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Exotic Spectroscopy: A Lattice QCD perspective

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We review recent results for exotic states with beauty quarks from Lattice QCD, focusing on heavy-light tetraquarks. In particular, there is a growing consensus of deeply-bound, strong-interaction stable T_{bb} and T_{bbs} four-quark states. Beyond these the possibility of further bound states where either the light quarks are made heavier, or the heavy quarks lighter are being explored, such as possible T_{bc} and T_{bs} bound states. We also compare predictions for positive-parity B_s-mesons, which, in analogy to the $D_{s0}^*(2317)$ and $D_{s1}(2460)$, may feature a large four-quark component. We conclude by briefly discussing searches for other heavy multi-quark states.

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1. Introduction and Motivation

At the dawn of QCD, Gell-Mann pointed out the possibility of bound multi-quark hadrons [1] such as tetra-, penta-, and hexa- quark states. Together with glueballs and hybrid mesons, such states are often referred to as *exotics*, which we will interpret loosely as hadrons that do not fit in the naive quark model of quark-antiquark mesons and three-quark baryons. For light-quark systems the unambiguous identification of such states is difficult; from the early 2000s such hadrons were discovered experimentally with heavy flavors. Examples include the X, Y, and Z states in the charmonium spectrum as well as the unexpected strange-light $D_{s0}^*(2317)$ and $D_{s1}(2460)$. As the ability to probe heavy-quark physics increases, more and more unusual exotic states have been discovered. It appears that the landscape of hadrons with heavy quarks is far richer than one may have first surmised.

Lattice Quantum Chromodynamics, a regularization of QCD on a 4D Euclidean space-time lattice, provides an ab-initio way to calculate QCD correlation functions as path integrals which are determined numerically using Markov-Chain Monte-Carlo methods. It can be used to probe various hadrons and, along with experiment and phenomenological modeling, proves to be an invaluable tool in determining the properties of exotic hadrons. In this work we will focus on reviewing the state of theoretical calculations for a particular brand of heavy-light tetraquarks. We will also present predictions for exotic mesons with b quarks and conclude with a quick look at fully-bottom hadrons and at two-baryon states with multiple heavy quarks.

2. Heavy-light four-quark states $(qq'QQ')$

Both phenomenologically and directly through lattice simulations the most deeply-bound candidate has been identified as the axial-vector like $I(J^P) = 0(1^+)$ T_{bb} tetraquark of flavor content $ud\bar{b}\bar{b}$. This state is the beauty-analog of the T_{cc} [2] observed by LHCb. From studies of the heavy quark mass-dependence of the binding of these states [3] we see the behaviour of this state fits a diquark-antidiquark picture, with a color-Coulomb attraction from the two heavy (anti) diquarks and a so-called *good* light diquark configuration of the u and d quarks. Other heavy-light four-quark states are expected to be penalised in this picture, either by making one or both of the heavy quarks lighter or by making one or both of the light quarks heavier.

2.1 The T_{cc} $(ud\bar{c}\bar{c})$

The T_{cc} is known experimentally to be very shallowly bound (by 273(63) KeV) [2] with respect to the relevant DD^* threshold. There were several attempts to investigate the relevant quantum numbers using Lattice QCD before its discovery. One (albeit at heavy pion mass) finding little evidence of binding [4], and one finding a relatively deep binding of ≈ 23 MeV below threshold [5]. Since its discovery there is continuing work in post-dicting this state e.g. in [6]. Although this state has no beauty quarks, it may be considered as part of a family of heavy-light tetraquarks that have been investigated on the lattice.

2.2 The T_{bb} $(ud\bar{b}\bar{b})$

Fig. 1 shows the current (as of writing) state of lattice determinations of the binding energy of the T_{bb}, i.e. the energy difference $\Delta_{ud\bar{b}\bar{b}}$ between the mass of the T_{bb} and the BB^* threshold.

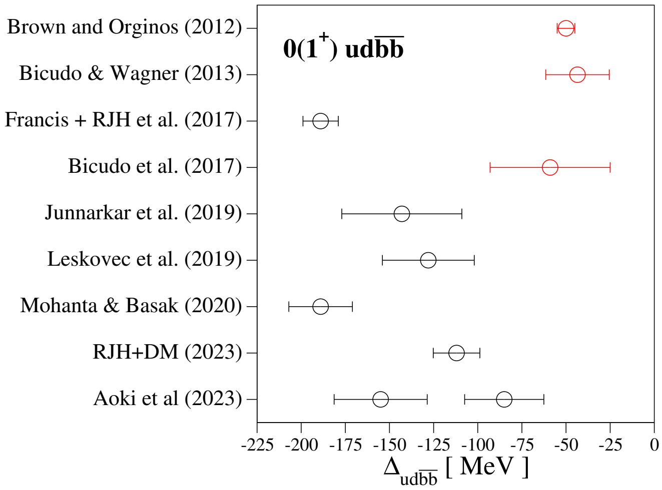

Figure 1: Overview of Lattice QCD binding energies for the $0(1^+)$ T_{bb} from [5, 7–13]. Red circles indicate evaluations using static b-quarks whereas black circles indicate the use of lattice NRQCD. For the last result their single and coupled analyses are presented.

Although there is some scatter, it appears that consensus is achieved in identifying this state's deeply-bound nature.

In our recent work [12] we focused on addressing open systematics of previous calculations. We tuned our lattice-NRQCD Hamiltonian entirely nonperturbatively unlike previous studies that used either only tree-level coefficients or a few one-loop coefficients determined from lattice perturbation theory. Our nonperturbative tuning demands the matching to continuum low-lying 1S- and 1P splittings of bottomonia, tuning the parameters of the action via a neural network (as previously detailed for charm in [14]). We also investigated the volume dependence of the T_{bb} by performing calculations on 10 different ensembles, spanning a large range in $m_\pi L$. In addition two of these ensembles have a different approach to the physical strange-quark mass, allowing for improved control over the quark-mass dependence of the results.

Fig. 2 shows the pion mass-dependence of the binding of the T_{bb}; with lighter pion mass giving deeper binding. We display lines of constant volume ($m_\pi L$) illustrating that the infinite-volume limit also yields deeper binding. We noted several other systematics that pose challenges in calculating this quantity using lattice-NRQCD; the largest of these was the uncertainty from discretisation effects as lattice-NRQCD, being an effective theory, has no formal continuum limit. We furthermore noted that obtaining the correct $B^* - B$ splitting is important, with a larger splitting giving shallower binding. At the order at which we performed our tuning in NRQCD it is however not possible to simultaneously match the bottomonia spectrum and $B^* - B$ splitting to experiment, so we needed to correct for this effect.

All of the Lattice QCD studies reported on in these proceedings imply that the T_{bb} is a narrow, reasonably long-lived state; being strong-interaction stable, it will decay only weakly. A likely decay is given by $T_{bb} \to B^+ \bar{D}^0$ [9]. It is unfortunately unlikely that the T_{bb} will be found at the LHC anytime soon.

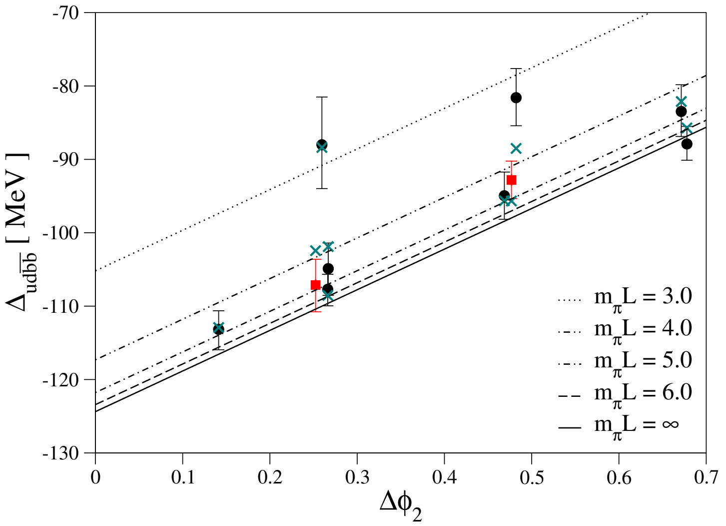

Figure 2: Quark-mass ($\phi_2 = 8t_0 m_\pi^2$) and finite-volume dependence of the binding energy of our T_{bb}. Black circles lie on a trajectory with constant trace of the quark-mass matrix and red squares indicate a trajectory of fixed physical strange-quark mass. Green crosses display the central values of our fit result evaluated at the respective point.

2.3 The T_{bbs} $(\ell s\bar{b}\bar{b})$

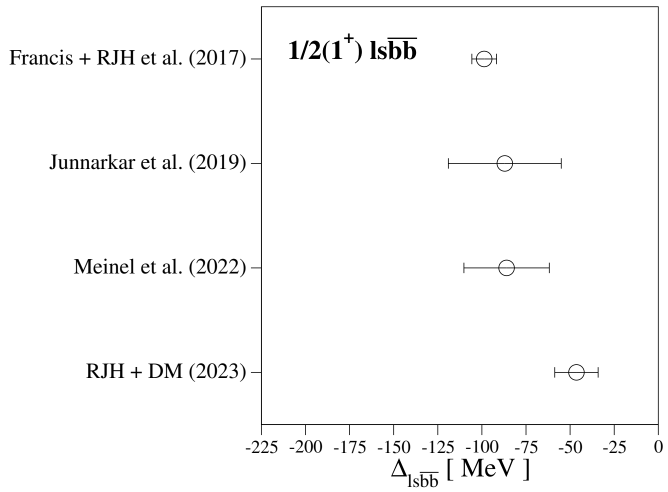

Figure 3: Review of lattice determinations of the binding energy of T_{bbs}, data collected from [5, 9, 10, 12].

We now move to our recent determination of the $\frac{1}{2}(1^+)$, T_{bbs}. As one would expect from the diquark picture the ls good diquark configuration is less attractive. A challenge is posed by the fact that expected non-interacting thresholds are very close-by (the $B_s B^*$ and the BB_s^*), requiring more care in disentangling these states from the tetraquark. Again, as we can see from Fig. 3 the consensus seems that this state is bound by around 50 MeV or more, albeit with fewer studies performed than for the T_{bb}. The shallowest binding reported appears to be near the $B - B^*$ splitting and pessimistically this state could decay electromagnetically.

2.4 Further tetraquarks with heavy quarks: $T_{bc}, T_{bs}, T_{bcs}, T_{cs}, T_{bbc}, T_{bbcs}$

Beyond these well-established states, several other quantum numbers have been studied using Lattice QCD.

2.4.1 Scalars and Axial-vectors ($Q' \neq Q$)

When the two heavy quarks have different masses a new set of quantum numbers, the scalar $0(0^+)$ is also possible along with the $0(1^+)$. As there is no obvious reason why the scalar would be less bound than the axial-vector both sets of quantum numbers should be considered. From a naive diquark prescription one would expect the binding energy to be proportional to the reduced mass of the heavy anti-diquark constituents, with the expectation being a binding energy closer to that of the T_{cc} rather than that of the T_{bb}.

Originally [3] indicated that at light pion masses there was evidence for a bound $0(1^+)$ tetraquark somewhere between 61 and 15 MeV below threshold. A later study by the same group [15] of the $0(0^+)$ and $0(1^+)$ T_{bc} upon improving some of the systematics of the previous study (more operators, larger volume, higher statistics, better sink treatment) found no evidence of binding in either channel. Likewise [16] found little evidence for a bound state in either channel but the authors could not rule out a shallowly-bound state within their systematic uncertainties. If such a $0(1^+)$ state exists it seems that it will likely decay electromagnetically to $DB\gamma$.

In [15] the investigation of further states that had been proposed to be deeply-bound in the literature, the T_{bs}, T_{bcs}, and T_{cs}, was also detailed, and little evidence of deep binding was found. This brings into question the use of Chiral Quark models used to predict the behaviour of these states and strongly prefers simple non-chiral models with color-Coulomb interaction, one gluon exchange hyperfine-interactions, such as the "AL1" from [17].

2.4.2 *Good* diquarks with heavier quarks ($q' = c$)

The remaining heavy-light candidates that studies have looked at are the T_{bbc} and T_{bbcs} in [5] and [15]. Both results illustrate that any binding has to be small, with the former suggesting these states are shallowly bound (of the order of a few MeV) and the latter reference suggesting that they are in fact unbound and that an additional state above threshold might exist in both spectra.

3. B_{s0}^* and B_{s1}: Regular mesons or meson molecules/tetraquarks?

We now briefly review recent results for the positive-parity scalar and axial vector B_s-mesons. These are the b-quark cousins of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$, whose unexpected properties helped spark renewed interest in the spectroscopy of heavy-quark hadrons. In a recent study we used simple single-meson operators and lattice-NRQCD for the b-quarks, unlike the scattering study of [18] which employed more sophisticated techniques, and used a relativistic heavy-quark action for the the b-quarks. However, it will turn out that the two approaches gave consistent results with our newest study being more precise. Fig. 4 illustrates the current model landscape (squares) and the few lattice calculations (circles). The two most-recent calculations indicate the B_{s0}^* and

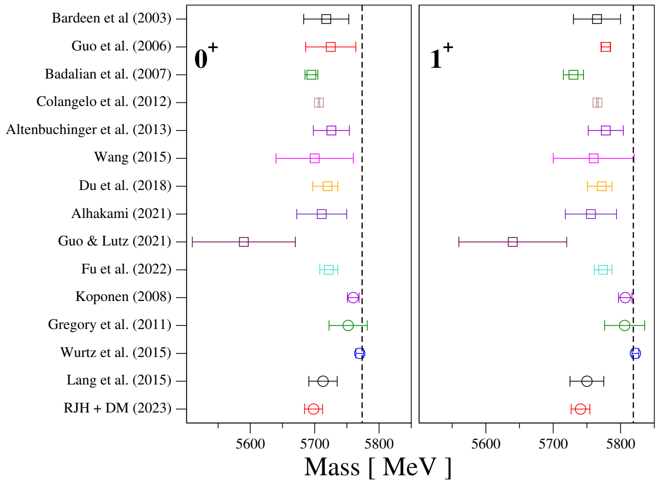

Figure 4: Phenomenological and lattice results for the B_{s0} and B_{s1} mesons.

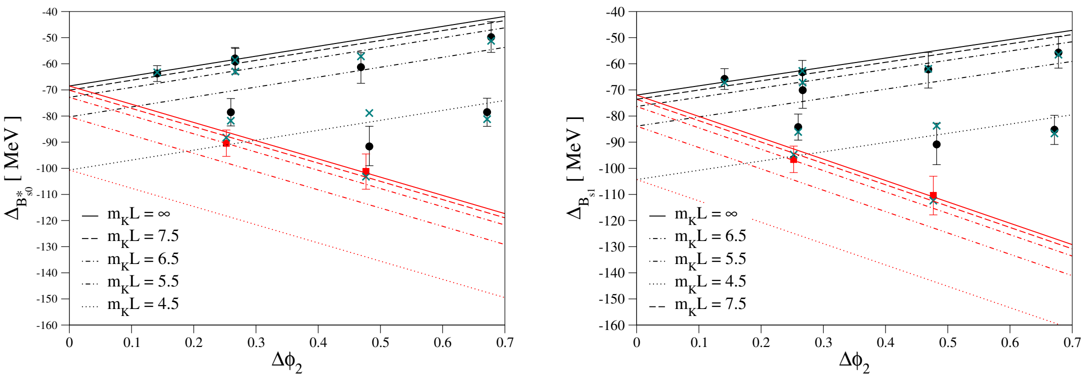

Figure 5: (Left) chiral and finite-volume dependence of B_{s0} binding energy $\Delta_{B_s 0^*}$, (Right) same for the binding energy of the B_{s1}.

B_{s1} lie considerably below the BK and B^*K thresholds respectively. Note that most relativistic quark-model calculations predicted these states well above the respective thresholds.

In Fig. 5 we show our combined chiral and infinite-volume extrapolations for the B^*_{s0} (left) and B_{s1} (right) mesons. We see large finite-volume effects (parameterised by $m_K L$ with the approach to the infinite volume yielding states less bound with regard to the respective thresholds.

4. Beauty-full multi-quark states

Several phenomenological models have predicted very deeply-bound $bb\bar{b}\bar{b}$ tetraquarks. However, the lattice study of [19] indicated no binding whatsoever in comparison to the respective non-interacting two-meson threshold. A plot of the binding from various channels and a comparison against phenomenological predictions is shown in Fig. 6.

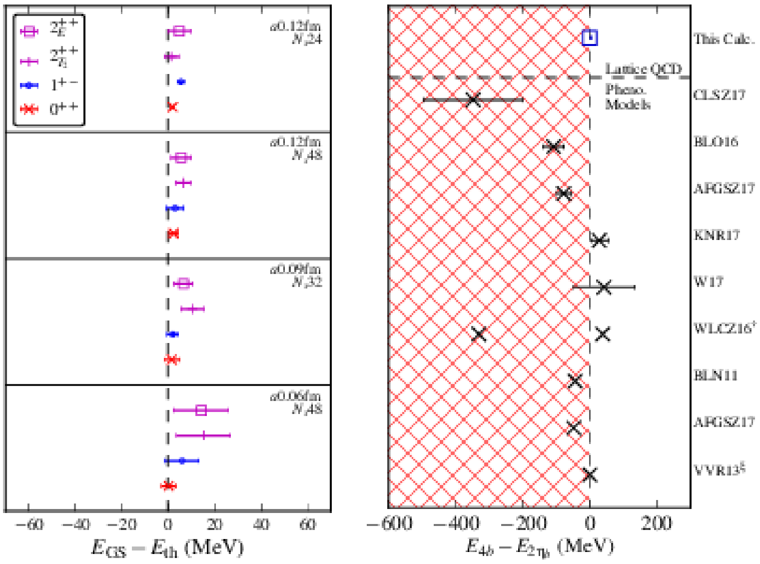

Figure 6: (Left) lattice results for various quantum numbers. (Right) comparison of the lattice determination of 0^+ $bb\bar{b}\bar{b}$ against model predictions. From [19].

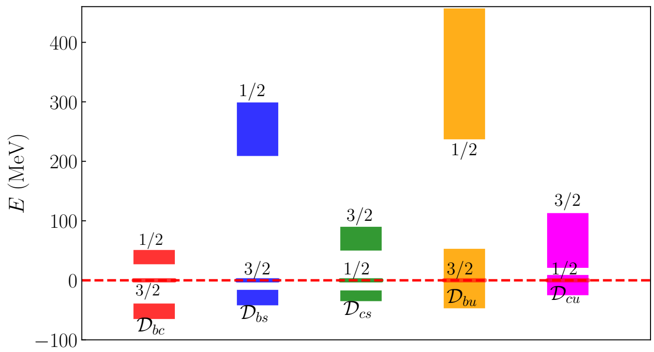

Figure 7: Binding energies of various deuteron-like dibaryons. From [20].

Finally we take a look at one of the most surprising recent developments of lattice multi-quark states; the deeply-bound nature of an all-beauty (beauty-full!) six-quark bound dibaryon [21]. Another calculation [20] suggests that that many more heavy, and somewhat deeply-bound, multi-baryon states are to be found in the future. Fig. 7 shows the binding energies observed for various dibaryon flavor contents.

5. Conclusions

Lattice QCD is provides a first-principles way to investigate the properties of exotic hadrons, and to predict their existence for channels where a direct experimental determination is not yet possible. It also has the ability to investigate the binding mechanism of these exotic hadrons in a systematically-improvable way. We have several indications of where to look for heavy-light four-quark states and a clarification of the existence of the T_{bc} is of most importance, as it is expected to be at an energy more accessible to current experiments.

The calculations of exotic hadrons discussed here pave the way for further improvements to phenomenological models such that the lattice data can be well described. For heavy-light tetraquarks, Chiral Quark models tend to predict unphysically-large bindings for many of the possible candidates, whereas simple non-relativistic models with one gluon exchange interactions fair better. Likewise for the exotic B_{s0} and B_{s1} hadrons the relativistic quark model fails to describe the lattice results whereas effective theories and models based on chiral symmetry and heavy-quark symmetry fair better. Similarly the apparent non-existence of the fully-beauty tetraquark casts into doubt several other model prescriptions, although further lattice studies are warranted for this state.

There are indications from the lattice for deeply-bound heavy di-baryons. This is indicative of what we are already seeing from experiment: the landscape of hadrons with heavy flavors is far richer than naively expected. This has implications for future experiments and may be a general feature of strongly-interacting gauge theories, also beyond the Standard Model.

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