PROCEEDINGS OF SCIENCE

Track reconstruction of charged particles using a 4D quantum algorithm

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Particle track reconstruction plays a crucial role in the exploration of new physical phenomena, particularly when rare signal tracks are obscured by a significant background. In muon colliders where beam muons interacting with the detector produce secondary and tertiary background particles, track reconstruction can be computationally intensive due to the large number of detector hits. The formulation of the reconstruction task as quadratic unconstrained binary optimisation (QUBO) enables the use of quantum computers, which are believed to offer an advantage over classical computers in such optimisation scenarios. The QUBO parameters are determined by combining spatial and temporal information from detector hits, resulting in a 4D quantum algorithm. To demonstrate the effectiveness of this approach, the quantum algorithm is used to reconstruct signal tracks from samples consisting of Monte Carlo simulated charged particles overlaid with background hits for a Muon Collider tracking detector. We will present the obtained reconstruction performance and discuss possible paths for further improvements.

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1. Introduction

In this work, we present a pattern recognition algorithm for charged particle tracking using a quadratic unconstrained binary optimisation(QUBO) formulation. The cost function of the QUBO, which is computationally equivalent to an Ising model, can be optimised using a quantum computer, potentially providing an advantage over classical algorithms.

The track reconstruction challenge in a muon collider experiment involves identifying a relatively small number of signal tracks within a significant beam induced background (BIB). This background arises from beam muons decaying into secondary and tertiary particles, which subsequently interact with the detector, generating extensive particle showers. Addressing this issue requires the incorporation of timing information as a vital component. Therefore, it is essential to have comprehensive timing data available throughout the entire detector region. This additional information can be integrated into existing track reconstruction algorithms to enhance their performance and effectively evolve them into 4D algorithms. In this study, we present a first investigation of how timing information, utilising a QUBO formulation [1], enhances track reconstruction performance in a muon collider environment.

2. Experimental setup

For this study, we utilised the tracking detector properties of a future muon collider, as described in Ref. [2]. The tracker consists of a vertex detector, an inner and outer tracker, each comprising

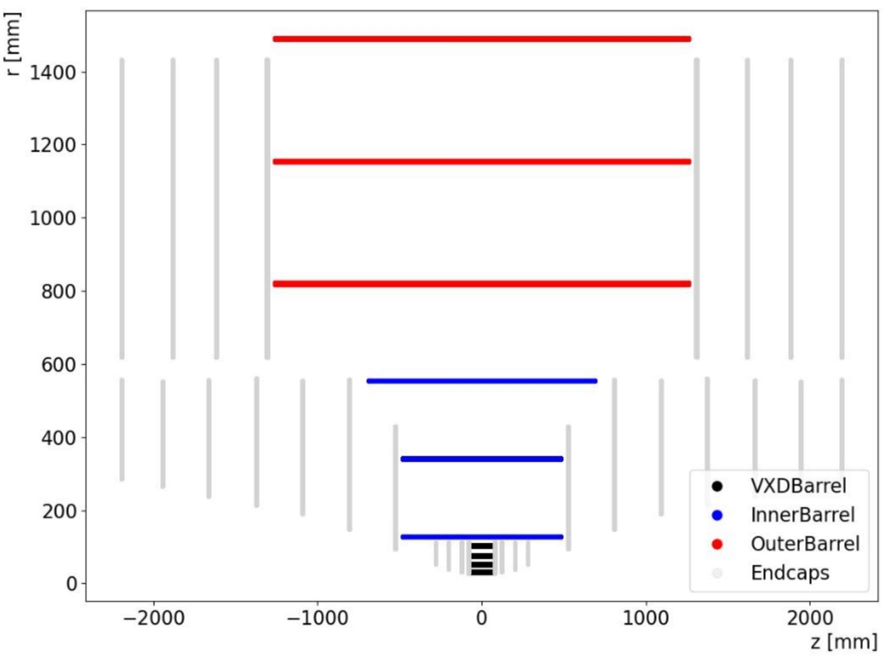

Figure 1: Muon collider tracker system in the r-z plane as described in [2].

barrel and endcap sections. The vertex detector consists of four double layers, while the inner and outer tracker each consist of three single layers. An overview of the detector shape and dimensions is provided in Figure 1. The vertex detector is equipped with pixels measuring 25 × 25 μm^2 in size and offers a time resolution of 30 ps. The inner tracker utilises asymmetric macropixels measuring 50 × 100 μm^2 with a time resolution of 60 ps. The outer tracker is assumed to have either macropixels or microstrips measuring 50 μm × 10 mm and offers a time resolution of 60 ps. The magnetic field in the tracker is 3.57 T.

3. Data sets and selection

We utilised simulated single muon tracks overlaid with a 1.5 TeV simulated BIB, using the Mars15 model [3]. Muon events, originating from the source and exhibiting a p_T within the range of 0.5 GeV to 5.0 GeV, were generated, whereas p_z is set to zero, allowing the tracks to only populate the transverse plane. Hits in the tracker are considered for the track reconstruction task if they fall within the time range of -3σ_t to +5σ_t relative to the bunch crossing time, with σ_t representing the time resolution of the corresponding detector part.

The pattern recognition task begins with triplets, which consist of three hits from consecutive detector layers. To manage the computational load and maintain high efficiency (around 100%) for transverse momentum (p_T) values greater than 1 GeV, a pre-selection process is applied to the initial doublet candidates, effectively reducing the number of combinatorial candidates. Doublets are formed if they are compatible with the primary vertex in the r-z plane. Triplets, on the other hand, are created by combining two doublets while ensuring compatibility with the primary vertex in the transverse plane. To control computational costs, the construction of doublets and triplets is constrained to a narrow region along the longitudinal plane surrounding the simulated muon track. The pre-selection values were determined by optimising cut values on 10^4 independent signal muon tracks.

4. Methodology

The pattern recognition algorithm is designed to identify triplets originating from the signal muon track, with the goal of finding a complete chain of partly overlapping triplets throughout the detector. To select the correct triplets, the algorithm optimises the QUBO cost function:

$$O = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i \tag{1}$$

Here, T_i and T_j represent triplets of hits, and a_i and b_{ij} are real coefficients. The binary variables T_i and T_j determine whether each triplet is considered false and rejected (set to zero) or true and selected (set to one). In this study, the linear term is disregarded for simplicity. The coefficient b_{ij} describes the relationship between two triplets. For two triplets that do not share any hits, the value is set to zero, effectively making the summand vanish. If it is possible to construct a quadruplet from the two triplets, a negative connection coefficient is assigned. Otherwise a conflict term (+1) is applied. The connection coefficient consists of either a spatial term, a temporal term, or a combination of both. The spatial term is calculated as the average of:

- The maximum angle difference in the r-z plane between two consecutive doublets of the quadruplet, with a function to normalise the results to the [0, 1] interval across the entire data set.
- The difference in curvature of the two triplets, using the formula: $0.5 \cdot (c - \frac{\min(p_{T,t_1}, p_{T,t_2})}{\max(p_{T,t_1}, p_{T,t_2})})$ for t_1, t_2 as triplets with $c = 1$ for a compatible curvature of the triplets and $c = 2$ else, which also maps the results to the [0, 1] range.

The time of recorded interactions between particles and detectors undergoes pre-processing prior to being employed in the computation of QUBO coefficients. This involves recalculating the time in relation to the bunch crossing time of the $\mu^- \mu^+$ beams at the interaction point. The final temporal coefficient is then calculated as the standard deviation of these normalised time values, which is subsequently divided by a constant factor to ensure it falls within the [0, 1] range. This normalisation procedure is crucial to ensure that all components can be uniformly merged into a single b_{ij} value. Finally, the values are re-scaled to fit within the interval [-1.0, -0.9]. This procedure ensures that all components have a noticeable impact on the final b_{ij} value and prevents connections from excessively compensating for the conflict term. The choice of the higher limit in the interval is unlikely to affect the optimisation process unless it becomes positive, signifying a conflict term. This is because the relative differences among the b_{ij} values remain constant.

Minimising the QUBO is equivalent to finding the ground state of the corresponding Ising Hamiltonian. To be computationally feasible on a quantum computer, the optimisation problem has to be divided into sub-problems, referred to as sub-QUBOs. This is necessary because the number of triplets (corresponding to the number of required qubits on a quantum computer) in the QUBO for this study is on the orders of 10^4, which is two orders of magnitude higher than the number of qubits currently available on quantum computing devices. The sub-QUBOs are solved sequentially and then combined to form a global solution.

To optimise a QUBO cost function on a quantum computer, it can be mapped to an Ising Hamiltonian with $T_i \rightarrow (1 + Z_i)/2$, where Z_i represents the third Pauli matrix. This transformed problem can then be optimised using a Variational Quantum Eigensolver (VQE), a hybrid classical-quantum algorithm [4]. In this context, we utilised the Qiskit implementation of VQE [5]. The hardware efficient quantum circuit (ansatz) and the optimiser [6] were adopted from our previous studies [7] on reconstructing charged particle tracks in the LUXE experiment [8], as it proved to be effective.

5. Results

In this work, the results are computed using the matrix diagonalisation method, providing an analytical solution. A sub-QUBO size of 18 is chosen, and the selected triplets are combined into tracks by linking overlapping triplets. To be considered, a reconstructed track must have a minimum of 6 hits. If the majority of these hits originate from the signal muon, the track is deemed matched. In Figure 2, the efficiency and fake rate of the reconstructed tracks are presented. When using only temporal information for the QUBO coefficients, the overall efficiency is relatively low. The spatial-only approach, on the other hand, achieves an efficiency of over 80% for transverse momenta starting from 1 GeV. For particles with a transverse momentum less than 1 GeV, a significant improvement is observed when combining temporal and spatial information. In the region above 1 GeV, there is no significant improvement. An improvement on the fake rate was also noticed for the sub 1 GeV p_T region while both spatial and the combined spatial and temporal approach remain compatible for higher p_T particles.

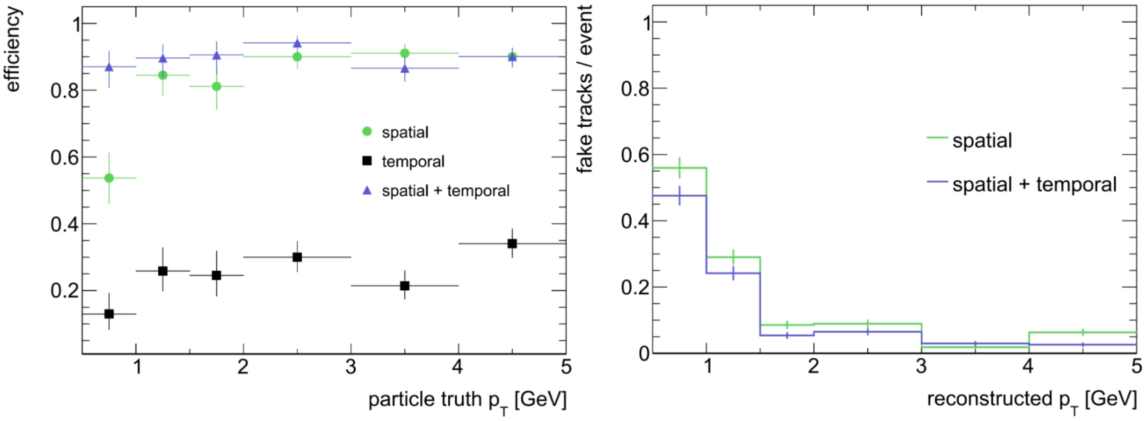

Figure 2: Left: Efficiency with respect to the transverse momentum for coefficients with spatial, temporal and a combined approach. Right: Fake rate per event with respect to the transverse momentum for coefficients with spatial and a combined approach.

To provide a basis for comparison, we compared the results of VQE with the analytical solution for a sub-QUBO size of 7. It was observed that the reconstruction efficiency of VQE was approximately 20% lower in total than that of the analytical solution. This discrepancy arose because VQE sometimes failed to find the ground state, leading to incorrect solutions for some of the corresponding sub-QUBOs. The performance of the VQE depends on the choice of the quantum circuit and the optimisation strategy. Given that VQE is a heuristic method, customisation tailored to the specific problem is vital. Further refinement of the VQE settings is needed to bring it up to a comparable level with the analytical solution.

In this track reconstruction approach, tracks are required to traverse detector layers without the option to skip one. This restriction leads to a reduction in efficiency when compared to state-of-the-art algorithms like the Combinatorial Kalman Filter [9, 10] which is not affected by that. To fairly compare algorithms, it's essential to combine the 4D quantum algorithm with proper track fitting techniques, using accepted triplets, which will part of our future work.

6. Summary

This study explored the application of a 4D quantum algorithm for tracking charged particles in a muon collider environment. In this initial investigation, results were derived using an analytical method. For particles with a momentum exceeding 1 GeV, the performance of both parameter settings, spatial-only and combined spatial and temporal information for the b_{ij} coefficients, showed compatibility within the associated uncertainties. Notably, for particles with momenta between 0.5 and 1 GeV, the addition of temporal information led to a significant improvement. This could be a characteristic of the combined spatial and temporal approach, which is especially intriguing for low-p_T tracking and potentially for tracking heavy particles with low relativistic velocity. Future work will concentrate on expanding this study to cover the entire detector region, including the endcaps.

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