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Excess-path-length distribution of shower electrons at radially distant points from the shower axis

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Excess-path-length distribution of shower electrons at radially distant points from the shower axis is derived as the product of 1-dimensional excess-path-length distribution of shower electrons with their lateral distribution integrated and lateral distribution of shower electrons with their excess-path-lengths fixed, the former of which is derived analytically and the latter of which is derived by Monte Carlo analyses. The result will give a method to determine the shower size (the total number of shower electrons) from the arrival-time-distribution of shower electrons at radial distance of r from the shower axis, another than the traditional method from the lateral distribution of those.

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1. Introduction

At the traversed thickness of t, shower electrons show the lateral distribution and the longitudinal distribution (or the excess-path-length distribution) due to the multiple Coulomb scattering of electrons with traversed materials. Though the former distribution is analytically investigated enough by Kamata and Nishimura [1, 2] and applied in the analyses of observed data, the latter has not been almost analytically investigated yet and observed data have not been enough analyzed yet.

So we have proceeded our Monte Carlo and analytical investigations of excess-path-length distribution for shower electrons [3, 4] and derived structure functions of the distribution. Our results are compared with observed arrival time distributions of shower electrons [5, 6]. Consequently, a new method is proposed to determine the shower size or the total number of shower electrons from the arrival time distribution of shower electrons at radial distance of r from the shower axis.

2. The structure functions of excess-path-length distribution for shower electrons

We express the probability density of shower electrons to show their excess-path-length Δ at the radial distance of $\vec{r}$ from the shower axis;

$$\frac{d^2\rho}{d\Delta d\vec{r}} = \frac{dP(\Delta;t)}{d\Delta}\frac{dQ(r;\Delta)}{d\vec{r}}, \tag{2.1}$$

where $dP(\Delta;t)/d\Delta$ denotes the probability density to delay Δ with the radial spread $\vec{r}$ integrated (the longitudinal distribution) and $dQ(r;\Delta)/d\vec{r}$ denotes the areal probability density for the spread $\vec{r}$ with the delay Δ fixed (the radial distribution). So that the densities denoted as the "structure functions" composed of the longitudinal distribution and the radial distribution are normalized as

$$\int_0^\infty \frac{dP(\Delta;t)}{d\Delta}d\Delta = 1. \quad \int_0^\infty \frac{dQ(r;\Delta)}{d\vec{r}}2\pi r dr = 1, \quad \text{and} \quad \int_0^\infty\int_0^\infty \frac{d^2\rho}{d\Delta d\vec{r}}d\Delta 2\pi r dr = 1. \tag{2.2}$$

We introduce new normalized variables,

$$u = 2\Delta/r_{\mathrm{M}}^2, \qquad v = (r/r_{\mathrm{M}})^2 \qquad \text{with} \qquad r_{\mathrm{M}} \equiv E_{\mathrm{s}}/\varepsilon, \tag{2.3}$$

for the excess-path-length and the square of radial spread. Then as they satisfy $r^2/(2t\Delta) = v/(tu)$ and $dv = d(\pi r^2)/\pi r_{\mathrm{M}}^2 = d\vec{r}/\pi r_{\mathrm{M}}^2$, we have

$$dQ = \frac{dQ}{d(v/tu)}d\left(\frac{v}{tu}\right) = h\left(1-\frac{v}{tu}\right)^{h-1} d\left(\frac{v}{tu}\right) \qquad \text{with} \quad 0 \le \frac{v}{tu} \le 1 \tag{2.4}$$

according to the predicted result in Monte Carlo analyses reported in this conference [3], so that Eq. (2.1) gives

$$d^2\rho = \frac{dp(u;s)}{du}du\, h\left(1-\frac{v/t}{u}\right)^{h-1}\frac{d(v/t)}{u} \qquad 0 < \frac{v}{t} < u, \tag{2.5}$$

where we apply the analytically predicted probability density $dP(\Delta;t)/d\Delta$ reported in this conference [4], satisfying

$$\frac{dP(\Delta;t)}{d\Delta}d\Delta = \frac{dp(u;s)}{du}du. \tag{2.6}$$

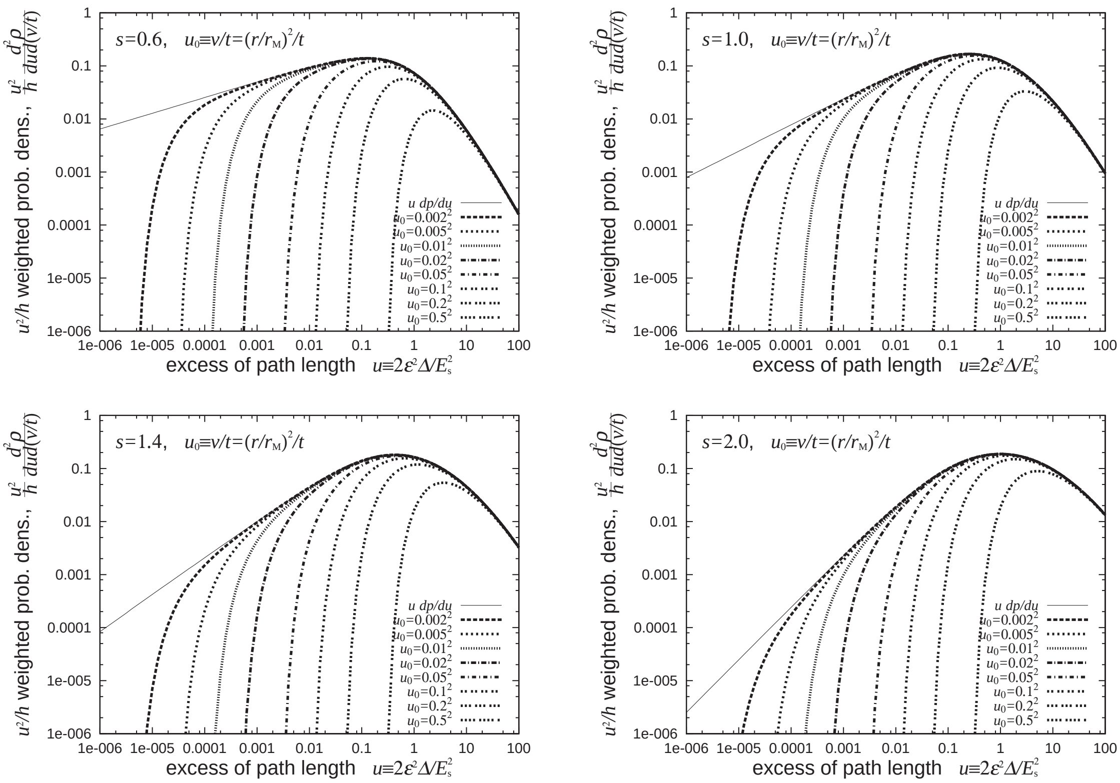

Figure 1: (u^2/h)-weighted probability densities, $(u^2/h)d^2\rho/du/d(v/t)$, for excess-path-length of shower electrons at radial distance of r from the shower axis, for respective ages of s and parameters of $u_0 \equiv v/t = (r/r_{\mathrm{M}})^2/t$ (dot lines). They approach to udp/du (thin solid line), with the approach of r to 0. We put h = 9.

3. Excess-path-length distribution of shower electrons at radially distant points from the shower axis

The probability density of shower electrons to exceed u of path length at v of squared radial spread is expressed as

$$\frac{d^2\rho}{dud(v/t)} = \frac{dp(u;s)}{du}\frac{h}{u}\left(1-\frac{v/t}{u}\right)^{h-1} \quad \text{with} \quad \frac{v}{t} \equiv u_0 < u. \tag{3.1}$$

We indicate in Fig. 1 the (u^2/h)-weighted probability densities, $(u^2/h)d^2\rho/du/d(v/t)$, at the shower ages s of 0.6, 1.0, 1.4, 2.0, and for radial distances r included in the parameter of

$$u_0 \equiv v/t = (r/r_{\mathrm{M}})^2/t. \tag{3.2}$$

We find the probability density starts at the geometrical front (the surface of the sphere departing t from the start point of the shower), delaying u_0 from the tangential plane at the shower front. Note that the longitudinal and radial probability density $d^2\rho/du/d(v/t)$ at the radial distance r of 0 is expressed by the longitudinal probability density with the radial distribution integrated, $dp(u;s)/du$ predicted in the preceding report [4], and the former at r of finite distance is also expressed by the latter for enough large excess u of path-length. The correction factor for the probability density at the radial distance r is indicated in Fig. 2.

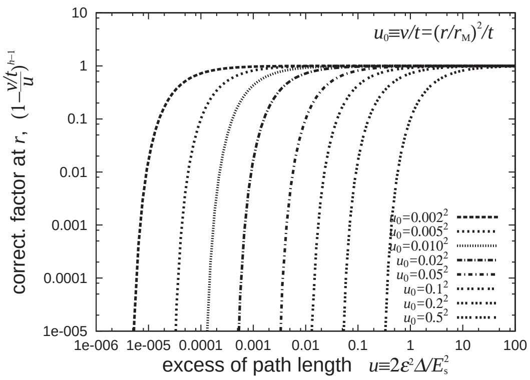

Figure 2: Correction factors of $(1-v/t/u)^{h-1}$ for the probability densities $d^2\rho/du/d(v/t)$ of excess-path-length defined as the ratio of the densities at radial distance of r to that at the shower axis, for respective values of the parameter $u_0 \equiv v/t = (r/r_{\rm M})^2/t$. We put h = 9.

Similarities in dependence on the shower age s were investigated between the lateral distribution and the temporal distribution of shower electrons, in EAS-TOP experiment [6]. As indicated in Fig. 1 with (u^2/h)-weighted, our u-weighted probability densities $ud^2\rho/du/d(v/t)$ for $r \to 0$ show power-law of $u^{s/2-1}$ at $u \ll 1$, just as the lateral distributions $f(r/r_1)$ of Nishimura-Kamata-Greisen shows power-law of $(r^2/r_1^2)^{s/2-1}$ at $r/r_1 \ll 1$ [1]. Though it must be taken much care about our $ud^2\rho/du/d(v/t)$ show considerable differences from $u^{s/2-1}$ for finite (not infinitesimal) $r/r_{\rm M}$, as understood from Figs. 1 and 2.

4. Comparison of our distribution with the observed results

The probability density (2.1) of shower electrons is changed to the particle density of $d^2n/(dudv)$ by multiplying the total number $\Pi(s,t)$ of shower electrons of the age s at the traversed thickness of t;

$$\frac{d^2n}{dudv} = \Pi(s,t)\frac{d^2\rho}{dudv} = \Pi(s,t)\frac{dp}{du}\frac{h/t}{u}\left(1-\frac{v/t}{u}\right)^{h-1} \quad \text{with} \quad \frac{v}{t} \equiv u_0 < u, \tag{4.1}$$

where

$$\begin{aligned}\Pi(s,t) &\equiv K(s,-s)\frac{1}{2\pi i}\int\frac{ds}{s}\left(\frac{W_0}{\varepsilon}\right)^s\frac{1}{2\pi i}\int e^{t\lambda}\phi_{00}(s,\lambda)d\lambda \\ &= K(s,-s)\frac{(W_0/\varepsilon)^s e^{\lambda_1(s)t}}{\sqrt{2\pi\{1+s^2\lambda_1''(s)t\}}}\frac{B(s)}{\lambda_1(s)-\lambda_2(s)},\end{aligned} \tag{4.2}$$

$$\text{with} \quad \lambda_1'(s)t = -\ln\frac{W_0}{\varepsilon}+\frac{1}{s}. \tag{4.3}$$

We compare our predicted density of excess-path-length u at radial distance of r with observed results. The abscissa u is proportional to the arrival time of τ and the ordinates $d^2n/(dudv)$ and $d^2\rho/(dudv)$ are proportional to the observed particle density of $d^2n/(dud\vec{r})$;

$$u = 2\frac{c\tau}{X_0}/r_{\rm M}^2 = \tau/\frac{r_{\rm M}^2X_0}{2c} \simeq \tau/35\text{ns}, \tag{4.4}$$

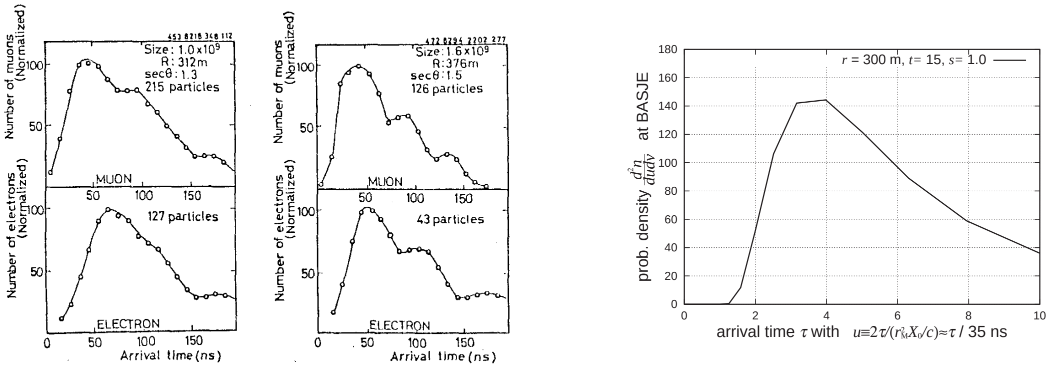

Figure 3: BASJE observations of arrival-time distribution for shower electrons (left) and reproduction by our $d^2n/(dudv)$ with the radial distance r of 300 m, the traversed thickness t of 15 cu, and the shower age s of 1.0 (right).

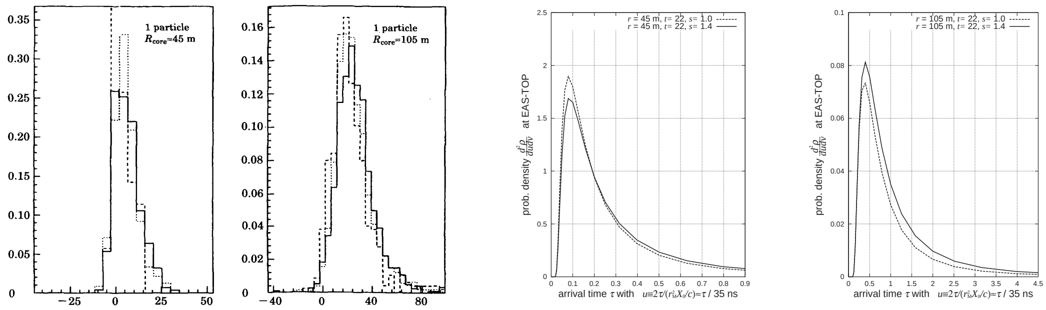

Figure 4: Examples of EAS-TOP observation of arrival-time distribution for shower electrons (left) and reproduction by our $d^2\rho/(dudv)$ with the radial distances r of 45 and 105 m, the traversed thickness t of 22 cu, and the shower ages s of 1.0 and 1.4 (right).

$$\frac{d^2\rho}{dudv} = r_{\mathrm{M}}^2 \frac{d^2\rho}{dudr^2} = \pi r_{\mathrm{M}}^2 \frac{d^2\rho}{du2\pi rdr} = \pi r_{\mathrm{M}}^2 \frac{d^2\rho}{dud\vec{r}}. \tag{4.5}$$

BASJE group observed arrival time distributions of shower electrons [5] as indicated in Fig. 3, where the ordinate values are normalized as the peak values to be 10. We compare our arrival time distribution predicted at the radial distance of 300 m, the traversed thickness of 15 cu, and the shower age s of 1.0, neglecting the arrival directions of θ. We find the both agree well, considering we took our abscissa starting from the tangential plane at the shower front so that the distribution started about 40 ns later at the geometrical front.

EAS-TOP group indicated observed examples of arrival time distribution of EAS electrons in his Fig. 4 [6] as indicated in our Fig. 4. We compare our arrival time distribution predicted at the radial distances of 45 and 105 m, the traversed thickness of 22 cu, and the shower ages s of 1.0 and 1.4, neglecting the arrival directions of θ. We find the both agree well, confirming the longer tail of distribution for larger age s of 1.4 than that for age s of 1.0.

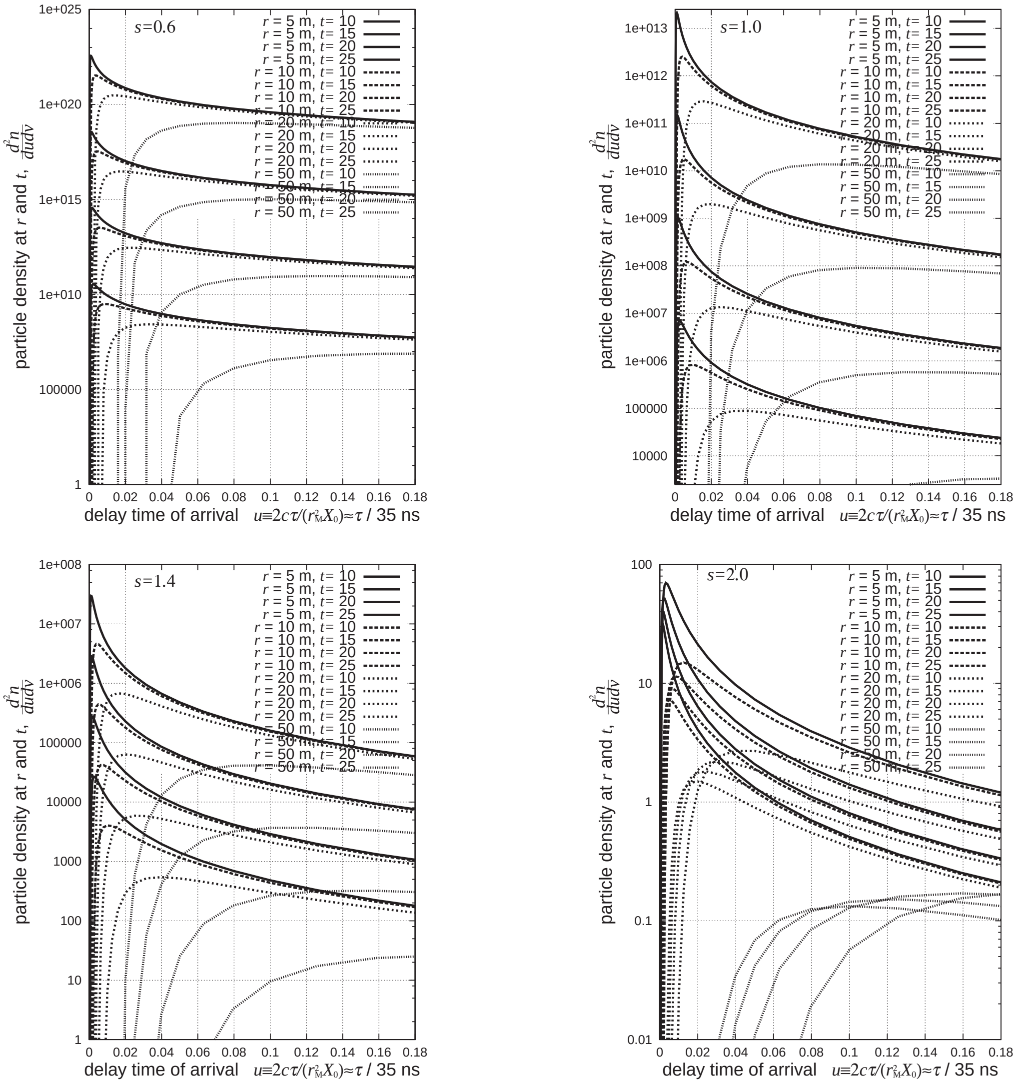

Figure 5: Particle densities $d^2n/(dudv)$ of shower electrons to show excess of path-length at traversed thicknesses of t and radial distance of r from the shower axis, for respective age s of 0.6, 1.0, 1.4, 2.0, respective radial distance r of 5, 10, 20, 50 m, and respective thickness t of 10, 15, 20, 25 cu. We put h = 9.

5. Evaluation of the total number of shower electrons from the excess-path-length distribution at radial distance of r from the shower axis

We propose a method to evaluate the total number of shower electrons from the excess-path-length distribution observed at the radial distance of r from the shower axis.

We can determine the shower age s and the radial distance r (so that the radial parameter v) of observation from the lateral distribution near the shower axis. As we do not directly observe the traversed thickness of t. we draw the expected particle densities $d^2n/(dudv)$ of Eq. (4.1) for assumed traversed thicknesses of t with the known s and v fixed, as indicated e.g. in Fig. 5. Then comparing them with the observed particle densities $d^2n/(dudv) = \pi r_M^2 d^2n/(dud\vec{r})$, we can estimate the traversed thickness t so that evaluate the incident energy W_0 and the total number of

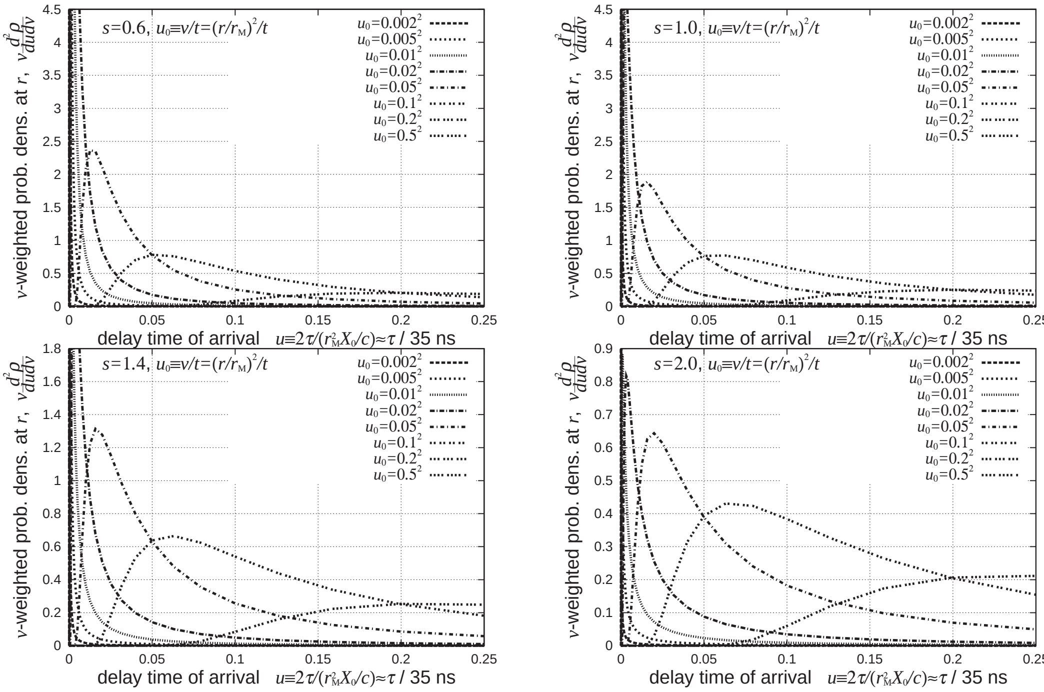

Figure 6: v-weighted probability densities $vd^2\rho/(dudv)$ for excess-path-length of shower electrons at radial distance of r from the shower axis, for respective ages of s and parameters of $u_0 \equiv v/t = (r/r_{\rm M})^2/t$. We put h = 9.

shower electrons $\Pi(s,t)$ from Eqs. (4.2) and (4.3).

We can propose another practical method to determine the traversed thickness of t. We draw the v-weighted probability densities $vd^2\rho/(dudv)$ derivable from Eq. (3.1) for assumed parameters of u_0 with the known ages of s, as indicated e.g. in Fig. 6. Then comparing them with the observed v-weighted particle densities $vd^2n/(dudv) = \pi r^2 d^2n/(dud\vec{r})$ with the ordinate renormalized as the latter is larger by the factor of yet unknown $\Pi(s,t)$, we can estimate the parameter u_0 so that the traversed thickness $t = v/u_0$. Thus we can evaluate the incident energy W_0 and the total number of shower electrons $\Pi(s,t)$ from Eqs. (4.2) and (4.3), as above. Note that the abscissa u in our figures is measured from the tangential plane at the shower front.

This method could realize effective observation of extensive air showers by the excess-path-length distribution to determine the incident energy comparatively near the shower axis, rather than the traditional method by the lateral distribution requiring wide area of array to observe particle densities at large radial distance from the shower axis.

6. Conclusions and discussions

The structure functions to describe the longitudinal (excess-path-length) and the radial distributions at the traversed thickness of t are proposed, based on the Monte Carlo and analytical investigations (Section 2). The excess-path-length distribution at radial distance r from the shower axis is predicted from the structure functions (Section 3), which well reproduced the observed arrival time distributions at BASJE and EAS-TOP (Section 4).

A new method to evaluate total number of shower electrons is proposed. It could realize more effective observation of extensive air showers to determine the incident energy near the shower axis, compared with the traditional method by the lateral distribution requiring wide observation area (Section 5).

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