

Recent Belle II results on hadronic B decays

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We report on recent measurements of B meson decays to hadronic final states using electron-positron collisions at the $\Upsilon(4S)$ resonance collected from 2019 to 2022 by the Belle II experiment and corresponding to an integrated luminosity of 362 fb^{-1} . We present new results to determine the quark-mixing parameter γ , performed combining Belle II data with the full Belle data set, and analyses of two-body decays relevant for the determination of α . We also present recent results on branching ratios and direct CP -violating asymmetries of several B decays, which result in a competitive standard-model test based on the $K\pi$ isospin sum rule and first observations of three new $B \rightarrow D^{(*)} K K_S^0$ decays.

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1. Introduction

Hadronic B decays provide precise constraints on the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix and are sensitive probes of physics beyond the Standard Model (SM). In addition, measurements of new decay channels expand our knowledge of the flavour sector. Belle II [1] is a particle detector designed to study 7-on-4 GeV e^+e^- collisions at the energy of the $\Upsilon(4S)$ resonance, produced at very high luminosity by the SuperKEKB collider located at the KEK laboratory [2]. The $\Upsilon(4S)$ decays almost exclusively into $B\bar{B}$ pairs, resulting in low backgrounds.

Belle II consists of several subdetectors, arranged hermetically in a cylindrical geometry around the interaction point. The innermost detector is a silicon tracker that provides a decay-position (vertex) resolution of about 20–30 μm . A large-radius wire drift chamber measures charged-particle charges, momenta with 0.4% resolution, and dE/dx with about 7% resolution. A time-of-propagation Cherenkov detector and an aerogel ring-imaging Cherenkov detector surround the drift chamber and provide charged-particle identification information, allowing a separation of kaons from pions of up to 4 GeV/ c momentum, with typical 90% efficiency and 5% misidentification rate. A CsI(Tl)-crystal electromagnetic calorimeter measures the energy of electrons and photons, with 1.6%–4% resolution. Layers of plastic scintillators and resistive-plate chambers alternated with iron plates provide muon and K_L^0 reconstruction and identification.

Belle II, which started collecting physics data in March 2019 and aiming to accumulate about 50 ab^{-1} in the next decade, has competitive, unique, or world-leading reach in many key quantities associated with hadronic B decays. The CKM angle $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ is a fundamental constraint for charge-parity (CP) violation in the SM and can be reliably determined in tree-level processes, with negligible loop-amplitude contributions. The angle $\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$ is a limiting factor of the precision of non-SM tests based on global fits of the quark-mixing matrix unitarity and is best determined through measurement of $B \rightarrow \rho\rho$ and $B \rightarrow \pi\pi$ decays. For $B \rightarrow K\pi$ decays, an isospin sum rule combines the branching fractions and CP -violating asymmetries of the decays, providing a null test of the SM. [3] The capability of investigating all the related final states jointly, under consistent experimental settings, is unique to Belle II.

The challenge in analysing these channels lies in the large amount of background coming from $e^+e^- \rightarrow q\bar{q}$ events, where q is a u , d , c , or s quark, called “continuum”. We suppress it combining information on the spatial distribution of momentum and energy in the event in binary classifiers C based on boosted decision trees. [4] These exploit the fact that continuum-background events produce particles collimated into back-to-back jets, while $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ events have a more spherical shape. The determination of the signal yields is mainly based on observables that exploit the specific properties of at-threshold production: the energy difference between the B candidate and the beam energy, $\Delta E = E_B^* - E_{\text{beam}}^*$, and the beam-constrained mass $M_{\text{bc}} = \sqrt{E_{\text{beam}}^*/c^2 - (\mathbf{p}_B^*/c)^2}$, where E_B^* and \mathbf{p}_B^* are the energy and momentum of the B candidate, respectively, and E_{beam}^* is the beam energy, all in the centre-of-mass frame.

2. Determination of CKM angle γ

The angle γ is studied through the interference of $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ transition amplitudes in tree-level B hadronic decays. The current world average $\gamma = \left(65.9^{+3.3}_{-3.5}\right)^\circ$ is dominated by LHCb measurements. [8, 9] The angle γ is determined through different approaches featuring different D final states from B decays into charmed final states. The most precise Belle II result is obtained with D decays in self-conjugate final states $K_S^0 h^+ h^-$, where $h = K, \pi$, where several intermediate resonances are involved in D decays, resulting in variation of the CP -violating asymmetry over the phase space. [10] Two other approaches have also been pursued: one with Cabibbo-suppressed D decays, and the other with the D meson decaying to two-body CP eigenstates. In both analyses it is required that $M_{bc} > 5.27 \text{ GeV}/c^2$, $|\Delta E| < 0.15 \text{ GeV}$ ($\Delta E > -0.13 \text{ GeV}$ for the latter), and a loose requirement on C removes about 60% of the continuum background. The signal yields are extracted with fits of ΔE and the continuum-suppression classifier. In both these approaches the precision is limited by the sample size.

2.1 Cabibbo-suppressed channels

GLS method measures γ with singly Cabibbo-suppressed decays of D mesons, $B^\pm \rightarrow D(\rightarrow K_S^0 K^\pm \pi^\mp) h^\pm$, where D is the superposition of D^0 and \bar{D}^0 mesons. [11] The B^\pm meson can have the same sign (SS) or opposite sign (OS) with respect to the K^\pm from the D decay. The information about the dynamic properties of the D decays is included with external inputs from CLEO [13].

Four CP -violating asymmetries and three branching-fraction ratios are measured from a simultaneous fit to ΔE and the continuum suppression classifier. [12] Fitting is performed in both the full D phase space and in the K^* region, where the invariant mass $m(KK_S^0)$ is close to that of $K^*(892)^0$, thus enhancing the interference and the precision on γ due to the large strong-phase difference in $D \rightarrow K_S^0 K^\pm \pi^\mp$ decays. Combining Belle (711 fb^{-1}) and Belle II (362 fb^{-1}) data, the results (shown in Table 1) are consistent, though not yet competitive to the results from LHCb. [14] They provide a constraint on γ when combined with results from other measurements.

Table 1: CP -violating asymmetries and branching-fraction ratios of $B^\pm \rightarrow D(\rightarrow K_S^0 K^\pm \pi^\mp) h^\pm$ decays obtained combining Belle (711 fb^{-1}) and Belle II (362 fb^{-1}) data for the full D phase space and the K^* region. The first uncertainties are statistical and the second are systematic.

	Full D phase space	K^* region
\mathcal{A}_{SS}^{DK}	$-0.089 \pm 0.091 \pm 0.011$	$0.055 \pm 0.119 \pm 0.020$
\mathcal{A}_{OS}^{DK}	$0.109 \pm 0.133 \pm 0.013$	$0.231 \pm 0.184 \pm 0.014$
$\mathcal{A}_{SS}^{D\pi}$	$0.018 \pm 0.026 \pm 0.009$	$0.046 \pm 0.029 \pm 0.016$
$\mathcal{A}_{OS}^{D\pi}$	$-0.028 \pm 0.031 \pm 0.009$	$0.009 \pm 0.046 \pm 0.009$
$\mathcal{R}_{SS}^{DK/D\pi}$	$0.122 \pm 0.012 \pm 0.004$	$0.093 \pm 0.012 \pm 0.005$
$\mathcal{R}_{OS}^{DK/D\pi}$	$0.093 \pm 0.013 \pm 0.003$	$0.103 \pm 0.020 \pm 0.006$
$\mathcal{R}_{SS/OS}^{D\pi}$	$1.428 \pm 0.057 \pm 0.002$	$2.412 \pm 0.132 \pm 0.019$

2.2 CP eigenstates

GLW method uses D meson decays to K^+K^- (CP -even) or $K_S^0\pi^0$ (CP -odd) eigenstates to extract the angle γ . [15, 16] Currently, the $K_S^0\pi^0$ eigenstate is accessible to B -factories only. Belle (711 fb^{-1}) and Belle II (189 fb^{-1}) data are combined to give the final result, which is consistent but not competitive with results from BaBar [17] and LHCb [18]. The branching fraction ratios ($\mathcal{R}_{CP\pm}$) and CP -violating asymmetries ($\mathcal{A}_{CP\pm}$) are

$$\begin{aligned}\mathcal{R}_{CP+} &= 1.164 \pm 0.081 \pm 0.036, \\ \mathcal{R}_{CP-} &= 1.151 \pm 0.074 \pm 0.019, \\ \mathcal{A}_{CP+} &= 0.125 \pm 0.058 \pm 0.014, \\ \mathcal{A}_{CP-} &= -0.167 \pm 0.057 \pm 0.060,\end{aligned}$$

where the first uncertainties are statistical and the second are systematic. The results constrain γ in combination with other measurements. In particular, evidence for \mathcal{A}_{CP+} and \mathcal{A}_{CP-} having opposite signs is observed, showing clearly the effect of CP violation.

3. Toward CKM angle α

With an uncertainty of about 5%, the CKM angle α is currently among the major limiting factors on the global precision of the Unitarity Triangle. Belle II has the unique capability of measuring all $B \rightarrow \rho\rho$ and $B \rightarrow \pi\pi$ decays, from which α is determined. The combined information from these decays exploits isospin symmetry, reducing the effect of hadronic uncertainties. In these analyses, the B candidates are required to have $M_{bc} > 5.27 \text{ GeV}/c^2$ and $|\Delta E| < 0.15 \text{ GeV}$ ($< 0.30 \text{ GeV}$ for final states involving neutral pions), followed by continuum suppression that removes 90%-99% of continuum background.

3.1 $B \rightarrow \rho\rho$ decays

The measurement of $B \rightarrow \rho\rho$ decays requires a complex angular analysis. Signal yields are determined with an unbinned maximum-likelihood fit of ΔE , C , two dipion masses and the two cosines of helicity angles of the ρ candidates. The helicity angle is the angle between the momentum difference of the ρ final states in the frame of the ρ , and the momentum of the ρ in the lab frame. The distributions of ΔE and cosine of the helicity angle of the ρ^+ candidates for $B^0 \rightarrow \rho^+\rho^-$ decays are shown in Fig. 1. The preliminary Belle II results of $B^0 \rightarrow \rho^+\rho^-$ and $B^+ \rightarrow \rho^+\rho^0$ decays using data corresponding to 189 fb^{-1} [19, 20] are on par with the most precise results from Belle [21, 22] and BaBar [23, 24]. Results are listed in Table 2.

3.2 $B \rightarrow \pi\pi$ decays

Measurements of $B^0 \rightarrow \pi^+\pi^-$ and $B^+ \rightarrow \pi^+\pi^0$ decays are based on Belle II data corresponding to 362 fb^{-1} . The fit is performed over ΔE and C , and the dominant systematic uncertainty arises from the π^0 efficiency. The first measurement of $B^0 \rightarrow \pi^0\pi^0$ at Belle II is also reported, using data corresponding to 189 fb^{-1} . [4] This decay is both CKM- and colour-suppressed, and has only photons in the final state, making it experimentally challenging to measure. The result obtained

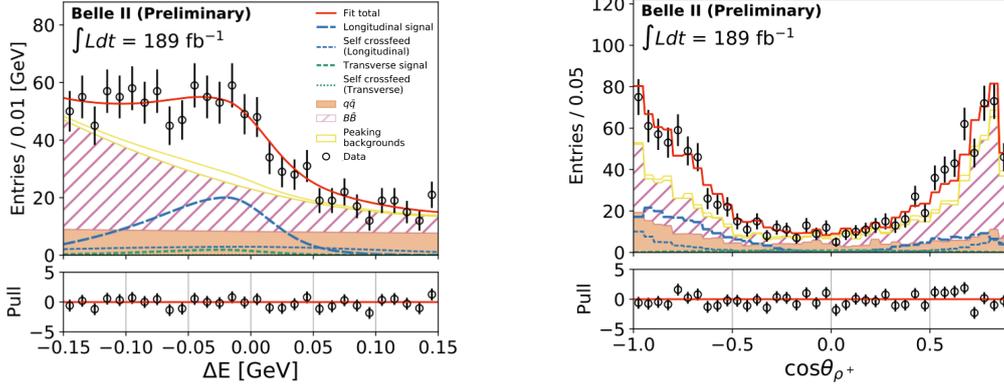


Figure 1: ΔE (left) and ρ^+ cosine of the helicity angle (right) distributions of $B^0 \rightarrow \rho^+ \rho^-$ candidates reconstructed in Belle II data, with fit overlaid.

from a fit to M_{bc} , ΔE , and C , achieves Belle's precision despite using a dataset that is only one fourth of the size. This is due to the dedicated π^0 selection and continuum suppression studies that yield a much higher π^0 efficiency. Results are listed in Table 2.

Table 2: $B \rightarrow \rho\rho$ and $B \rightarrow \pi\pi$ results. The first uncertainties are statistical and the second are systematic.

	$\mathcal{B} [10^{-6}]$	\mathcal{A}_{CP}	f_L
$B^0 \rightarrow \rho^+ \rho^-$	$26.7 \pm 2.8 \pm 2.8$	–	$0.956 \pm 0.035 \pm 0.033$
$B^+ \rightarrow \rho^+ \rho^0$	$23.2^{+2.2}_{-2.1} \pm 2.7$	$-0.069 \pm 0.068 \pm 0.060$	$0.943^{+0.035}_{-0.033} \pm 0.027$
$B^0 \rightarrow \pi^+ \pi^-$	$5.83 \pm 0.22 \pm 0.17$	–	–
$B^+ \rightarrow \pi^+ \pi^0$	$5.10 \pm 0.29 \pm 0.32$	$-0.081 \pm 0.054 \pm 0.008$	–
$B^0 \rightarrow \pi^0 \pi^0$	$1.38 \pm 0.27 \pm 0.22$	$0.14 \pm 0.46 \pm 0.07$	–

4. $K\pi$ isospin sum rule

The isospin sum rule $I_{K\pi}$ is defined by

$$I_{K\pi} = \mathcal{A}_{K^+ \pi^-} + \mathcal{A}_{K^0 \pi^+} \cdot \frac{\mathcal{B}_{K^0 \pi^+} \tau_{B^0}}{\mathcal{B}_{K^+ \pi^-} \tau_{B^+}} - 2\mathcal{A}_{K^+ \pi^0} \cdot \frac{\mathcal{B}_{K^+ \pi^0} \tau_{B^0}}{\mathcal{B}_{K^+ \pi^-} \tau_{B^+}} - 2\mathcal{A}_{K^0 \pi^0} \cdot \frac{\mathcal{B}_{K^0 \pi^0}}{\mathcal{B}_{K^+ \pi^-}}, \quad (1)$$

where $\mathcal{B}_{K\pi}$ and $\mathcal{A}_{K\pi}$ are the branching fractions and the CP -violating asymmetries, and $\tau_{B^0}/\tau_{B^+} = 0.9273 \pm 0.0033$ is the ratio of B^0 and B^+ lifetimes. [8] The SM prediction of the sum rule is zero, with a precision of better than 1%, in the limit of isospin symmetry and no electroweak penguins contributions. Any large deviation from the SM prediction is an indication of anomalously enhanced amplitudes or non-SM physics. The experimental precision of the sum rule, about 11%, is limited by $\mathcal{A}_{K^0 \pi^0}$, which has an uncertainty of 13%. [8]

We report the measurement of the branching fraction and CP -violating asymmetry of all the final states associated with the sum rule: $B^0 \rightarrow K^+ \pi^-$, $B^+ \rightarrow K_S^0 \pi^+$, $B^+ \rightarrow K^+ \pi^0$, and $B^0 \rightarrow K_S^0 \pi^0$ using Belle II data corresponding to 362 fb^{-1} . The analyses of the various decays follow a similar strategy, with common selections applied to the final states particles. Reconstructed B candidates

are required to satisfy $5.272 < M_{bc} < 5.288 \text{ GeV}/c^2$, $|\Delta E| < 0.3 \text{ GeV}$, and a loose requirement of C that suppresses 90%-99% of continuum background. A fit is performed on the ΔE and C distribution. To determine the CP -violating asymmetry in $B^0 \rightarrow K_S^0 \pi^0$ decays, where we have no information from the final states about the initial B flavour, we use a tagging algorithm to determine it by using the final states of the second B meson produced in the event. [5] The ΔE distributions are shown in Fig. 2. The measured branching fractions and CP -violating asymmetries, as well as the sum rule calculated using these measurements, are listed in Table 3. They agree with the world averages and have competitive precisions. [8] In particular, the time-integrated and time-dependent results of $B^0 \rightarrow K_S^0 \pi^0$ are combined to achieve the world's best result for $\mathcal{A}_{K^0 \pi^0}$, and consequentially for $I_{K\pi}$ a competitive precision that is limited by the statistical uncertainty.

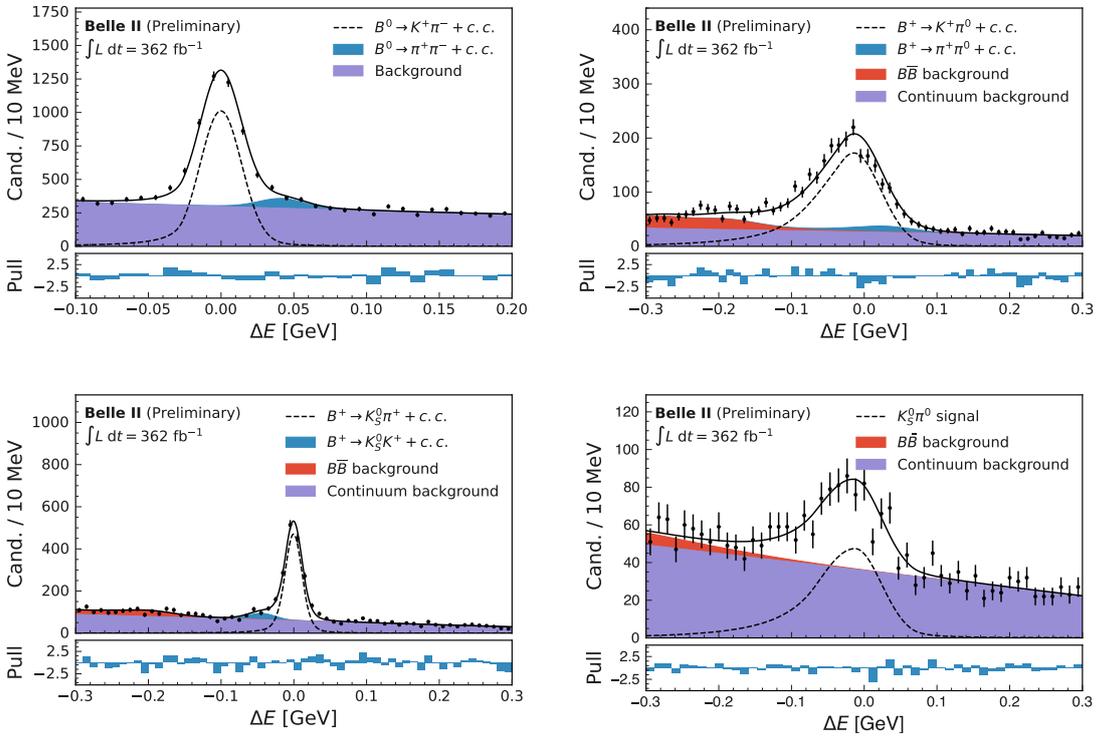


Figure 2: ΔE distributions of $B^0 \rightarrow K^+ \pi^-$ (upper left), $B^+ \rightarrow K^+ \pi^0$ (upper right), $B^+ \rightarrow K_S^0 \pi^+$ (lower left) and $B^0 \rightarrow K_S^0 \pi^0$ (lower right) candidates reconstructed in Belle II data, with fit overlaid.

5. $B \rightarrow D^{(*)} K K_S^0$ decays

The composition of 40% of the B hadronic width is unknown, which limits significantly our capability to model B decays in simulation, impacting the precision of many measurements. Belle II is pursuing a systematic program of exploration of hadronic B decays to reduce this uncertainty. We report a preliminary measurement of the branching fraction of $B^- \rightarrow D^0 K^- K_S^0$ decay [6], with a precision that is three times better than the previous best results [7], and first observation of three new decay channels ($\bar{B}^0 \rightarrow D^+ K^- K_S^0$, $B^- \rightarrow D^{*0} K^- K_S^0$, and $\bar{B}^0 \rightarrow D^{*+} K^- K_S^0$) [6]. The branching

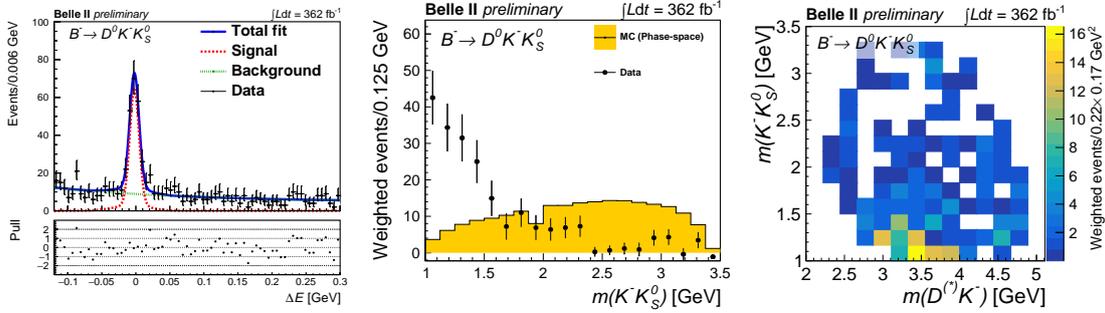
Table 3: $B \rightarrow K\pi$ results using Belle II data corresponding to 362 fb^{-1} . The first uncertainties are statistical and the second are systematic.

	$\mathcal{B} [10^{-6}]$	\mathcal{A}_{CP}
$B^0 \rightarrow K^+\pi^-$	$20.67 \pm 0.37 \pm 0.62$	$-0.072 \pm 0.019 \pm 0.007$
$B^+ \rightarrow K^+\pi^0$	$13.93 \pm 0.38 \pm 0.84$	$0.013 \pm 0.027 \pm 0.005$
$B^+ \rightarrow K^0\pi^+$	$24.40 \pm 0.71 \pm 0.86$	$0.046 \pm 0.029 \pm 0.007$
$B^0 \rightarrow K^0\pi^0$	$10.16 \pm 0.65 \pm 0.67$	$-0.006 \pm 0.15 \pm 0.05$
$I_{K\pi}$	$-0.03 \pm 0.13 \pm 0.05$	

fractions

$$\begin{aligned} \mathcal{B}(B^- \rightarrow D^0 K^- K_S^0) &= (1.89 \pm 0.16 \pm 0.10) \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \rightarrow D^+ K^- K_S^0) &= (0.85 \pm 0.11 \pm 0.05) \times 10^{-4}, \\ \mathcal{B}(B^- \rightarrow D^{*0} K^- K_S^0) &= (1.57 \pm 0.27 \pm 0.12) \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \rightarrow D^{*+} K^- K_S^0) &= (0.96 \pm 0.18 \pm 0.06) \times 10^{-4}, \end{aligned}$$

are extracted from a 362 fb^{-1} Belle II sample using likelihood fits to the unbinned distributions of the energy difference ΔE , where the first uncertainties are statistical and the second are systematic. The invariant mass $m(KK_S^0)$ of the two kaons is also investigated. For all four channels, the $m(KK_S^0)$ distribution exhibits a peaking structure in the low-mass region, which departs from the three-body phase space distribution. Structures are also observed in the Dalitz distributions (Fig. 3).

**Figure 3:** ΔE (left), $m(KK_S^0)$ (middle), and Dalitz (right) distributions of $B^- \rightarrow D^0 K^- K_S^0$ decay.

6. Summary

We report on precise measurements of hadronic B decays at the Belle II experiment. We present the Belle-Belle II combined measurement of the CKM angle γ with the GLS and GLW methods, with precision significantly improved with respect to the previous Belle measurements. We present measurements of two-body decays that contribute to the determination of α , the measurement of the $K\pi$ isospin sum rule with a competitive precision to the world's best result, and the first observation of three new $B \rightarrow D^{(*)} K K_S^0$ decays.

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