PROCEEDINGS OF SCIENCE

BSD-COBI: New search pipeline to target inspiraling light dark compact objects

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Dark compact objects, like primordial black holes, can span a large range of masses depending on their time and mechanism of formation. In particular, they can have subsolar masses and form binary systems with an inspiral phase that can last for long periods of time. Additionally, these signals have a slow increase of frequency, and, therefore, are well suited to be searched with continuous gravitational waves methods. We present a new pipeline called COmpact Binary Inspiral (COBI), based on the Band Sampled Data (BSD) framework, which specifically targets these signals. We describe the method and propose a possible setup for a search on LIGO-Virgo O3 data. We characterize the pipeline performance in terms of sensitivity and computing cost of the search.

The European Physical Society Conference on High Energy Physics (EPS-HEP2023)
21-25 August 2023
Hamburg, Germany

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PoS(EPS-HEP2023)067

1. Introduction

The study of continuous gravitational waves (CW) stands at the forefront of current gravitational waves searches, offering a unique way to explore the universe. These waves, originated from various sources [1], hold the promise of unraveling profound cosmic phenomena, including the existence and characteristics of dark compact objects, notably primordial black holes. The pursuit of detecting CW signals is a current mathematical and computational challenge as the signal is extremely faint when compared to the levels of noise of the detectors [2]. Despite this, several searches have been carried out on LIGO and Virgo data (see Ref. [3] for a review). These waves can also be used to detect light compact binaries [4, 5].

The present proceedings serve as a summary of the COmpact Binary Inspiral (COBI) pipeline, a new code developed to specifically target CW signals from the inspiral phase of binary compact objects. We provide a description of the signal model used in Sec. 2, a summary of the method in Sec. 3, followed by an explanation of how to construct the search grid and an estimation of the sensitivity of the search in Sec. 4.

2. Signal

The signal searched can be modeled using the 0PN approximation. This is justified by the small masses of the compact objects that are considered and because we are interested in the phase where they are far enough to ignore higher order effects. The spin-up of the gravitational wave emitted during the inspiral phase for a circularized system can be described by a power law of the form $\dot{f} = kf^n$, where $n = 11/3$ and

$$k = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} , \tag{1}$$

being $\mathcal{M}_c$ is the chirp mass of the binary. Integrating this differential equation leads to the frequency evolution of the signal of

$$f_{\text{emission}}(t) = \left[f_0^{1/\alpha} + (1-n)k(t-t_0)\right]^{\alpha} , \tag{2}$$

being $\alpha = \frac{1}{1-n}$ and f_0 the frequency at the reference time t_0.

Additionally, signals reaching the detector are subject to the Doppler effect, which produces a frequency shift dependent on the relative motion between the source and observer. This is given by [6]

$$f(t) = f_{\text{emission}}(t)\left(1 + \frac{\vec{v}\cdot\hat{n}}{c}\right) , \tag{3}$$

where $\vec{v}$ is the detector velocity and $\hat{n}$ the unitary vector pointing to the source, both expressed in the solar barycenter system.

3. Method

Having a model of the signal allows us to use a method to enhance the signal-to-noise (SNR) ratio called heterodyne correction [7]. We assume that the strain in the detector takes the form of

$h(t) = A(t)e^{i\phi(t)} + n(t)$, with $A(t)$ the amplitude of the signal, $\phi(t)$ its phase and $n(t)$ the noise of the detector. The heterodyne correction is based on multiplying this strain by a factor $e^{-i\phi_{\rm corr}(t)}$. The term $\phi_{\rm corr}(t)$ is the modulation of the signal and it is related to the frequency evolution as

$$f(t) = \frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t} \,. \tag{4}$$

Ideally, $\phi_{\rm corr}(t) = \phi(t) + 2\pi f_{\rm ref} t$. If the modeled $\phi(t)$ matches the one of the signal, then the result of the heterodyne correction is to have a purely monochromatic signal at a frequency $f_{\rm ref}$, which can be arbitrarily chosen. If there is any deviation between the model used and the true signal, then there will be some residual frequency variations.

The data is processed to obtain the so-called peakmap as explained in Ref. [6]. This peakmap is constructed setting a critical parameter called $T_{\rm fft}$ which defines the length of the data of which each fast Fourier transform (FFT) is performed. A plot with data containing a signal before and after the heterodyne correction is applied is displayed in Fig. 1. The peakmap without the correction, despite the fact that also contains the signal, does not show at first sight its presence. On the other hand, after the heterodyne correction is performed the signal becomes clearly visible in the peakmap.

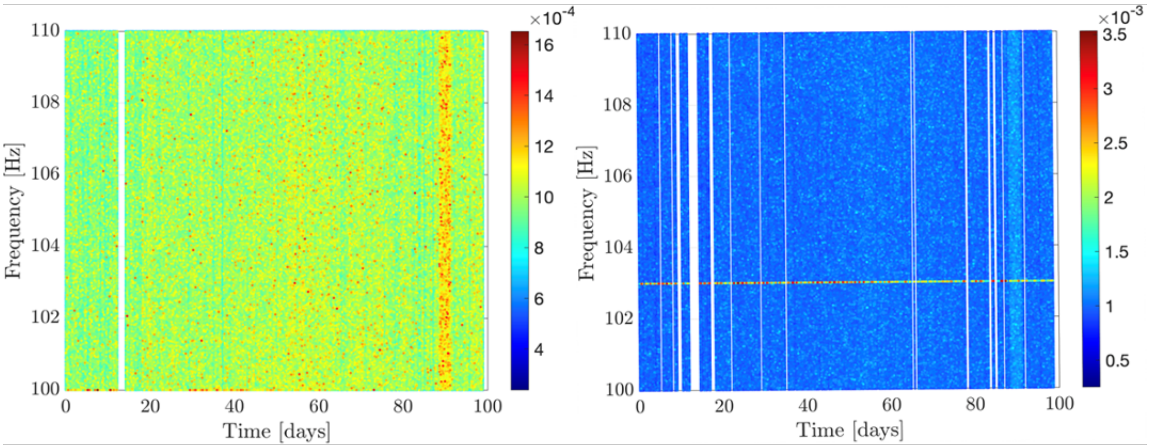

Figure 1: Peakmap of an injected signal with a chirp mass of $10^{-4}\ M_\odot$ at 1 kpc and with a reference frequency of 103 Hz with no correction (left) and after the heterodyne correction (right) with O3b data of LIGO Livingstone. The colorbar shows the amplitude of the FFT transform in 1/Hz. The time is expressed with respect to the initial time of O3b, 1st of November of 2019 at 15:00 UTC.

Projecting the peaks of the peakmap onto the frequency axis a histogram can be made. Calling N the number of counts of peaks inside each histogram frequency bin, μ the mean of N across all the frequency range (excluding the bin being considered) and σ its standard deviation (also excluding the bin being considered) a statistic, called critical ratio (CR), can be created by computing

$$\mathrm{CR} = \frac{N-\mu}{\sigma} \,. \tag{5}$$

The threshold of this CR determines when the candidates are selected provided a level of confidence. We use a threshold of $\mathrm{CR}_{\rm thr} = 5$ as it provides a false alarm probability corresponding to a $5-\sigma$ confidence level when assuming Gaussian noise [6].

For this work, the Band Sampled Data (BSD) framework is used [7]. It consists of a set of band-limited time series, down-sampled and divided in sub-bands, that have been subjected to partial cleaning of instrumental disturbances. The main advantage is that it allows for a fast and flexible data management and a series of tools for CW data analysis are provided, like the ones used for the peakmap creation..

4. Construction of the search grid

To perform the search, the heterodyne correction should be performed for all the possible combinations of $(f_0, \mathcal{M}_c)$ inside the parameter space that wants to be probed. This is typically done by constructing a two-dimensional grid. Usually, the ideal search would cover a wide portion of the parameter space and using a fine grid to make it as deep as the method gets. This requires a large number of points in the grid making it computationally unfeasible so, typically, only one of the two can be achieved.

Therefore, a way to reduce the computational cost of the overall search is introduced. This can be achieved by creating a new variable that maps each point of the $(f_0, \mathcal{M}_c)$ plane into a single dimensional one. The transformation is

$$\xi(f_0, \mathcal{M}_c, T_{obs}) = \left[f_0^{1/\alpha} + (1-n)k(\mathcal{M}_c)T_{\rm obs}\right]^{\alpha} - f_0 \,, \tag{6}$$

where $T_{\rm obs}$ is the observing time that is chosen to be analyzed. This variable exploits the intrinsic degeneracy of both parameters that define the signal to construct a single variable parameter space instead of a two-dimensional one.

Using the new variable defined by Eq. 6 this grid is only one-dimensional and determined by $\xi_{\rm min}$ and $\xi_{\rm max}$. This variable, by definition, determines the amount of frequency variation that a signal will undergo during a period of time $T_{\rm obs}$. Therefore, the minimum and maximum values can be chosen by the computing power available as well as the region of the parameter space that wants to be covered. Then, we can define the following function

$$\mathcal{G}(\xi, f_0, f_0') = \max_t |f(t) - f'(t) - f_0 + f_0'| \,, \tag{7}$$

which quantifies the maximum variation of frequency between two signals with the same ξ but different frequencies, f_0 and f_0', in the time interval $t \in [t_0, t_0 + T_{\rm obs}]$. The grid can be obtained by applying the following steps:

1. Define the limits $\xi_{\rm min}, \xi_{\rm max}, f_0^{\rm min}, f_0^{\rm max}$ according to the region that has to be probed and the computational power available. Set $\xi_0 = \xi_{\rm min}$ and $i = 0$.

2. Solve the following constrained minimization problem for a given ξ_i:

$$\begin{aligned} \min_{f_0} \quad & \max(\mathcal{G}(\xi_i, f_0, f_0^{\rm min}), \mathcal{G}(\xi_i, f_0, f_0^{\rm max})) \\ \text{s.t.} \quad & f_0 \in [f_0^{\rm min}, f_0^{\rm max}] \end{aligned}$$

3. Set $T_{\rm fft}^i = 1/\mathcal{G}_{\rm max}$.

4. Set $i = i + 1$ and $\xi_i = \xi_{i-1} + 1/T^i_{\rm fft}$.
5. If $\xi_i < \xi_{\rm max}$ go to step 2.

This procedure sets one single point in the $(f_0, \mathcal{M}_c)$ plane that is able to test all the iso-ξ hypersurface that crosses it. To achieve having a single point that covers such broad parameter space, the $T_{\rm fft}$ has been left as a free parameter that is set during the construction of such grid.

An example of a grid constructed using this method is displayed in Fig. 2 for a part of the parameter space between $f_0 = 20$ Hz and $f_0 = 90$ Hz. This grid contains only about 10^4, points which is lots of orders of magnitude less when compared to a typical wide parameter space grid used in other CW searches [1–3]. Then, this grid should be used for the desired chunks of data of length $T_{\rm obs}$.

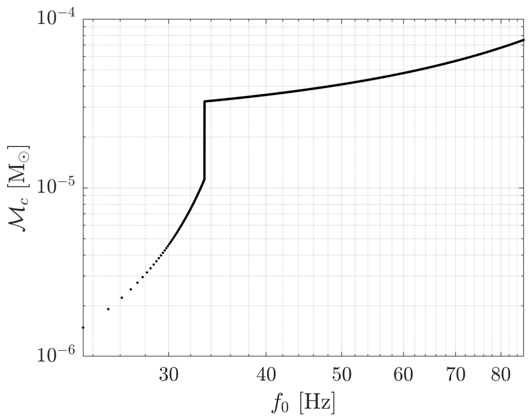

Figure 2: Example of a grid constructed using a variable $T_{\rm fft}$ between the frequencies $f_0 = 20$ Hz and $f_0 = 90$ Hz.

An estimation of the sensitivity, understood as the maximum reachable distance, can be obtained following the procedure explained in Ref. [6]. This is, for a peakmap based search,

$$d_{\rm max} = \frac{4}{\mathcal{B}} \left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3} \left(\frac{\pi f_0}{c}\right)^{3/2} \left(\frac{T_{\rm obs}}{T_{\rm fft}}\right)^{1/4} \sqrt{\frac{T_{\rm fft}}{S_n(f)}} \left[{\rm CR_{thr}} - \sqrt{2}{\rm erfc}^{-1}(2\Gamma)\right]^{-1/2}, \tag{8}$$

where $\mathcal{B}$ is a parameter that depends on the interferometer used and which value can be found in Ref. [8], $S_n(f)$ is the power spectral density of the strain noise and $\Gamma = 0.95$ is the confidence level chosen.

In Fig. 3, the maximum reacheable distance for LIGO Livingstone with the O3b noise curve, $T_{\rm obs} = 0.5$ days, $T_{\rm fft} = 10^4$ s, is displayed. This result indicates that the galactic center, located at $d_{\rm GC} = 8$ kpc, can be reached and its population might be probed.

5. Conclusions

To address the challenge of detecting faint CW coming from the inspiral of compact objects, we have outlined a method based on the heterodyne correction that uses a model of the spin-up of the signal determined by the 0PN approximation.

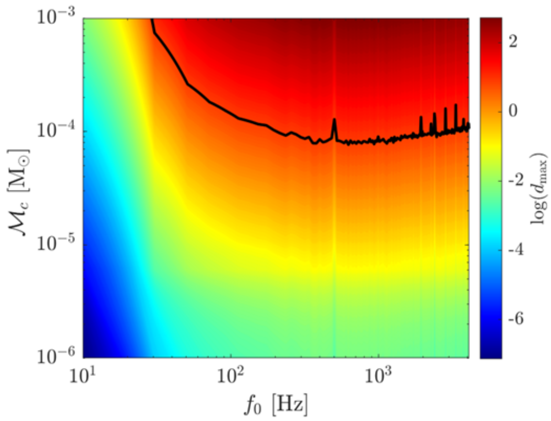

Figure 3: Maximum reachable distance in kpc inside the parameter space. The black line indicates the distance at which the galactic center is located. The noise curve used is that of LIGO Livingstone for O3b, $T_{\rm obs} = 0.5$ days, $T_{\rm fft} = 10^4$ s and $\Gamma = 0.95$.

The core of our proceedings focus on the COBI pipeline, specifically designed to enhance the detectability of these CW signals. COBI is constructed on top of the the Band Sampled Data framework, and exploits a parameter degeneracy to lower the computing cost needed for the search. By defining a one-dimensional search grid through a transformation variable, we significantly streamline the analysis process. This approach offers a remarkable reduction in computing time, while efficiently exploring the parameter space. Furthermore, a possible grid construction strategy has been presented showing how exploiting the degeneracy of parameters allows for a more efficient grid to probe a large parameter space.

Finally, we have presented analytical sensitivity estimates, outlining the minimum detectable strain at a given confidence level, which translates into the maximum reachable distance. We have shown that, for a particular region of the parameter space and $T_{\rm fft}$, the galactic center can theoretically be reached.

Acknowledgments

This material is based upon work supported by NSF's LIGO Laboratory which is a major facility fully funded by the National Science Foundation. MAC is supported by the 2022 FI-00335 grant.

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