

Nonlinear feedback of the electrostatic instability on the blazar-induced pair beam and GeV cascade

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The missing GeV gamma-ray cascade signal of distant Blazars implies either the deflection of the pair beam by intergalactic magnetic fields or, alternatively, an energy loss of the beam due to the beam-plasma instability. A recent study demonstrated that the instability feedback on a simplified 1D beam profile reduces the instability growth severely. Here we study the feedback on the realistic 2D beam distribution. We solve the Fokker-Planck diffusion equation with initially dominant angular diffusion, then we analyse the other terms using the widened beam distribution. We found that the energy loss of the beam due to the growth of the instability and its momentum diffusion feedback on the beam is small compared to the inverse Compton cooling rate. We found also that another angular diffusion term, which is initially negligible, might become relevant and narrows the beam particles with Lorentz factors less than 10^6 . Finally, we have included the unstable waves saturate so that the beam widening balances the injection at the production angles and the beam keeps expanding. However, it is essential to include the inverse Compton cooling in the beam transport equation to understand the physical effects of this balance.

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1. Introduction

Blazars are active galactic nuclei with their jets pointing toward Earth. Their very highenergy gamma rays interact with the extragalactic background light (EBL), producing focused electron-positron pair beams, which are anticipated to dissipate their energies via inverse Compton scattering on the cosmic microwave background (CMB) [7]. However, the inverse Compton cooling cascade seems to be absent from the γ -ray spectra of certain blazars [10] and possibly the isotropic gamma-ray background [3].

One possible solution for the absence of the GeV cascade emission from the γ -ray spectra of blazars is that the pair beam is significantly deflected by the intergalactic magnetic fields (IGMF) [10, 11, 19]. In this case, the observed blazar spectra are used to put lower limits on the strength of the IGMF. An alternative solution is energy loss of the pair beam by the collective beam-plasma instabilities faster than the inverse Compton cooling. [1, 2, 4–6, 8, 9, 12, 13, 15–18, 21, 22].

Perry & Lyubarsky [12] studied the instability feedback for the first time considering the angular diffusion of the instability on a simplified one-dimensional angular distribution. Their findings imply that the back reaction of the unstable waves on the pair beam increases the angular spread of the beam by around one order of magnitude without any significant energy loss.

GeV cascade is emitted by pairs with Lorentz factors of 10^6 and a bit higher. In order to study the feedback impact on those pairs it's more comprehensive to use the realistic momentum beam distribution rather than the integrated one. Since the global picture of the simplified distribution does not necessarily reflect the impact on those pairs. Besides that, Vafin et al. [22] demonstrated that the realistic distribution of the beam is crucial for the growth rate of the instability itself.

The primary goal of this paper is to investigate the influence of electrostatic waves on the two-dimensional (angular and momentum) realistic pair beam distribution, specifically, using the beam profile found in Vafin et al. [22] at a distance of 50 Mpc from the blazar.

Given that angular diffusion is the dominant process initially, we have performed a simulation of the angular scattering feedback of the instability on the realistic 2D spectrum of the beam. We then used the widened beam 2D profile to calculate the energy loss/gain of the momentum diffusion of the instability feedback. We also compared the initial subdominant angular diffusion term with the dominant widening term as the beam widens. Finally, we instigated the effect of the pairs injection by the gamma-ray annihilation with the EBL on the widening feedback of the instability.

A detailed description of this research will follow in the main paper that is in preparation.

2. Quasilinear theory of the beam-plasma system

The feedback of the instability on the beam is described by the Fokker-Planck diffusion equation [14]

$$\frac{\partial f_b(p,\theta,t)}{\partial t} = \frac{1}{p^2 \theta} \frac{\partial}{\partial \theta} \left(\theta D_{\theta \theta} \frac{\partial f_b}{\partial \theta} \right) + \frac{1}{p \theta} \frac{\partial}{\partial \theta} \left(\theta D_{\theta p} \frac{\partial f_b}{\partial p} \right)
+ \frac{1}{p^2} \frac{\partial}{\partial p} \left(p D_{p \theta} \frac{\partial f_b}{\partial \theta} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{p p} \frac{\partial f_b}{\partial p} \right),$$
(1)

where the diffusion coefficients are defined as the following integral involving the resonance condition

$$\begin{cases} D_{pp} \\ D_{p\theta} \\ D_{\theta\theta} \end{cases} = \frac{\pi m_e \omega_p^2}{n_e} \int_{\omega_p/c}^{\infty} k^2 dk \int_{\cos\theta_1'}^{\cos\theta_2'} d\cos\theta' \frac{W(\boldsymbol{k},t)}{kc(1-\frac{1}{2\gamma^2})} \\ \times \frac{1}{\sqrt{(\cos\theta'-\cos\theta_1')(\cos\theta_2'-\cos\theta')}} \begin{cases} 1 \\ \xi \\ \xi^2 \end{cases},$$
(2)

where

$$\xi = \frac{\cos\theta \frac{\omega_P}{kc(1-\frac{1}{2\gamma^2})} - \cos\theta'}{\sin\theta},\tag{3}$$

and the boundaries of the $\cos \theta'$ integration are fixed by the resonance condition

$$\cos\theta_{1,2}' = \frac{\omega_p}{kc(1-\frac{1}{2\gamma^2})} \left[\cos\theta \pm \sin\theta \sqrt{\left(\frac{kc}{\omega_p}\right)^2 - 1}\right].$$
(4)

The angles θ and θ' are defined respectively as the beam particle angle and the unstable plasma waves wave vector angle with the beam propagation axis. We implicitly assumed the azimuthal symmetry of the pair-beam distribution function and we have a symmetry between $D_{p\theta}$ and $D_{\theta p}$ $(D_{p\theta} = D_{\theta p})$.

In the linear phase of the instability growth, the unstable resonance wave spectrum evolves in time as

$$\frac{\partial W(\boldsymbol{k},t)}{\partial t} = 2(\omega_i(\boldsymbol{k},t) + \omega_c(\boldsymbol{k}))W(\boldsymbol{k}), \tag{5}$$

where $\omega_i(\mathbf{k}, t)$ is the linear growth rate of the unstable waves given by eq(17) in Vafin et al. [22] and it depends on the distribution function $f(p, \theta, t)$ at time t. For the collisional damping rate of the electrostatic waves ω_c , we use the estimate given by eq(45) in Tigik et al. [20]. This estimate is based on an enhanced formulation of the collisional damping rate that takes into account the microscopic wave-particle interactions. This estimate is approximately 20 times smaller compared to the previous estimation employed in Miniati & Elyiv [9], Perry & Lyubarsky [12], Vafin et al. [21]. For the intergalactic medium temperature, we used $T_e = 10^4 K$ and for the density number of the intergalactic background electrons, we used $n_e = 10^{-7}(1+z)^3$ cm⁻³ with a redshift of 0.15.

The direct energy loss rate of the beam due to the growth of the waves at time t is given by [22]

$$\frac{dU_{\rm b}}{dt}(t) = -8\pi \int dk_{\perp} k_{\perp} \int dk_{||} W(k_{\perp}, k_{||}, t) \omega_i(k_{\perp}, k_{||}, t).$$
(6)

3. Results

A full discussion of the numerical method and the results will follow in the main paper that is in preparation. Here we summarize the main findings.



Figure 1: The total beam energy transferred to the growth of unstable waves (black dashed line), the total beam energy loss $(p\theta)$ and gain (pp) due to the momentum diffusion of the instability feedback.

We solved numerically the system of the angular diffusion of the instability feedback including only the first term on the right-hand side of eq.1

$$\frac{\partial f_b(p,\theta,t)}{\partial t} = \frac{1}{p^2 \theta} \frac{\partial}{\partial \theta} \left(\theta D_{\theta\theta} \frac{\partial f_b}{\partial \theta} \right),\tag{7}$$

and the unstable wave spectrum evolution equation (eq.5) with the pair beam spectrum found in Vafin et al. [22] at 50 Mpc as an initial condition for the distribution function. We used the Crank-Nicolson scheme to solve the diffusion equation with time steps of the maximum linear growth rate inverse $\omega_{i,\text{max}}^{-1}$ and the fastest rate of change of the beam distribution in the diffusion equation from the last time step. We have verified that with shorter time steps up to a factor of 10^{-1} .

In agreement with the results of Perry & Lyubarsky [12], we found that the angular diffusion of the instability feedback severely limits the growth of the unstable resonant waves. As the unstable waves generated by beam particles with certain beam angles grow, those resonant waves scatter the beam particles to larger angles reducing the linear growth rate. This process happens at different time scales for beam particles with different Lorentz factors.

We have calculated the total energy transferred from the pair beam to the unstable waves up to the time *t* by integrating eq.6 over the simulation time until the time *t*. The black dashed line in Fig.1 shows the result, where we see that by the IC cooling time for the pairs with Lorentz factor of 10^7 that is around 4×10^{12} seconds for redshift 0.15, the beam lost less than 1% of its total energy due to the instability development.

3.1 2D analysis of the diffusion equation

Using the time dependant pair beam 2D spectrum in the angular diffusion simulation, we check for the possible feedback of the other diffusion terms as the beam widens. We do not include those



Figure 2: The ratio between $I_{\theta p}$ in eq.10 and $I_{\theta \theta}$ in eq.11, the ratio is larger than one in the red area and it's less than 0.1 in the dark blue one. The drop after 10^{11} seconds is due to an increase in the widening as a result of waves' growth outside the initial resonance region. After that, the collisional damping effectively damps the waves and both $I_{\theta \theta}$ and $I_{\theta p}$ decrease.

terms in the simulation setup but rather analyse their possible impacts on the beam as it widens in time due to the angular feedback (eq.7).

For the momentum diffusion terms (the third and the fourth terms on the right-hand side of eq.1), we found the energy loss/gain rate that those terms impact the beam with.

The rate of change in the total beam energy due to the momentum diffusion by the term $(p\theta)$ at time *t* is given by the following formula after integrating by part

$$\frac{dU_b}{dt}\Big|_{p\theta}(t) = 2\pi m_e c^2 \int d\theta\theta \int dp p^2 \gamma \frac{df_b}{dt}\Big|_{p\theta}(p,\theta,t) = -2\pi c \int d\theta\theta \int dp p D_{p\theta} \frac{\partial f_b}{\partial \theta}(p,\theta,t),$$
(8)

and for the term (pp)

$$\left. \frac{dU_b}{dt} \right|_{pp}(t) = -2\pi c \int d\theta \theta \int dp p^2 D_{pp} \frac{\partial f_b}{\partial p}(p,\theta,t).$$
(9)

The sign of the diffusion coefficient $D_{p\theta}$ is negative where it's positive for D_{pp} . Since we have a Gaussian beam angular profile then the angular derivative is always negative. This means that the overall sign of eq.8 is negative and hence the term $(p\theta)$ imposes an energy loss of the beam's total energy. As for the term (pp), it is dependent on the sign of the momentum derivative of the beam profile. The realistic beam momentum profile declines for beam Lorentz factors above 10^5 [22] and therefore the impact of the term (pp) in this range is an energy gain of the beam.

We have integrated eq.8 and eq.9 over the simulation time finding the energy loss/gain from the beam's initial energy due to the instability feedback. The result is shown in Fig.1, we see that

both of them affect a negligible fraction of the total beam energy by the time the system reaches the inverse Compton cooling time of pairs with Lorentz factors of 10^7 .

Now we take a look at the second term on the right-hand side of eq.1, this term involves an angular diffusion with the opposite direction of the angular widening of the first term. We see that the diffusive flux of this term is proportional to the momentum gradient of the beam distribution, whereas the widening diffusive flux is proportional to the angular gradient. When the beam widens the angular gradient of the beam profile decreases at the beam's inner angles whereas the momentum gradient stays constant since we don't allow for momentum diffusion. Therefore, even though the term $(\theta \theta)$ is dominant initially, the term (θp) might become significant as the beam spreads.

In order to investigate this, we found the total rate of change that the term (θp) affects the distribution function at certain beam momentum

$$I_{\theta p}(p,t) = \int d\cos\theta \left| \frac{df_b}{dt} \right|_{\theta p} \right| = \int d\cos\theta \left| \frac{1}{p\theta} \frac{\partial}{\partial\theta} \left(\theta D_{\theta p} \frac{\partial f_b}{\partial p} \right) \right|,\tag{10}$$

we have also calculated the same for the term $(\theta\theta)$ using the rate of changes during simulation time steps

$$I_{\theta\theta}(p,t) = \int d\cos\theta \left| \frac{df_b}{dt} \right|_{\theta\theta} \right|.$$
(11)

The result of the ratio is shown in Fig.2. We see that Lorenz factors less than 10^6 , the ratio is 10% or higher at times much earlier than the IC cooling time that is larger than 10^{13} seconds for those pairs. The drop after 10^{11} seconds is due to the widening increases as a result of accumulating unstable outside the initial resonance region. After that time the collisional damping effectively damps the waves and both $I_{\theta\theta}$ and $I_{\theta p}$ decrease.

The GeV cascade needs IC emission by pairs with Lorentz factors of around $10^6 - 4 \times 10^6$. Therefore the feedback of the term (θp) might influence slower pairs, but its feedback is not that relevant for the missing GeV cascade.

3.2 Simulation with beam injection

Finally, we have added a source term for the production of new pairs due to the gamma-ray annihilation with the EBL photons. The distribution of the pairs found in Vafin et al. [22] is the accumulated production of the pairs for the time 7.7×10^{12} seconds, we used the initial beam distribution divided by this time interval as a constant production rate Q_{ee}

$$\frac{\partial f_b(p,\theta)}{\partial t} = \frac{1}{p^2\theta} \frac{\partial}{\partial \theta} \left(\theta D_{\theta\theta} \frac{\partial f_b}{\partial \theta}\right) + Q_{ee}.$$
 (12)

We solved eq.12 and eq.5 numerically as we did for the angular diffusion simulation. We found that the beam widening balance the pairs injection at the production angles leading to a quasi-steady-state wave spectrum where the linear growth rate balances the collisional damping rate. In this configuration, the beam keeps expanding due to the steady state unstable wave spectrum. In the end, it's essential to include the inverse Compton cooling in the beam distribution evolution equation to understand the physical impact of this balance between the injection and the feedback widening.

4. Conclusion

We have studied the quasi-parallel and oblique plasma instability feedback on the 2D realistic spectrum of the blazar-induced pair beam. We found that this instability is incapable of taking a significant fraction of the beam energy before the inverse Compton cooling time due to the suppression of the wave's growth by the feedback spread. However, we found that including only the widening feedback does not necessarily reflect the overall impact of the instability on the beam. Since as the beam expands, there is another diffusion term that becomes relevant for pairs with Lorentz factors less than 10⁶. We found also that under a more realistic situation that includes the beam particle injection, the unstable waves do not decay after the beam expands, but saturate at a level that their wedding feedback balances the injection at the production angles and the beam keeps expanding. However, it is essential to include the IC cooling rate in the transport equation to understand the physical impact of this process.

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References

- [1] Alawashra, M., & Pohl, M. 2022, The Astrophysical Journal, 929, 67, doi: 10.3847/ 1538-4357/ac5a4b
- [2] Alves Batista, R., Saveliev, A., & de Gouveia Dal Pino, E. M. 2019, Mon. Not. Roy. Astron. Soc., 489, 3836, doi: 10.1093/mnras/stz2389
- Blanco, C., Ghosh, O., Jacobsen, S., & Linden, T. 2023, arXiv e-prints, arXiv:2303.01524, doi: 10.48550/arXiv.2303.01524
- [4] Broderick, A. E., Chang, P., & Pfrommer, C. 2012, The Astrophysical Journal, 752, 22, doi: 10.1088/0004-637x/752/1/22
- [5] Broderick, A. E., Pfrommer, C., Puchwein, E., & Chang, P. 2014, 790, 137, doi: 10.1088/ 0004-637X/790/2/137
- [6] Chang, P., Broderick, A. E., Pfrommer, C., et al. 2016, Astrophys. J., 833, 118, doi: 10.3847/ 1538-4357/833/1/118
- [7] Gould, R. J., & Schréder, G. P. 1967, Physical Review, 155, 1408, doi: 10.1103/PhysRev. 155.1408
- [8] Kempf, A., Kilian, P., & Spanier, F. 2016, Astronomy & Astrophysics, 585, A132, doi: 10. 1051/0004-6361/201527521

- [9] Miniati, F., & Elyiv, A. 2013, The Astrophysical Journal, 770, 54, doi: 10.1088/0004-637x/ 770/1/54
- [10] Neronov, A., & Semikoz, D. V. 2009, Phys. Rev. D, 80, 123012, doi: 10.1103/PhysRevD. 80.123012
- [11] Neronov, A., & Vovk, I. 2010, Science, 328, 73, doi: 10.1126/science.1184192
- [12] Perry, R., & Lyubarsky, Y. 2021, , 503, 2215, doi: 10.1093/mnras/stab324
- [13] Rafighi, I., Vafin, S., Pohl, M., & Niemiec, J. 2017, Astronomy & Astrophysics, 607, A112, doi: 10.1051/0004-6361/201731127
- [14] Rudakov, L. I. 1971, Soviet Journal of Experimental and Theoretical Physics, 32, 1134
- [15] Schlickeiser, R., Ibscher, D., & Supsar, M. 2012, The Astrophysical Journal, 758, 102, doi: 10.1088/0004-637x/758/2/102
- [16] Shalaby, M., Broderick, A. E., Chang, P., et al. 2020, Journal of Plasma Physics, 86, 535860201, doi: 10.1017/S0022377820000215
- [17] Sironi, L., & Giannios, D. 2014, The Astrophysical Journal, 787, 49, doi: 10.1088/ 0004-637x/787/1/49
- [18] Supsar, M., & Schlickeiser, R. 2014, The Astrophysical Journal, 783, 96, doi: 10.1088/ 0004-637x/783/2/96
- [19] Taylor, A. M., Vovk, I., & Neronov, A. 2011, 529, A144, doi: 10.1051/0004-6361/ 201116441
- [20] Tigik, S. F., Yoon, P. H., & Ziebell, L. F. 2019, Plasma Physics and Controlled Fusion, 61, 125008, doi: 10.1088/1361-6587/ab4aad
- [21] Vafin, S., Deka, P. J., Pohl, M., & Bohdan, A. 2019, The Astrophysical Journal, 873, 10, doi: 10.3847/1538-4357/ab017b
- [22] Vafin, S., Rafighi, I., Pohl, M., & Niemiec, J. 2018, The Astrophysical Journal, 857, 43, doi: 10.3847/1538-4357/aab552