PROCEEDINGS OF SCIENCE

Neutrino oscillations revisited and neutrinoless double beta decay

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The theory of neutrino oscillations remains to be an open problem. To address this, a new quantum field theory formalism of neutrino oscillations is advocated. This formalism includes neutrino emission and detection in a second-order Feynman diagram, demonstrates entanglement between the two processes and leads to the expected production rate for Racah's process, which involves the oscillation of neutrinos into antineutrinos (and vice versa). Further, a link between the Majorana masses of neutrinos, which govern the neutrinoless double-beta decay and the process based on neutrino-antineutrino oscillation, is established. It is argued that further investigation is necessary to explore the potential of Racah's process in determining the Majorana nature of neutrinos.

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1. Introduction

Neutrino oscillations have provided strong evidence that neutrinos oscillate from one flavor to another due to neutrino mixing. These oscillations have been observed in experiments involving atmospheric, solar, reactor and accelerator neutrinos. It was found that neutrinos have small masses, which can offer insights into new physics beyond the Standard Model. All the neutrino oscillation data are conventionally analyzed on the basis of the quantum-mechanical concept of neutrino oscillation via the oscillation probability

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) \equiv \left|\langle\nu_\beta|\nu_\alpha\rangle\right|^2 = \left|\sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i m_j^2 L/(2E_\nu)}\right|^2. \quad (1)$$

Here, E_ν is the energy of the neutrino, m_j (j=1,2,3) denotes the mass of the j-th neutrino, $U_{\alpha j}$ ($\alpha =$ e, μ and τ) is the element of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and L is the distance between the source and the detector. It is assumed that the neutrino emission in the source **S**, its propagation in space and its detection in the detector **D** are three independent processes: $S \to S' + \ell^+_\alpha + \nu_\alpha$, $\nu_\alpha \to \nu_\beta$ and $\nu_\beta + D \to D' + \ell^-_\beta$. The propagation of neutrinos in a space is described by plane waves.

Recently, neutrino oscillations was reformulated in Quantum Field Theory (QFT) using a second-order Feynman diagram which describes the process $S + D \to \ell^+_\alpha + \ell^-_\beta + S' + D'$ [1]. In this process, the intermediate neutrinos are described by the usual QFT propagators for the neutrino mass eigenstates. For the differential rate the L-dependent Master Formula was derived by exploiting the S-matrix formalism. It was found that the Master Formula is process-dependent and cannot be reduced to the conventional approach. However, for a specific choice of the underlying process, it approximately coincides with the conventional result under the given realistic assumptions. The QFT oscillation probability is given by

$$\mathcal{P}^{\rm QFT}_{\alpha\beta}(E_\nu, L) = \frac{1}{2}\sum_{km} U_{\alpha k} U^*_{\beta k}\, U^*_{\alpha m} U_{\beta m}\, e^{i(p_k - p_m)L}\left(1 + \frac{p_k p_m}{E_\nu^2}\right) \quad (2)$$

with $p_j = \sqrt{E_\nu^2 - m_j^2}$. For $p_k p_m/E^2_\nu \simeq 1$ the probability in Eq. (1) is reproduced.

The proposed QFT formalism was extended to neutrino-antineutrino oscillations, which underly, in particular, the lepton number violating process $S + D \to \ell^+_\alpha + \ell^+_\beta + S' + D'$ with on-shell Majorana neutrinos oscillating in flight between the source and the detector. This reaction was proposed by Gulio Racah for experimental verification of hypothesis of Majorana neutrino soon after its appearance in 1937. The neutrino to antineutrino oscillation probability takes the form

$$\mathcal{P}^{\rm QFT}_{\alpha\overline{\beta}}(E_\nu, L) \equiv \left|\langle\nu_\beta|\overline{\nu}_\alpha\rangle\right|^2 = \left|\sum_{j=1}^{3} U^*_{\alpha j} U^*_{\beta j} \frac{m_j}{E_\nu} e^{-i m_j^2 L/(2E_\nu)}\right|^2 = \frac{(m^L_{\alpha\beta})^2}{E^2_\nu}. \quad (3)$$

We note that $\mathcal{P}^{\rm QFT}_{\alpha\overline{\beta}}(E_\nu, L)$ differs from $\mathcal{P}_{\alpha\beta}(E_\nu, L)$ in Eq. (1) by a presence of the suppression factor m_j/E_ν and due to the replacement $U_{\beta j} \to U^*_{\beta j}$.

In this article, we investigate the relationship between the effective Majorana masses m^L_{ee} and $m_{\beta\beta}$, which govern the neutrino-antineutrino oscillations and the neutrinoless double-beta decay ($0\nu\beta\beta$-decay) processes, respectively.

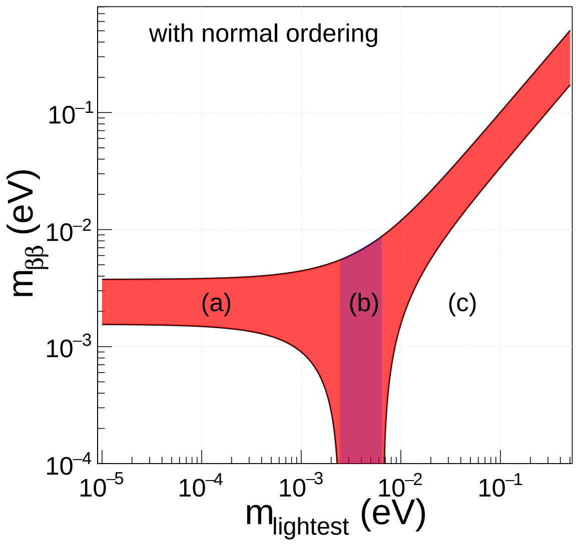

Figure 1: The Majorana neutrino mass $m_{\beta\beta}$ as function of the lightest neutrino mass $m_{\rm lightest} = m_1$. Normal ordering of neutrino masses ($m_1 < m_2 < m_3$) is assumed. In region (b), which corresponds to 2.5 meV $< m_{\rm lightest} <$ 6 meV, the lowest value of $m_{\beta\beta}$ is zero.

2. Majorana masses related to neutrino-antineutrino oscillations and $0\nu\beta\beta$-decay

Detecting $0\nu\beta\beta$-decay events will prove the Majorana nature of neutrinos and reveal Majorana mass $m_{\beta\beta}$, which depends on Majorana phases ϕ_1 and ϕ_2. We have

$$m_{\beta\beta} = \left|\sum_j^3 U_{ej}^2\, m_j\right| = \left|\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3\right|, \qquad (4)$$

where $\rho_1 = c_{12}^2 c_{13}^2 m_1$, $\rho_2 = s_{12}^2 c_{13}^2 m_2$ and $\rho_3 = s_{13}^2 m_3$. $c_{12,13} \equiv \cos\theta_{12,13}$, $s_{12,13} \equiv \sin\theta_{12,13}$ and θ_{12} and θ_{13} are two of tree mixing angles which parametrize the PMNS matrix.

The maximal value of $m_{\beta\beta}$ is $m_{\beta\beta}^{\rm max} = \rho_1 + \rho_2 + \rho_3$ ($\phi_1 = \Phi_2 = 0$). For the lowest value of $m_{\beta\beta}$ we have: i) $m_{\beta\beta}^{\rm min} = |\rho_2 - \rho_3| - \rho_1$, if $\rho_1 < |\rho_2 - \rho_3|$ (region I, $\Phi_1 = 0$ and $\Phi_2 = \pm\pi/2$); ii) $m_{\beta\beta}^{\rm min} = 0$ if $|\rho_2 - \rho_3| \le \rho_1 \le \rho_2 + \rho_3$ (region II, Majorana phases determined in [2]); iii) $m_{\beta\beta}^{\rm min} = \rho_1 - (\rho_2 + \rho_3)$ if $\rho_2 + \rho_3 < \rho_1$ (region III, $\Phi_1 = \pm\pi/2$ and $\Phi_2 = 0$).

In Fig. 1 we show $m_{\beta\beta}$ as a function of the lightest neutrino mass $m_{\rm lightest} = m_1$ (the normal ordering of neutrino masses (NO)). The used values of the neutrino oscillation parameters are $\theta_{12} = 34°$, $\theta_{13} = 8.5°$, $\Delta m_{21}^2 = 7.5 \times 10^{-5}$ eV^2, and $\Delta m_{31}^2 = 2.5 \times 10^{-3}$ eV^2. In the range of 2.5 meV $< m_{\rm lightest} <$ 6.5 meV (region II), $m_{\beta\beta}^{\rm min} = 0$. A strong suppression of $m_{\beta\beta}$ can prevent the observation of the $0\nu\beta\beta$-decay in current and future experiments.

The Majorana neutrino mass m_{ee}^L associated with the neutrino-antineutrino oscillations between the source and detector with distance L is [2]

$$m_{ee}^L = \left|U_{e1}^2\, m_1 + U_{e2}^2\, m_2\; e^{-i\frac{\Delta m_{21}^2}{2E}L} + U_{e3}^2\, m_3\; e^{-i\frac{\Delta m_{31}^2}{2E}L}\right|. \qquad (5)$$

For NO and L/E = 300 m/MeV (or km/GeV) the dependence of m_{ee}^L on $m_{\rm lightest}$ is displayed Fig. 2. We notice that $m_{\beta\beta} = 0$ is compatible with $m_{ee}^L = m_{\beta\beta}^{\rm max}$ for a given value of L/E [2].

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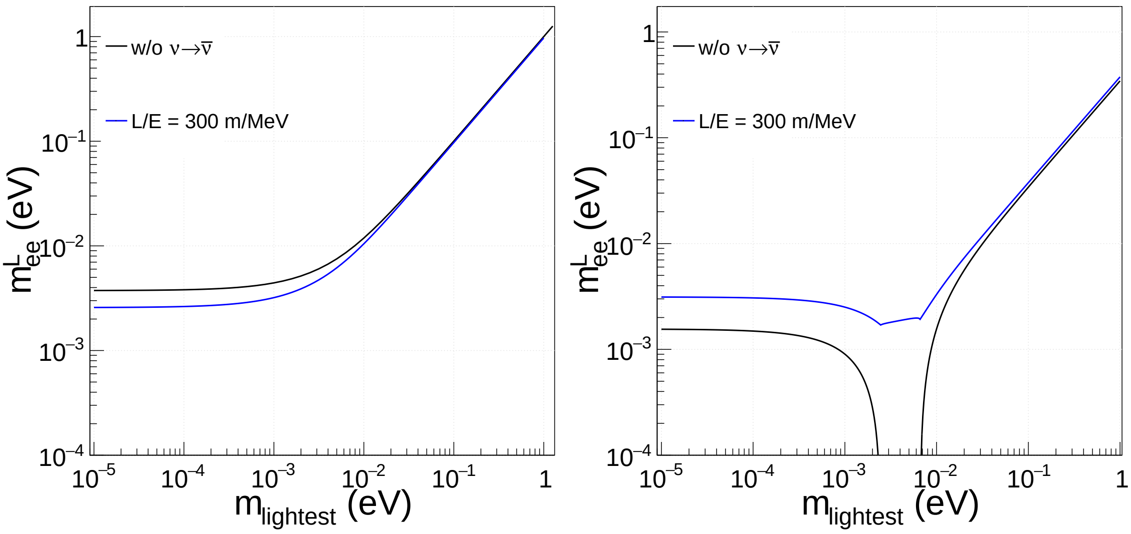

Figure 2: Variation of m_{ee}^L with lightest mass (m_1 for NO) with Majorana phases corresponding to highest and lowest values of $m_{\beta\beta}$ in left and right panels, respectively. L/E=300 m/MeV (or km/GeV) is assumed.

3. Conclusions

In summary, a new QFT formalism for neutrino oscillations that work for both (anti)neutrino-(anti)neutrino and neutrino-antineutrino oscillations (and vice versa) is highlighted. Further, a study was conducted on the relationship between Majorana masses associated with neutrino-antineutrino oscillations and $0\nu\beta\beta$-decay. We conclude that it is important to explore the potential of total lepton number-violating processes with on-shell intermediate neutrinos, as a strong suppression of $0\nu\beta\beta$-decay rate is possible.

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