

## *CP* violation in *b*-decays

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An important open question in fundamental physics is the origin of the matter-antimatter asymmetry of the Universe. This is connected to the breaking of the fundamental *CP* symmetry. Measurements over the last decades by the *b*-factories and LHCb have validated the Standard Model picture of *CP* violation to high precision. In these proceedings we review the status of measurements as of Summer 2023 and discuss future prospects.

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### 1. Introduction

One of the most important open questions in particle physics is the origin of the matter-antimatter asymmetry of the Universe. This question is related via the Sakharov conditions [1] to the breaking of CP symmetry. In the Standard Model, CP-violation arises in the quark sector from the complex phase in the CKM matrix [2],

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

It is customary to exploit the observed experimental hierarchy of the CKM matrix and use the Wolfenstein parameterization [3],

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

with parameters  $\rho, \eta, \lambda$  and  $A$ . The unitarity of the CKM matrix leads to relations of the form  $\sum_k V_{ij}V_{jk}^* = 0$ , which correspond to a set of six triangles in the  $\rho - \eta$  plane. The area of each triangle is  $J_{CP}/2$  where  $J_{CP} = \lambda^6 A^2 \eta$  is the Jarlskog invariant [4]. Two of the relations

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,$$

and

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0,$$

lead to triangles that have sides of similar magnitude (Fig. 1), allowing them to be explored experimentally. The triangle defined by the first relation, with angles  $\alpha, \beta$  and  $\gamma$ , is the most well known and often referred to as the unitarity triangle.

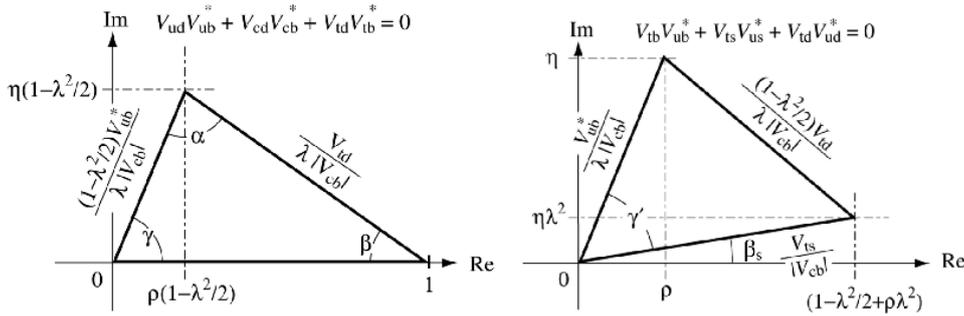
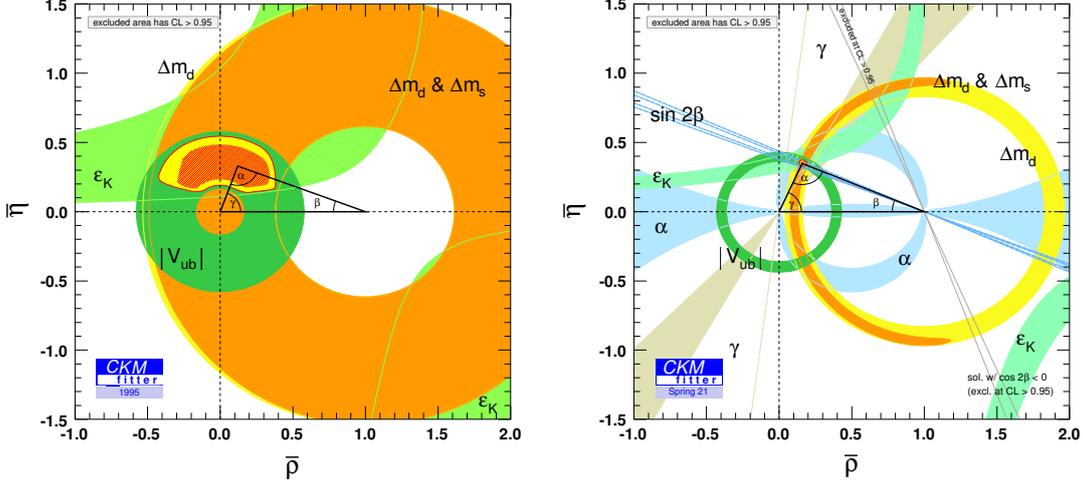


Figure 1: Unitarity triangles for the two relations given in the text.

From the discovery of CP violation in kaon decays in 1964 [5] until the late 1990s, experimental knowledge of CP violation was limited and alternative models to the CKM picture were viable [6]. The measurements made first by the b-factories in the 2000s and more recently by LHCb have

transformed our understanding and confirmed the correctness of the CKM picture [7]. This can be seen in Fig. 2 which shows how our knowledge of the unitarity triangle has evolved from 1995 to 2021. However, the level of CP-violation in the CKM matrix is not sufficient to explain the dominance of matter in the Universe that we observe today, motivating the need for more precise measurements.



**Figure 2:** (left) Summary of the knowledge of the unitarity triangle parameters in 1995 and (right) 2021 made by the CKMFitter group [8].

## 2. CP violation in $B^0$ mixing

Interference between  $B^0$  decays with and without mixing leads to a large CP violating asymmetry in tree-level  $b \rightarrow c\bar{c}s$  transitions. Assuming negligible CP violation in mixing and decay, the asymmetry is

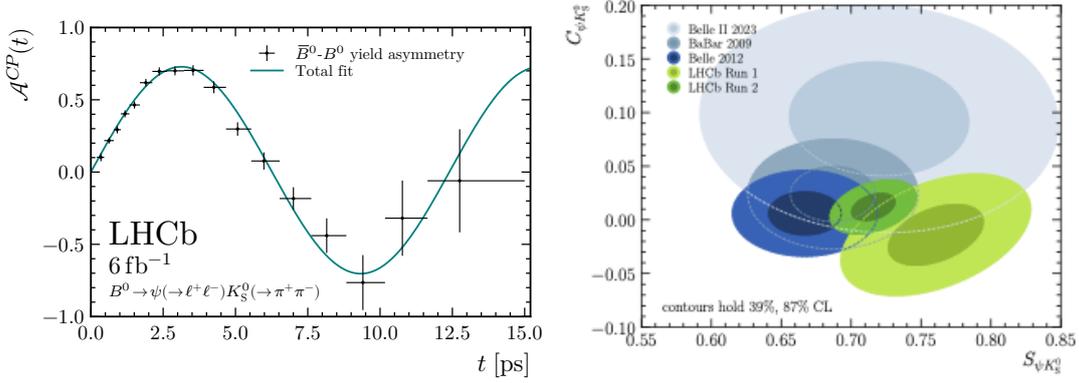
$$A_{CP}(t) = \eta_f \sin 2\beta \sin(\Delta m_d t) \tag{1}$$

where  $\eta_f$  is positive (negative) for a CP even (odd) eigenstate and  $\Delta m_d$  is the mass difference between the mass eigenstates.

The golden mode for determining  $\sin 2\beta$  in tree-level decays is the decay  $B^0 \rightarrow J/\psi K_s$ . The precise determination of  $\sin 2\beta$  made using this mode was one of the most important results obtained by the Babar and Belle experiments. Both Babar and Belle achieved a precision of 0.03 on  $\sin 2\beta$  [9, 10]. A similar precision was achieved by the LHCb experiment using a dataset corresponding to  $3 \text{ fb}^{-1}$  collected at  $\sqrt{s} = 7, 8 \text{ TeV}$  during Run 1 [11].

Recently, LHCb has presented results using the  $6 \text{ fb}^{-1}$  dataset collected at  $\sqrt{s} = 13 \text{ TeV}$  during Run 2 [12, 13]. The larger dataset and increased cross-section for  $b$ -production allows the precision achieved at the  $b$ -factories to be surpassed. The observed CP asymmetry and the fit projection are shown on the left of Fig. 3. The mixing-induced violation parameter CP is measured as  $S = \sin 2\beta = 0.717 \pm 0.013 \text{ (stat)} \pm 0.008 \text{ (syst)}$ . The measurement is dominated by the statistical

uncertainty with the most important systematic uncertainties arising from the calibration of the flavour tagging and the knowledge of  $\Delta\Gamma)d$  Combining the LHCb results from Run 1 and Run 2 gives  $\sin 2\beta = 0.724 \pm 0.014$ , improving on the precision achieved at the  $b$ -factories. The precision of the LHCb result is close to that which was assumed during the early stages of the experimental design [14]. The LHCb result is shown in Fig. 3 (right) together with the results of the  $b$ -factories. Also included on this plot is the first measurement by Belle II using  $190 \text{ fb}^{-1}$ ,  $\sin 2\beta = 0.72 \pm 0.06 \text{ (stat)} \pm 0.016 \text{ (syst)}$  [15]. With its full dataset of  $50 \text{ ab}^{-1}$  [16], Belle II will achieve a precision of 0.005. LHCb will achieve a similar precision with its Upgrade I dataset of  $50 \text{ fb}^{-1}$  [17]. With the LHCb Upgrade II dataset of  $300 \text{ fb}^{-1}$ , which will be collected by the end of Run 6 a precision of 0.003 will be achieved [18].



**Figure 3:** (left) Asymmetry between  $B_s^0$  and  $\bar{B}_s^0$ -tagged decays as function of the decay-time using Run 2 data [26] overlaid with the fit projection. (right) Comparison of measurements of  $\sin 2\beta$  from different experiments.

Measurements in decays dominated by loops have also been made. At this conference, Belle II presented a result in the loop-dominated mode  $B^0 \rightarrow \eta' K_S$  decay [19]. Theoretically, the  $CP$  asymmetry in this mode  $S_{CP}$  is predicted to be equal to  $\sin 2\beta$  with an uncertainty of about 1% [20]. Using a multivariate analysis, Belle II observes a clear signal of  $829 \pm 35$  events and measures  $S_{CP} = 0.67 \pm 0.1 \pm 0.04$  in good agreement with previous measurements [21] and the  $S_{CP} \sim \sin 2\beta$ .

### 3. CP violation in $B_s^0$ mixing

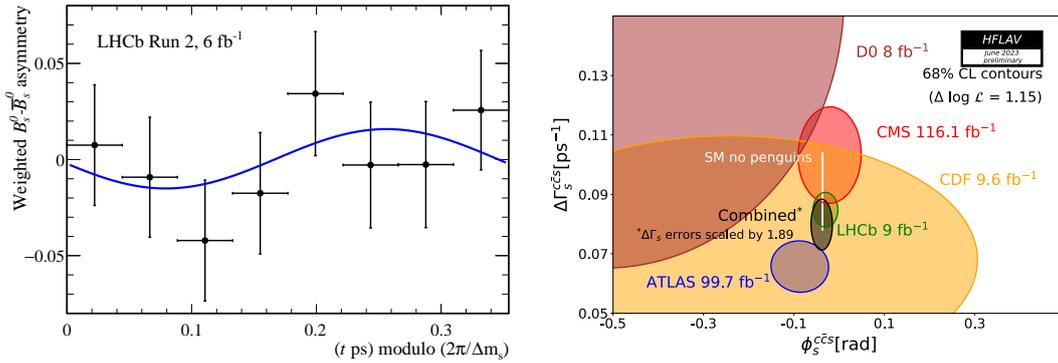
The corresponding variable to  $\beta$  in the  $B_s^0$  sector is  $\phi_s = 2\beta_s$ . The prediction of the Standard Model, [22], indirectly determined from a global fit to the experimental data, is  $\phi_s = -0.0368^{+0.009}_{-0.008}$  rad. Experimentally,  $\phi_s$  can be measured from the time-dependent  $CP$  asymmetry in  $B_s^0$  decays to  $CP$  eigenstates via  $b \rightarrow c\bar{c}s$  transitions. The golden mode for these studies is the decay  $B_s^0 \rightarrow J/\psi \phi$  which has a relatively high branching ratio and clean experimental signature. As this mode is not a  $CP$  eigenstate an angular analysis is needed to disentangle the  $CP$  components and measure  $\phi_s$ . The ATLAS, CMS and LHCb collaborations [23–25] have all performed studies with this mode determining  $\phi_s$  as well as the decay-width,  $\Gamma_s$  and the decay-width difference between the light and heavy eigenstates,  $\Delta\Gamma_s$ .

Measurement	$\phi_s$ [rad]	$\Delta\Gamma_s$ [ $\text{ps}^{-1}$ ]	$\Gamma_s$ [ $\text{ps}^{-1}$ ]
ATLAS	$-0.087 \pm 0.0361 \pm 0.021$	$0.0657 \pm 0.043 \pm 0.0037$	$0.6703 \pm 0.0014 \pm 0.0018$
CMS	$-0.021 \pm 0.044 \pm 0.01$	$0.1032 \pm 0.009 \pm 0.0048$	$0.6531 \pm 0.0042 \pm 0.0026$
LHCb (Run 1)	$-0.058 \pm 0.049 \pm 0.006$	$0.00805 \pm 0.0091 \pm 0.0032$	$0.6603 \pm 0.0027 \pm 0.0015$
LHCb (Run 2)	$-0.039 \pm 0.022 \pm 0.006$	$0.0845 \pm 0.0044 \pm 0.0024$	$0.6527 \pm 0.0030$

**Table 1:** Measurements of  $\phi_s$ ,  $\Delta\Gamma_s$  and  $\Gamma_s$  by the LHC collaborations in the  $B_s^0 \rightarrow J/\psi \phi$  decay mode. *Nota Bene*, LHCb reports  $\Gamma_s - \Gamma_d$  which is converted here to  $\Gamma_s$  using  $\tau_d = 1.519 \pm 0.004$  ps.

Recently, LHCb has presented results using its full Run 2 dataset [13, 26]. The measured asymmetry is shown in Fig. 4 (right). It can be seen that, in contrast to the  $B^0$  case, the observed asymmetry is small as expected in the Standard Model. The study gives  $\phi_s = -0.039 \pm 0.022 \pm 0.006$  rad, the most precise measurement to date. The new HFLAV averages [21] are  $\phi_s = 0.050 \pm 0.017$  rad,  $\Delta\Gamma_s = 0.083 \pm 0.005 \text{ ps}^{-1}$  and  $\Gamma_s = 0.653 \pm 0.0023 \text{ ps}^{-1}$ . For  $\Gamma_s$  and  $\Delta\Gamma_s$  tensions between the values reported by the LHC experiments result in scale factors of around two in the HFLAV combination procedure (Fig. 4 (right)). To understand and resolve the experimental discrepancies further measurements are needed. The LHCb experiment has made measurements of the lifetimes in decays to  $CP$ -eigenstates, most recently determining  $\Delta\Gamma_s = 0.087 \pm 0.012 \pm 0.009 \text{ ps}^{-1}$  from the lifetimes in the  $B_s^0 \rightarrow J/\psi \eta'$  and the  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$  decay modes. This is in agreement with the LHCb measurement using the  $B_s^0 \rightarrow J/\psi \phi$  decay and the world average. It also agrees with the Standard Model prediction,  $\Delta\Gamma_s = 0.091 \pm 0.013 \text{ ps}^{-1}$  [27].

For the golden mode,  $B_s^0 \rightarrow J\psi \phi$  significant experimental progress is expected in the next decade. The current ATLAS and CMS analyses do not fully exploit the data they collected during Run 1 and Run 2. Since the measurements of  $\phi_s$  are so far limited by the dataset size further progress can be expected. All LHC experiments expect to achieve a statistical precision one order of magnitude better than the current one by the end of LHC operations in 2041 [18, 28, 29].



**Figure 4:** (left) Asymmetry between  $B_s^0$  and  $\bar{B}_s^0$ -tagged decays as function of the decay-time using Run 2 data [26] overlaid with the fit projection. (right) Comparison of the measurements of  $\phi_s$  and  $\Delta\Gamma_s$  from different experiments made by HFLAV [21].

Studies of  $CP$  violation can also be made in penguin dominated modes such as the decay  $B_s^0 \rightarrow \phi\phi$  which proceeds via a  $b \rightarrow s\bar{s}$  transition. In this case, the Standard Model expectation is

that  $CP$ -violation is very small ( $\phi_s^{s\bar{s}s} \sim 0$  rad). The most recent LHCb measurements makes use of the full LHCb Run 2 dataset and finds  $\phi_s^{s\bar{s}s} = -0.042 \pm 0.075 \pm 0.009$  rad. Combining with Run 1 data gives  $\phi_s^{s\bar{s}s} = -0.074 \pm 0.069$  rad in agreement with the Standard Model expectation.

#### 4. The angle $\gamma$

Until recently,  $\gamma = \arg(-V_{ud}V_{ub}^*)/V_{cd}V_{cb}^*$  was experimentally the least well determined angle of the unitarity triangle. It can be measured with negligible theoretical uncertainty through the interference of tree-level  $b \rightarrow c$  and  $b \rightarrow u$  transitions. A wide range of techniques have been developed to determine  $\gamma$  including the BBGGZW [30, 31] and GLW [32, 33] methods.

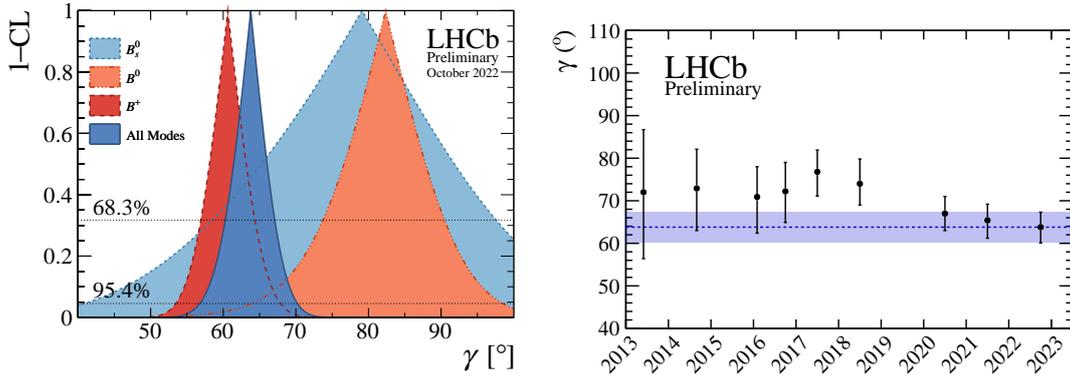
The LHCb collaboration has used a wide variety of  $b$ -hadron decay modes to measure  $\gamma$  using the data collected during LHC Run 1 and 2. The golden mode for these studies is  $B^+ \rightarrow Dh$ , where  $h$  is a charged kaon or pion and  $D$  is the superposition of a  $D^0$  and  $\bar{D}^0$  meson. Theoretical uncertainties in this decay mode are estimated to be  $O(10^{-7})$  [34], well below the level achievable experimental uncertainty now or in the future.

Ambiguities in the extraction of  $\gamma$  mean that the best precision comes from the combination of many modes. To fully exploit the experimental data, information on the hadronic phases of the  $D^0$ -meson from BESIII and CLEO-c are crucial. The world average for  $\gamma$  is dominated by LHCb, with the latest combination [35], from October 2022, giving  $\gamma = (63.8^{+3.5}_{-3.7})^\circ$ , in good agreement with the prediction from indirect measurements  $\gamma = (65.4^{+1.3}_{-1.2})^\circ$ . It is interesting to note that LHCb has already exceeded its goal of measuring  $\gamma$  to  $4^\circ$  precision using the Run 1&2 dataset [17]. Since the October 2022 combination, several new LHCb measurements have been released. Using the decay  $B^0 \rightarrow D^0 K^*$  with  $D^0 \rightarrow K_s h^+ h^-$  LHCb has measured  $\gamma = (49^{+23}_{-18})^\circ$ . Studies of the decay  $B^+ \rightarrow D^* h^+$  with  $D^* \rightarrow D^0 \pi^0(\gamma)$  and  $D^0 \rightarrow K_s h^+ h^-$  give  $\gamma = (65 \pm 14)^\circ$ . More details can be found in Ref. [36]

Belle II is beginning to produce results related to  $\gamma$  [37]. A BPGGZW study using  $128 \text{ fb}^{-1}$  of data collected with Belle II combined with a reanalysis of the Belle dataset ( $711 \text{ fb}^{-1}$ ) gives  $\gamma = (78.4(\text{stat}) \pm 11.4 \pm 0.5(\text{syst}) \pm 1.0(\text{ext}))^\circ$  [38], in good agreement with the current world average. Belle 2 have also presented results of a GLW analysis of  $B^+ \rightarrow D_{CP} K^\pm$  which gives  $\gamma \in [84.5 - 94.5]^\circ$  at 90% confidence level [39]. These are the first towards the Belle II goal to achieve a precision of  $\sim 1^\circ$  on  $\gamma$  with their full dataset of  $50 \text{ ab}^{-1}$  [16]. A similar precision will be achieved by the first phase of the LHCb upgrade program which aims to collect  $50 \text{ fb}^{-1}$  by the end of LHC Run 4 [17]. The ultimate precision on  $\gamma$  ( $\sim 0.4^\circ$ ) will be achieved by the second phase of the LHCb upgrade [18].

#### 5. Summary

The last decades have seen the confirmation of the CKM picture of  $CP$  violation in the quark sector. This transformation has been driven by the measurements made by the  $b$ -factories and LHCb. Though possible new physics is constrained by the current data, there is still room for new physics amplitudes to be present. Over the next decades the data collected by Belle II and with LHCb upgrades will allow to either confirm the Standard Model picture with higher precision or uncover the effects of new physics.



**Figure 5:** (left) LHCb Autumn 2022  $\gamma$  combination. (right) Evolution of the LHCb  $\gamma$  combination versus time (right). Both plots are taken from [35].

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