L. Del Debbio, J. M. Flynn, L. Lellouch, and J. Nieves, "Lattice-constrained parametrizations of form factors for semileptonic and rare radiative B decays," *Phys. Lett.* **B416** (1998) 392–401, arXiv:hep-lat/9708008.

[12] R. A. Briceño and M. T. Hansen, "Multichannel $0 \to 2$ and $1 \to 2$ transition amplitudes for arbitrary spin particles in a finite volume," *Phys. Rev. D* **92** no. 7, (2015) 074509, arXiv:1502.04314 [hep-lat].

[13] C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies, A. Pochinsky, G. Rendon, and S. Syritsyn, "P-wave $\pi\pi$ scattering and the ρ resonance from lattice QCD," *Phys. Rev. D* **96** no. 3, (2017) 034525, arXiv:1704.05439 [hep-lat].

[14] L. Leskovec, S. Meinel, M. Petschlies, J. Negele, S. Paul, A. Pochinsky, and G. Rendon, "A lattice QCD study of the $B \to \pi\pi\ell\bar{\nu}$ transition," *PoS* **LATTICE2022** (2023) 416, arXiv:2212.08833 [hep-lat].

[15] A. X. El-Khadra, A. S. Kronfeld, and P. B. Mackenzie, "Massive fermions in lattice gauge theory," *Phys. Rev. D* **55** (1997) 3933–3957, arXiv:hep-lat/9604004.

[16] P. Chen, "Heavy quarks on anisotropic lattices: The Charmonium spectrum," *Phys. Rev. D* **64** (2001) 034509, arXiv:hep-lat/0006019.

[17] L. Lellouch and M. Luscher, "Weak transition matrix elements from finite volume correlation functions," *Commun. Math. Phys.* **219** (2001) 31–44, arXiv:hep-lat/0003023.

[18] C. J. D. Lin, G. Martinelli, C. T. Sachrajda, and M. Testa, "$K \to \pi\pi$ decays in a finite volume," *Nucl. Phys. B* **619** (2001) 467–498, arXiv:hep-lat/0104006.

[19] C. J. D. Lin, G. Martinelli, E. Pallante, C. T. Sachrajda, and G. Villadoro, "$K^+ \to \pi^+\pi^0$ decays on finite volumes and at next-to-leading order in the chiral expansion," *Nucl. Phys. B* **650** (2003) 301–355, arXiv:hep-lat/0208007.

[20] N. H. Christ, C. Kim, and T. Yamazaki, "Finite volume corrections to the two-particle decay of states with non-zero momentum," *Phys. Rev. D* **72** (2005) 114506, arXiv:hep-lat/0507009.

[21] H. B. Meyer, "Lattice QCD and the Timelike Pion Form Factor," *Phys. Rev. Lett.* **107** (2011) 072002, arXiv:1105.1892 [hep-lat].

[22] R. A. Briceño, M. T. Hansen, and A. Walker-Loud, "Multichannel $1 \to 2$ transition amplitudes in a finite volume," *Phys. Rev. D* **91** no. 3, (2015) 034501, arXiv:1406.5965 [hep-lat].

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[23] R. A. Briceno, J. J. Dudek, and R. D. Young, "Scattering processes and resonances from lattice QCD," *Rev. Mod. Phys.* **90** no. 2, (2018) 025001, arXiv:1706.06223 [hep-lat].

[24] R. A. Briceño, J. J. Dudek, and L. Leskovec, "Constraining $1 + \mathcal{J} \to 2$ coupled-channel amplitudes in finite-volume," *Phys. Rev. D* **104** no. 5, (2021) 054509, arXiv:2105.02017 [hep-lat].

[25] M. Lüscher, "Two particle states on a torus and their relation to the scattering matrix," *Nucl. Phys. B* **354** (1991) 531–578.

[26] C. h. Kim, C. T. Sachrajda, and S. R. Sharpe, "Finite-volume effects for two-hadron states in moving frames," *Nucl. Phys. B* **727** (2005) 218–243, arXiv:hep-lat/0507006.

[27] L. Leskovec and S. Prelovsek, "Scattering phase shifts for two particles of different mass and non-zero total momentum in lattice QCD," *Phys. Rev. D* **85** (2012) 114507, arXiv:1202.2145 [hep-lat].

[28] W. I. Jay and E. T. Neil, "Bayesian model averaging for analysis of lattice field theory results," *Phys. Rev. D* **103** (2021) 114502, arXiv:2008.01069 [stat.ME].

[29] C. G. Boyd, B. Grinstein, and R. F. Lebed, "Constraints on form-factors for exclusive semileptonic heavy to light meson decays," *Phys. Rev. Lett.* **74** (1995) 4603–4606, arXiv:hep-ph/9412324.

[30] C. Bourrely, I. Caprini, and L. Lellouch, "Model-independent description of $B \to \pi\ell\nu$ decays and a determination of $|V(ub)|$," *Phys. Rev. D* **79** (2009) 013008, arXiv:0807.2722 [hep-ph]. [Erratum: Phys.Rev.D 82, 099902 (2010)].

[31] C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies, A. Pochinsky, G. Rendon, and S. Syritsyn, "$\pi\gamma \to \pi\pi$ transition and the ρ radiative decay width from lattice QCD," *Phys. Rev. D* **98** no. 7, (2018) 074502, arXiv:1807.08357 [hep-lat]. [Erratum: Phys.Rev.D 105, 019902 (2022)].

[32] E. J. Gustafson, F. Herren, R. S. Van de Water, R. van Tonder, and M. L. Wagman, "A model independent description of $B \to D\pi\ell\nu$ decays," arXiv:2311.00864 [hep-ph].

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