PROCEEDINGS OF SCIENCE

Analytic calculations of the spectra of cosmogenic neutrinos

Alessandro Cermenati,[a,*] **Roberto Aloisio,**[a,b] **Denise Boncioli**[c,b] **and Carmelo Evoli**[a,b]

[a] *Gran Sasso Science Institute, Viale Francesco Crispi 7, 67100, L'Aquila, Italy*
[b] *INFN/Laboratori Nazionali del Gran Sasso Via G. Acitelli 22, 67100, Assergi (AQ), Italy*
[c] *Dipartimento di Scienze Fisiche e Chimiche Università degli Studi dell'Aquila Via Vetoio, 67100, L'Aquila, Italy*

E-mail: alessandro.cermenati@gssi.it

We present an analytic derivation of the diffuse spectra for high energy neutrinos (E>PeV), produced by the propagation of ultra-high energy protons in astrophysical photon backgrounds (cosmogenic neutrinos). We address several scenarios of cosmological transport based on homogeneous distribution of sources with a generic cosmological evolution. By utilizing recent cross-section models, we compute the local neutrino emissivity and finally the expected neutrinos flux at redshift $z = 0$.

Beyond its pedagogical value, this result yields essential insights into estimating the uncertainties associated with these predictions. Furthermore, it serves as a reliable benchmark for more advanced calculations conducted using modern numerical codes such as SimProp and CRPropa.

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*Speaker

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1. Introduction

Ultra-high-energy cosmic rays (UHECR), with energies E>EeV, likely originate from extragalactic sources [1]. During their journey from the source to the Earth, UHECR are expected to interact with the diffuse extragalactic background photons, leading to energy loss and the production of secondary particles such as neutrinos and photons, with energies larger than 1 PeV [2]. Similar processes can occur in photon fields in the environment of the accelerator sites [3].

Experimental evidences show that UHECRs may consist of nuclei of hydrogen or heavier species [4]. In this work, we will focus on protons. The interaction between cosmic-ray protons and cosmological photon fields results in energy loss over distances shorter than the Hubble radius for proton energies above $\sim 5 \times 10^{19}$ eV, leading to the well-known Greisen-Zatsepin-Kuzmin (GZK) cutoff [5, 6]. These interactions are expected to generate substantial amounts of neutrinos (dubbed as *cosmogenic*) and photons. Moreover, neutrinos serve as excellent cosmic messengers because they are uncharged, therefore unaffected by magnetic fields. Additionally, their small cross sections allow them to travel over cosmological distances, carrying information from distant sources.

The cosmogenic neutrino flux, guaranteed by the photo-meson process, depends on the characteristics of the UHECR flux at the escape from their sources. Next-generation neutrino detectors such as the Askaryan Radio Array (ARA), the Antarctic Ross Ice-shelf ANtenna Neutrino Array (ARIANNA), the Giant Radio Array for Neutrino Detection (GRAND), and the Probe Of Extreme Multi-Messenger Astrophysics (POEMMA), have been specifically designed to achieve the sensitivity required to detect this flux.

Recent studies have focused on constraining the origin of UHECRs using neutrinos [7]. However, some physical quantities relevant to the production of cosmogenic neutrinos and photons remain poorly known, such as the spectral energy density of the background light and the interaction yields for photo-meson processes.

Many of these calculations rely on extensive numerical simulations performed using widely-used codes for UHECR propagation, such as CRPropa [8] and SimProp [9]. Consequently, exploring all the uncertainties associated with these calculations is challenging.

In this paper, we present an analytical procedure for performing calculations of cosmogenic neutrino production from UHECR protons. This approach allows for easy exploration of the parameter space associated with the assumptions underlying these calculations, with the aim of improving our understanding of this process.

Finally, it is remarkable to note that a detailed analytical derivation of the cosmogenic neutrino flux has not been published to date. Hence, these findings hold not only scientific significance but also offer valuable pedagogical insights.

2. UHECR transport equation in an expanding universe

In this section, we calculate the proton flux using the kinetic equation and trajectory calculations developed in [10, 11]. The calculated flux will be utilized in section 4 to determine the generation rate of neutrinos in the Universe.

The temporal evolution of the proton density, denoted as n_p (number of particles per unit volume and unit energy), is described by the following transport equation [10, 11]:

$$\frac{\partial n_p(E,t)}{\partial t} + 3H(t)n_p(E,t) - \frac{\partial}{\partial E}\left[b(E,t)n_p(E,t)\right] = \frac{\mathbf{Q}(E,t)}{a^3(t)} \tag{1}$$

In the above equation, $H(t)$ represents the Hubble rate, $a(t)$ corresponds to the cosmic scale factor, $b(E,t)$ describes the energy losses resulting from the Universe expansion (adiabatic losses), pair-production and photo-pion production on photon radiation, and $\mathbf{Q}$ represents the source generation rate per unit *comoving* volume.

Equation (1) is conveniently rewritten in terms of the evolution in redshift as

$$\frac{dn_p(E_g,z)}{dz} + \left[3H(z) - \frac{\partial b(E_g,z)}{\partial E_g}\right]\left|\frac{dt}{dz}\right| n_p(E_g,z) = \left|\frac{dt}{dz}\right|(1+z)^3\mathbf{Q}(E_g,z) \tag{2}$$

where the redshift z is given by $1+z = a^{-1}(t)$ and the jacobian by

$$\frac{dt}{dz} = -\frac{1}{H_0(1+z)\sqrt{\Omega_{\rm m}(1+z)^3+\Lambda}} \tag{3}$$

In the above expression, E_g denotes the *generation* energy, which is the solution to the characteristic equation derived from equation (1), specifically, $dE/dt = -b(E,t)$. Physically, E_g signifies the energy at which a particle must be generated at redshift z_g to be observed with energy E at redshift z.

Finally, the formal solution to equation (2) is given by

$$n_p(E,z) = (1+z)^3 \int_z^{\infty} dz_g \, \frac{\mathbf{Q}\left[E_g(E,z,z_g),z_g\right]}{(1+z_g)H(z_g)} \frac{dE_g(E,z,z_g)}{dE} \tag{4}$$

Here, the term dE_g/dE is computed as

$$\frac{dE_g(E,z,z_g)}{dE} = \frac{1+z_g}{1+z}\exp\left(\int_z^{z_g} ds \left|\frac{dt}{ds}\right| \left.\frac{db_{\rm int}(E',s)}{dE'}\right|_{E'=E_g(E,z,s)}\right) \tag{5}$$

It should be noted that for what concerns the transport of UHE protons, the energy losses on the EBL are negligible [10, 11]. Therefore, we are allowed to assume interaction with only the CMB, and the energy losses due to the interaction with the photon field, $db_{\rm int}/dE$, can be expressed accordingly as [12]:

$$\frac{db_{\rm int}}{dE} = (1+z)^3\left[\frac{db_0(E')}{dE'}\right]_{E'=(1+z)E_g(E,z)} \tag{6}$$

where $b_0(E)$ is the interaction energy loss at $z=0$.

Equation (4) represents our final result for computing the proton density at any given redshift, considering an arbitrary energy loss, $b_{\rm int}$, and source term, $\mathbf{Q}(z)$.

As an application of equation (4), we consider a generation rate of primary protons given by:

$$\mathbf{Q}_{\rm s}(E,z) = \begin{cases} \mathbf{L}_0 K q_{\rm inj}(E) S(1+z) & \text{for } z < z_{\rm max} \\ 0 & \text{for } z > z_{\rm max} \end{cases} \tag{7}$$

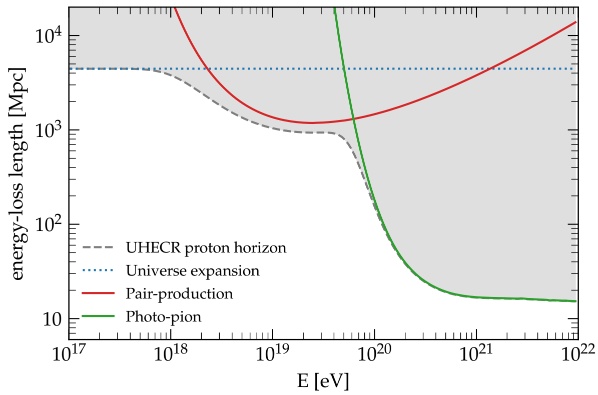

Figure 1: Proton energy loss lengths ($\lambda = c/dEdt$) for the different processes.

Here, $\mathbf{L}_0$ represents the luminosity density, i.e. the energy release per unit time and unit comoving volume, the factor $S(1+z) = (1+z)^m$ describes the potential cosmological evolution of sources up to a maximum redshift defined by $z_{\max}$, and $q_{\rm inj}(E) = (E/E_{\min})^{-\gamma}$ represents the power-law injection spectrum. Finally, the normalization constant $K = (\gamma-2)/E_{\min}^2$ is included.

Under this assumption, the proton intensity $I_p = \frac{c}{4\pi} n_p$ at redshift z can be obtained from the formal solution as:

$$I_p(E,z) = \frac{c}{4\pi}(1+z)^3 \mathbf{L}_0 K \int_z^{z_{\max}} dz_g \left|\frac{dt}{dz_g}\right| (1+z_g)^m q_{\rm inj}(E_g) \frac{dE_g}{dE}(E,z,z_g) \tag{8}$$

3. Proton energy losses

For UHE protons, it is crucial to account for energy losses resulting from pair production ($p+\gamma \to p + e^+ + e^-$) and pion production ($p + \gamma \to N + \pi$'s), where γ represents a low-energy background photon.

The energy loss per unit time for both processes is given by [10, 12]:

$$-\frac{1}{E}\frac{dE}{dt} = \frac{c}{2\Gamma^2} \int_{\epsilon'_{\rm th}}^{\infty} d\epsilon' \epsilon' Y(\epsilon')\sigma(\epsilon') \int_{\epsilon_{\min}} d\epsilon \frac{n_\gamma(\epsilon,z)}{\epsilon^2} \tag{9}$$

In the above equation, Γ represents the proton Lorentz factor, σ corresponds to the interaction cross-section, Y represents the inelasticity, ϵ and ϵ' represent the photon energy in the interaction rest-frame and the nucleus rest frame (NRF), respectively, and n_γ signifies the *proper* photon density.

To compute these quantities, we utilize the SimProp code [9]. Specifically, the pair production rate is calculated based on the method outlined in [13], while the photo-pion production employs the cross-section and inelasticity model computed using the SOPHIA code [14].

The tabulated rates for these processes at redshift $z = 0$ are illustrated in figure 1.

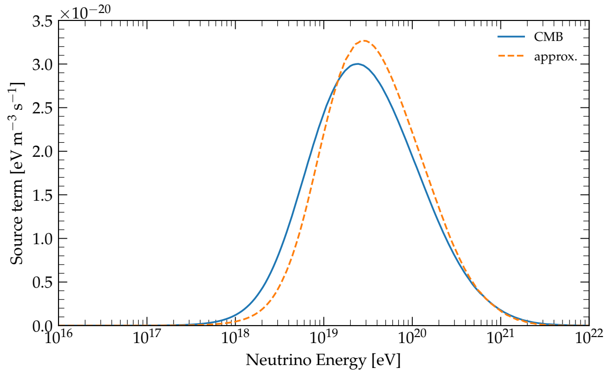

Figure 2: Neutrino emissivity at $z = 0$ on CMB.

4. Cosmogenic neutrinos

Neutrinos exhibit minimal interactions with other particles, and it is often assumed that the neutrino horizon is approximately equal to the Hubble horizon [15]. As a result, the only relevant energy losses for neutrinos are adiabatic losses.

Since cosmogenic neutrinos are primarily produced in GZK interactions, where UHE protons interact with low-energy photons, we adopt the following generation rate for neutrinos:

$$\mathbf{Q}_\nu(E_\nu, z) = \frac{c}{(1+z)^3} \int_{E_\nu}^{\infty} \frac{dE_p}{E_p} n_p(E_p, z) \int_0^{\infty} d\epsilon \, n_\gamma(\epsilon, z) \frac{E_p d\sigma_\nu}{dE_\nu}(E_p, \epsilon) \tag{10}$$

In the above equation, $\frac{d\sigma_\nu}{dE_\nu}$ represents the differential neutrino production cross-section. We adopt the form provided in [16], which was derived by fitting a practical parametrization to the results of extensive SOPHIA simulations.

Assuming $b_{\rm int} = 0$, the formal solution in equation (1) can be expressed as:

$$I_\nu(E_{\nu,0}) = \int_0^{z_{\rm max}} dz_g \left|\frac{dt}{dz_g}\right| \frac{1}{(1+z_g)^2} \int_{E_{\nu,0}(1+z_g)}^{\infty} \frac{dE_p}{E_p} I_p(E_p, z_g) \int_0^{\infty} d\epsilon \, n_\gamma(\epsilon, z_g)\, \Phi(\eta, x) \tag{11}$$

where $E_{\nu,0}$ is the observed neutrino energy. Here, using the notation from [16], we define $\Phi = c\frac{d\sigma}{dx}$, where $x = \frac{E_\nu}{E_p}$ and $\eta = \frac{4\epsilon E_p}{m_p^2}$.

Notice that the integration in equation 11 involves the proton *physical* intensity, $I_p(E_p, z)$, as derived in equation 8.

In figure 2, we illustrate the neutrino emissivity on the CMB at $z = 0$, considering a simple power-law proton spectrum $I_p \propto E_p^{-3}$ normalized to Auger data [17] at $E_p \simeq 10^{18}$ eV. We compare this with the well-known approximation where the photon field is assumed to be represented by a δ-function centered at $\epsilon_0 \simeq 2.75$ K, normalized to the total photon density $\tilde{n}_\gamma = \int d\epsilon n_\gamma(\epsilon) \simeq 410$ cm^{-3}. This plot reveals that, based on the chosen cross-section model, the cosmogenic production on the CMB yields secondary particles with energies greater than 10^{19} eV. To explore lower energy ranges within the window accessible to next-generation telescopes, it becomes necessary to consider production processes involving the EBL.

5. Results

We provide here an example of the application of our model. Specifically, we conduct a series of comparisons between different computations to evaluate the impacts of various source parameters and different models of the EBL.

For each injection scenario described below, we perform UHECR propagation calculations and compute the cosmogenic neutrino fluxes across the energy range from 10^{15} to 10^{20} eV.

We focus on models with a pure proton composition, characterized by the slope index γ, the maximum acceleration energy, and the cosmological evolution of the sources parameterized by $(1+z)^m$ with a fixed value of m and a maximal redshift.

γ	m	$\mathbf{L}_0$ [10^{45} erg Mpc^{-3} yr^{-1}]
2.4	0	3.0
2.2	0	2.5
2.6	0	4.5
2.4	-3	3.8
2.4	3	2.2

Table 1: Model parameters

We set the cutoff energy at 5×10^{19} eV and $z_{\rm max} = 10$, while allowing for variations in the other parameters, namely γ and m. The proton intensity at each cosmological epoch is calculated following the approach outlined in equation (8), where we select the luminosity $\mathbf{L}_0$ as the maximum value allowed based on the Auger data.

The reference model is identified by the injection slope $\gamma = 2.4$ and the evolution index $m = 0$. In figure 3 we compare different models in which we vary γ (left panel) and m (right panel). For each of these models the maximum allowed luminosity is reported in table **??**.

We proceed to compute the neutrino flux for each of the considered models, taking into account the production on both the CMB and the (EBL. The latter is expected to contribute to neutrinos in the energy range of approximately 10 PeV. This is illustrated in figure 4, where the curves represent the range of uncertainty resulting from variations in the considered parameters. In the right panel, we present calculations using different EBL models recently reported in the literature.

Our findings confirm previous results, indicating that the evolution of sources can have a significant impact on the level of the diffuse flux, potentially varying by more than one order of magnitude. Consequently, the detection of this observable presents an excellent opportunity to probe the cosmological evolution of UHECR sources.

6. Conclusions

In this paper, we have presented an analytical derivation of the cosmogenic neutrino diffuse flux, with the objective of examining the influence of various uncertainties on the production rates of secondary particles. Specifically, we have investigated the uncertainties arising from different models of the EBL spectrum and some key source parameters.

A forthcoming publication is anticipated, which will significantly expand the scope of assumptions considered in these calculations, particularly with regard to nuclear physics aspects.

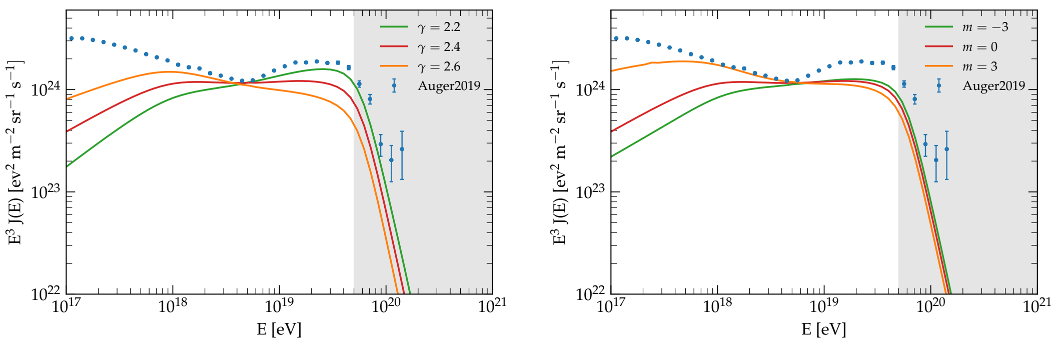

Figure 3: Fluxes of protons expected at Earth in proton-only scenarios with various models for the injection slope (left panel) and the cosmological evolution of source (right panel). The source luminosity is the maximum allowed by the Auger data [17] (shown as blue dots).

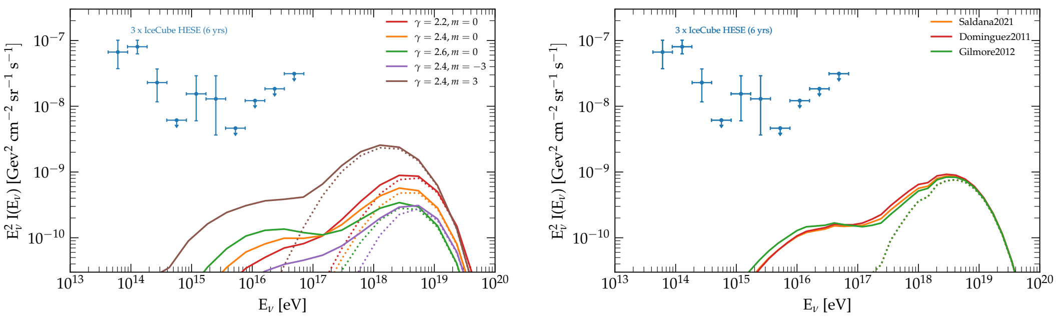

Figure 4: Fluxes of neutrinos computed in the different scenarios. In the right panel we show the comparison between the adopted EBL models. Data are from [18].

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