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Investigating the validity of straight radio signal propagation for very inclined air showers

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When calculating radio emission from an air shower, the standard approximation used in all current air shower radio software is to assume straight-line signal propagation. This approximation is expected to become less valid for very inclined geometries, but the magnitude of the error caused by the approximation is yet to be ascertained. Therefore, it is critical to understand the region of validity for this approximation as it could affect the design of next-generation radio-based detectors. To investigate the possible error introduced by the approximation, we present results obtained using a modified version of CoREAS combined with input data from ray tracing to more correctly describe signal propagation in a non-uniform atmosphere for very inclined geometries without performing the full ray tracing during simulation. We aim to determine geometries where the straight-line approximation might introduce significant errors.

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1. Very Inclined air showers

Very inclined air showers are air shower geometries which propagate nearly horizontally. In other words, geometries for which the zenith angle θ is large, with values between 80° and 90°.

These very inclined air shower geometries are characterized by a large radio signal footprint on the ground. This is advantageous as they could be detected by use of a sparse and relatively inexpensive radio array. These geometries are of particular interests for next-gen cosmic-ray or neutrino radio detectors such as AugerPrime [1] and GRAND [2].

Modelling the signal that one would observe from a very inclined air shower is however more complex than the analogous description for vertical air showers. Reasons for this are, among others:

- Earth curvature effects have to be taken into account.
- The signal travels through more of the atmosphere than would be the case for a vertical geometry. Because of this it is expected that small changes in the description of the non uniform atmosphere could have large effects on the signal that is received on the surface.

The focus of this article will be on the effect of the non-uniform atmosphere on the calculation of the amplitude associated with the signal from a single emitter as described by the end-point formalism. Important to note is that while an approximation of straight line propagation is often made when calculating this amplitude, the non uniformity of the atmosphere is taken into account for other aspects such as travel time of the signal. Previous work [3] indicates that because of a correct treatment of relative timings between emitters, a refractive like displacement is already present in CoREAS. This while the straight line approximation is used when calculating the amplitude. Our aim is to investigate if this straight line approximation could break down for very inclined air showers.

2. Modelling signal propagation

When describing signal propagation, a couple of different techniques are available:

- *Finite Difference Time Domain Techniques*: This approach works by solving the Maxwell equations on a grid. While very accurate these techniques are typically computationally expensive.
- *The parabolic equation method*: This technique uses an approximation of the full field to obtain the fields in a stepwise manner. While less accurate than FDTD this method is more computationally efficient. [4]
- *Raytracing*: This technique is currently the most widespread, as it is a well known method that is more computationally efficient than the alternatives listed above. A distinction is made between analytical raytracers, which use a fully derived expression of the ray path, and numerical raytracers which solve the raypath in a stepwise manner.

Analytical raytracers are considered to be very fast but they rely on a perfect knowledge of the medium as well as often approximations such as a flat Earth and a purely exponential index of refraction profile. While generally slower, the numerical raytracers are more versatile in the sense that one can more easily take into account arbitrary index of refraction profiles. Important to note is that raytracing implicitly assumes that any change in the medium happens over a scale which is large compared to the wavelength, this extra approximation is also what sets it apart from methods like FDTD.

For this article, results from a numerical raytracing simulation based on Fermat's principle will be used. The simulation takes into account Earth curvature effects as well as a multi-layer model for the atmosphere corresponding to what is used in current simulation codes [5] [6], with each layer being represented by an altitude-dependent exponential model. More details on the raytracer used can be found in [7]. As a summary: the ray is found by solving Hamilton's equations in 2D:

$$\dot{x} = \frac{p_x}{n(x,z)}, \tag{1}$$

$$\dot{z} = \frac{p_z}{n(x,z)}, \tag{2}$$

$$\dot{p}_x = \frac{\partial n}{\partial x}, \tag{3}$$

$$\dot{p}_z = \frac{\partial n}{\partial z}. \tag{4}$$

with n the index of refraction, x and z cartesian coordinates and $p_i = n\dot{x}_i$ the generalised momenta, which represents the direction in which the ray is developing. Given initial conditions and a differentiable index of refraction profile these differential equations can be solved numerically to find x_i, p_i along the raypath.

To obtain data from the raytracer for use with simulation software, one needs to take special care in defining the geometry. This geometry is explained below and a visual representation is given in figure (1). The origin of this geometry is defined as the intersection of a shower axis at a

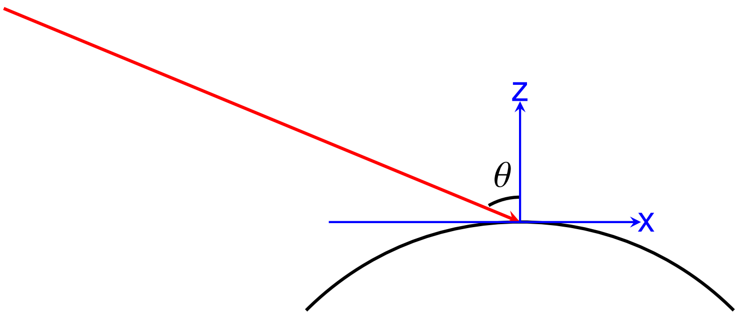

Figure 1: Geometry for the raytracer. The red line represents the line model of a cascade and θ denotes the zenith angle.

specified zenith angle θ with the 2D circle representing the earth at sea level. The shower is then modelled as a line of emitters, since for large θ any displacements from this central shower axis are expected to be small compared to the distance between emitter and receiver. Rays start from the receiver, which can have an arbitrary position within this 2D plane, and end on the line. The choice of this geometry allows for a straightforward translation to geometries in CoREAS, a simulation code that can be used as an option of the CORSIKA program to simulate radio emission from air showers [6].

3. Implementing tabulation in CoREAS

This section describes how we use the information obtained through raytracing within current state of the art simulation software. An approach with look-up tables allows us to investigate the effects of the non-uniform atmosphere without the computationally intensive task of performing the raytracing for each emission point. The implementation happens in a part of the code that computes the boost factor, a concept that is discussed in the next section.

3.1 The geometric boost factor: straight line vs curved path

When an emitter travels in a medium, a known phenomenon is that the emitted signal can have a propagation speed slower than the travel speed of the emitter. This allows a signal that was emitted over an emission time interval $\Delta t'$ to arrive in a shorter observer time interval Δt. This effect is most easily described by introducing the geometric boost factor: for a uniform medium with $n = constant$, it can be shown that [7]:

$$\frac{\mathrm{d}t}{\mathrm{d}t'} = 1 - n \cdot \beta \cdot \cos\theta_{BF}, \tag{5}$$

with t the observer time, t' the emitter time, β the velocity of the emitter divided by the speed of light in vacuum and θ_{BF} the viewing angle with respect to the momentum of the emitter, not to be confused with the zenith angle θ. When $\frac{\mathrm{d}t}{\mathrm{d}t'} = 0$, the signal emitted over some finite time interval arrives instantly at the receiver, creating a burst. Looking at Eq. (5) this would also mean $\cos\theta_{BF} = \frac{1}{\beta n}$ which is the description of the Cherenkov angle in uniform media. The advantage of thinking in terms of $\frac{\mathrm{d}t}{\mathrm{d}t'}$ is that this approach is also directly applicable in non-uniform media.

From a previous study [7], it was found that relation (5) still holds in non-uniform media, provided that one uses the index of refraction at the emission point and the initial launch angle of the ray connecting the emission point with the receiving point. An example of the difference between the uniform case with straight rays and the non uniform case with curved rays and their appropriate boost factor calculations is shown in figure (2). This example also illustrates an earlier result which indicated that calculating the boost factor with a straight line approximation potentially introduces an error for very inclined air showers.

The corrected boost factor calculation is currently also being used in other studies, notably for the signal from an air shower core which propagates into the ice[8] [9]. For this work instead the focus is on signal coming from very inclined air showers.

4. The end-point formalism and CoREAS

CoREAS [6] is a current state of the art simulation code to calculate the radio signal coming from air shower geometries. As an option of the CORSIKA program, it calculates the radio signal coming from air shower geometries by summing the contributions of many different tracks in what is called the end-point formalism [10].

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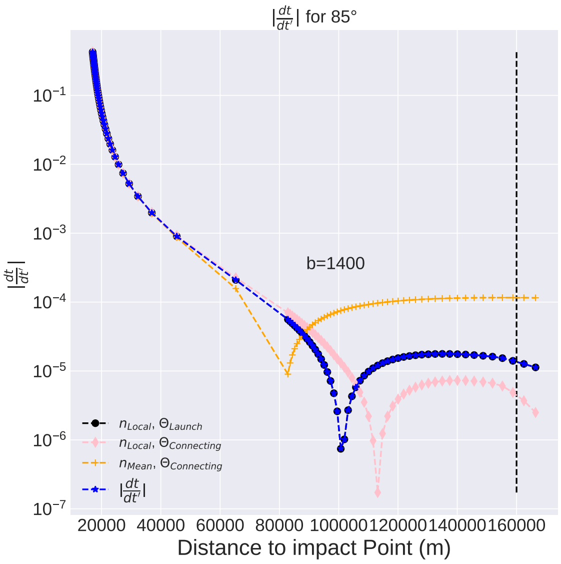


Figure 2: Example of boost factor calculations for an 85° zenith angle shower with the receiver at distance $b = 1400$ m from the shower axis. In blue is the numerically computed derivative. Black, yellow and pink represent different calculations of $1 - n\beta \cos\theta$. Black: n at emitter, θ initial launch angle of the ray. Pink:n at emitter, θ straight line angle. Yellow: $< n >$ along the path and straight line angle. The black estimator follows the derivative nicely. Note that the pink estimator is what is being used when assuming straight line propagation. The x axis represents distance along the shower axis and the vertical dotted line is a region from where one can expect signal.

The contribution of a single emitter in the end point formalism is [10]:

$$\vec{E}_{\pm} = \pm \frac{1}{\Delta t} \frac{q}{c} \left(\frac{\hat{r} \times [\hat{r} \times \vec{\beta}]}{(1 - n\vec{\beta} \cdot \hat{r})R} \right) \tag{6}$$

where often the straight line approximation is used such that $\hat{r}$ denotes a normalised vector pointing along a straight line from emitter to receiver. The goal now is to use a look up table to map the straight line vector $\hat{r}$ to the more correct launch vector $\hat{r}'$.

First, note that in general $\hat{r}$ is a three dimensional vector but that the problem can be reduced to a 2D geometry by assuming spherical symmetry of the earth. This 2D geometry can then be used for the raytracer. The steps for this transformation are outlined below. Perform a 2D rotation in the x, y plane with an angle δ so that $\hat{r}$ is transformed to L as follows:

$$\begin{pmatrix} L_x \\ 0 \\ L_z \end{pmatrix} = \begin{pmatrix} \cos(\delta) & -\sin(\delta) & 0 \\ \sin(\delta) & \cos(\delta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \hat{r}_x \\ \hat{r}_y \\ \hat{r}_z \end{pmatrix}$$

So that $r_z = L_z$. The other two equations are:

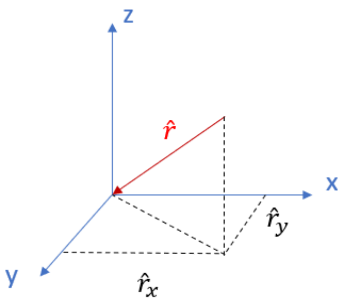
$$L_x = \hat{r}_x \cdot \cos(\delta) - \hat{r}_y \cdot \sin(\delta),$$
$$0 = \hat{r}_x \cdot \sin(\delta) + \hat{r}_y \cdot \cos(\delta).$$

So that:

$$L_x = \hat{r}_x \cdot \cos(\delta) - \hat{r}_y \cdot \sin(\delta),$$
$$\tan(\delta) = -\frac{\hat{r}_y}{\hat{r}_x}.$$

Using $\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$ and $\cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$ this becomes:

$$L_x = \sqrt{\hat{r}_x^2 + \hat{r}_y^2},$$



so that the x component of L is the original horizontal component, as can be seen in the diagram. This is as expected, as the rotation served to align the x-axis with the plane defined by the origin, the emission point and the middle of the Earth. Now L denotes a straight line vector in this 2D coordinate system. The next step is to replace this with the correct $\hat{r}'$ launch vector. To do this, the raytracer outlined in Eq. (1 - 4) is used to generate tables mapping L_x to $\hat{L}'_x$. The difference between $\hat{L}'_x$ and $\hat{r}'_x$ is that the inverse rotation still has to be applied to $\hat{L}'_x$ to return to the original frame. The z component can be found from the normalisation $\hat{L}'_z = \sqrt{1 - \hat{L}'^2_x}$. After this second rotation the final launch vector becomes:

$$\hat{r}'_x = \frac{\hat{r}_x}{\sqrt{\hat{r}_x^2 + \hat{r}_y^2}} L'_x$$
$$\hat{r}'_y = \frac{\hat{r}_y}{\sqrt{\hat{r}_x^2 + \hat{r}_y^2}} L'_x$$
$$\hat{r}'_z = \sqrt{1 - L'^2_z}$$

These components now allow us to use the correct $\hat{r}'$ for the calculation of the boostfactor. To summarize: we rotate the frame so that we can use the data from the 2D raytracer and afterwards preform the inverse rotation to then let CoREAS continue the calculations with these updated values.

5. Preliminary results

To investigate the effect of the corrected boost factor we compare output traces between a version of CoREAS with the tabulation procedure implemented and a version that has been unaltered.

When setting up the simulation with a one-to-one table, such that the table is a mapping of the identity relation, we note that the trace obtained with a one-to-one table is nearly identical to the trace obtained with an unaltered version of CoREAS, with a relative error of around 10^{-6}. This indicates that the tabulation procedure does not introduce any artifacts in the signal. When instead filling the tables with actual raytracing data we observe a slight discrepancy compared to the unaltered trace. An example of these comparisons for an 85° zenith angle geometry is given in figure 3, where the receiver position was chosen such that one could expect a strong signal.

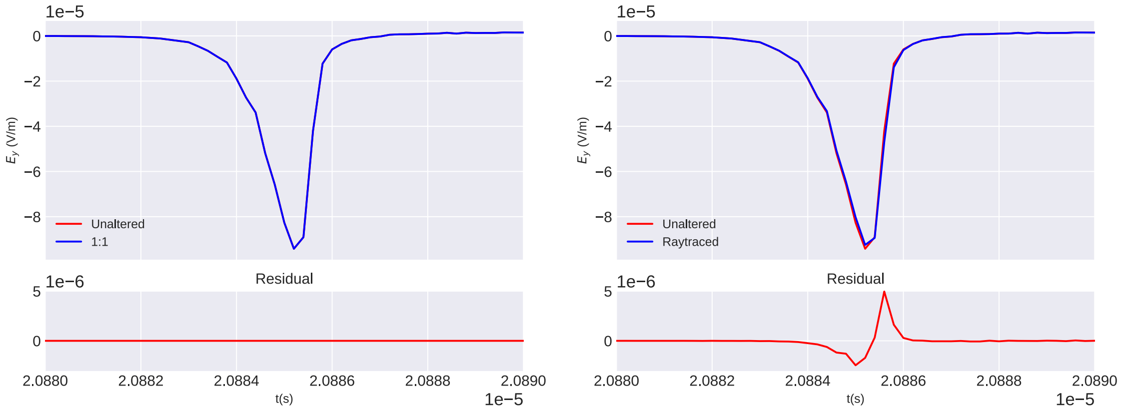


Figure 3: An example trace when running CoREAS with a one-to-one table versus the output trace of an unaltered CoREAS version. Left: The comparison between a trace obtained from an unaltered version of CoREAS and a trace obtained with a one-to-one table. The residuals here are of the order 10^{-11}. Right: A comparison with the same input parameters, except the table is filled with actual raytracing data. The residuals remain small, at percent level.

From this it seems that, while a small discrepancy is seen, the approximation does not break down spectacularly for a zenith angle of 85°. However, more positions and zenith angles need to be investigated in order to correctly estimate the error that one could introduce by approximating the boost factor.

6. Conclusion and summary

In this article we discussed the importance and challenges of accurately modelling signal coming from very inclined air showers. We mentioned how results from a raytracing study could allow for a correction in the geometric boost factor that is calculated when describing radiation through the end-point formalism. This correction came down to replacing the straight line vector with the initial launch vector of the ray connecting emitter and receiver. A tabulation procedure was outlined that would allow the comparison of traces between an unaltered version and a version that uses tabulated raytracing data. From preliminary results we saw that the straight line approximation seems to hold for 85°. More extensive checks are however needed to make a definitive claim.

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