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Projections of discovery potentials for future neutrinoless double beta decay experiments

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The most promising strategy for demonstrating the Majorana nature of neutrinos is to observe neutrinoless double beta decay. Measurement of the neutrinoless double beta decay lifetime will provide direct insight into the absolute mass scale of neutrinos and probe the neutrino mass ordering. The next generation of neutrinoless double beta decay experiments targets to probe the inverted mass ordering and enter the normal ordering regions. Estimation of the experimental specifications and their cost-effectiveness is becoming increasingly important as these experiments generally require tonne-scale of enriched isotopes and decade-long efforts to realize. We perform a quantitative study of the projected experimental sensitivities in terms of the discovery potentials – prior to the experiments are performed. The sensitivity of counting analysis is derived with complete Poisson statistics and compared with its continuous approximation. Additional measurable signature such as energy can boost the sensitivity and this is incorporated via a maximum likelihood analysis. The roles and effects of uncertainties in background predictions are examined. The results reinforce and quantify the vital role of background suppression in future neutrinoless double beta decay projects with sensitivity goals of approaching and covering normal ordering.

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1. Introduction

In searches for novel phenomena, background information with certain uncertainties is usually known before the experiments being conducted. At the design stage, *prior* to conducting experiments, the experimenters should make projections of the sensitivities, either in terms of signal discovery potentials or as exclusion limits, under specific statistical criteria they determine. Given the expected level of background, these answers will determine how much exposure (Σ = target size $\times$ data taking time) is required to achieve certain specific sensitivities. It directly relates to the investment in hardware, time, and manpower, the precise knowledge of which is increasingly important as experimental projects become more and more elaborate. It is important to know and compare the cost-effectiveness of achieving a certain scientific goal at the proposal stage, which can be a decade or more before actual data collection begins.

In this article, we address some of the key aspects of this problem. In the present work, the sensitivity of counting analysis based on complete Poisson statistics [1] and its continuous approximation [2, 3] is compared with that based on maximum likelihood, including additional measurables as signatures [4]. The methodology and results of this study have general validity to many different research areas, however we apply the developed formulations to experimental searches of neutrinoless double beta decay ($0\nu\beta\beta$).

2. Counting versus extended likelihood

2.1 Complete Poisson

Experimental sensitivity goals are typically defined in the literature [2] as: "Discovery potential at 3σ with 50% probability" $\mathrm{P}_{50}^{3\sigma}$ and "upper limits at 90% confidence level", which characterize possible positive and negative outcomes. Statistical analysis using Poisson distributions is essential for handling rare signal processes and low background. As an illustration, we construct the Poisson distribution $\mathrm{Poi}(i;\mathrm{B}_0)$ with mean μ=B_0 for a given real and positive input B_0. The observed count $N_{obs}^{3\sigma}$ is evaluated as:

$$\sum_{i=0}^{N_{obs}^{3\sigma}-1} \mathrm{Poi}(i;\mathrm{B}_0) \geq (1-0.00135) \ , \tag{1}$$

where 0.00135 is the fraction of a Gaussian distribution in the interval $[+3\sigma,\infty]$. It is the minimum integer number of observed events providing $\geq 3\sigma$ significance over a predicted average background B_0. The output S_0 represents the minimum signal strength, such that a Poisson distribution with μ=(B_0+S_0) would yield $N_{obs}^{3\sigma}$ events or more with 50% probability:

$$\sum_{i=N_{obs}^{3\sigma}}^{\infty} \mathrm{Poi}(i;[\mathrm{B}_0+\mathrm{S}_0]) = 0.5 \ \ . \tag{2}$$

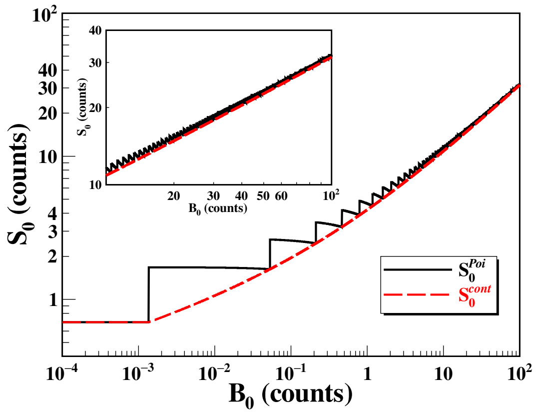
We define the signal (S_0)-to-background (B_0) criteria using Eqs. 1 & 2 [1]. This approach is represented in Fig. 1(a) (black solid line S_0^{Poi}), showing the qualifying S_0 sensitivities. In Fig. 1(b), the black solid line ($\mathscr{L}_C$) indicates the minimum $\mathrm{S}_0/\varepsilon_{\mathrm{RoI}}$ sensitivity necessary within the optimal Region of Interest (RoI). Clearly, it provides a thorough description, taking into account both the discrete nature of the problem and its inherent fluctuations.

2.2 Continuous approximation

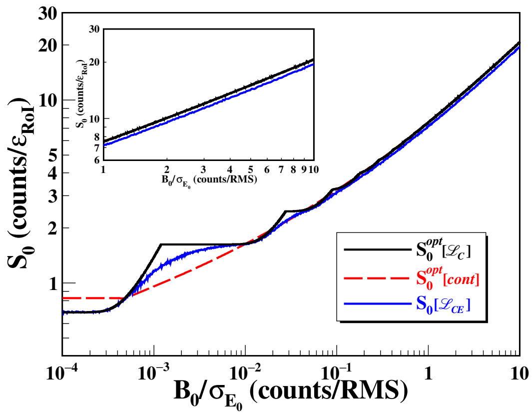
A continuous approximation to the Poisson distribution can be obtained by replacing the Cumulative Poisson Distribution [$\mathrm{CPoi}(\leq C;\mu) = \sum_{i=0}^{C}\mathrm{Poi}(i;\mu)$] with the regularized incomplete gamma function:

$$\mathrm{CPoi}(\leq C;\mu) = \frac{\Gamma(C{+}1;\mu)}{\Gamma(C{+}1)} \ , \tag{3}$$

where C is generalized to be a positive real variable. There is several literature [2, 3] that uses this method for determining sensitivity projections. The continuous approximation always underestimates the required signal strength S_0^{cont} to establish a positive signal as shown in Fig. 1(a). In cases of low background, the deviation compared to S_0^{Poi} can reach up to 60%, but when the background is substantial ($\mathrm{B}_0 \gtrsim 100$), this deviation is constrained to within 3%.



(a)



(b)

Figure 1: (a) A comparison of counting-only with complete Poisson (S_0^{Poi}) and continuous approximation (S_0^{cont}) in defining $\mathrm{P}_{50}^{3\sigma}$ sensitivity. (b) Sensitivities of $(\mathrm{S}_0/\varepsilon_{\mathrm{RoI}})$ as a function of $(\mathrm{B}_0/\sigma_{\mathrm{E}_0})$ on $\mathscr{L}_{CE}$ by LLR analysis with complete information incorporated, choosing $\mathrm{RoI}(\mathscr{L}_{CE})$=$\mathrm{E}_0 \pm 4\sigma_{\mathrm{E}_0}$. We compare these results with counting-only analysis via $\mathscr{L}_C$ and continuous approximation at the optimal RoI of $\mathrm{E}_0 \pm \mathrm{N}_\sigma^{opt}\sigma_{\mathrm{E}_0}$.

2.3 Extended likelihood

The likelihood function based solely on counting is expressed as:

$$\begin{aligned} \mathscr{L}_C &\equiv \mathscr{L}(S|N,B) \\ &= \frac{e^{-(B+S)}(B+S)^N}{N!} . \end{aligned} \tag{4}$$

Correspondingly, the log-likelihood-ratio (LLR), denoted by q_0, is defined as

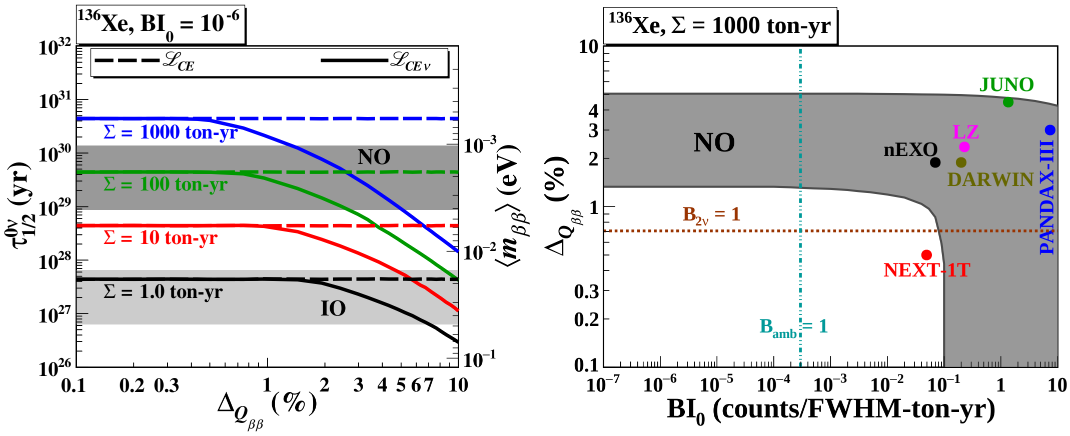
$$q_0 \equiv t(S{=}0) = -2\ln\left[\frac{\mathscr{L}(S{=}0)}{\mathscr{L}(\hat{S})}\right] , \tag{5}$$

in which $\hat{S}$ is the value of $S\in(0,\infty)$ that $\mathscr{L}(S)$ is maximized at given N & at a fixed B=B_0 value. This study aims to quantitatively determine the significance of measurements for supporting discovery scenario. Therefore, the data set must be tested against the *null hypothesis* (H_0) case of S=0. An *alternative hypothesis* (H_1) is defined as the case in which S=S_0>0, where S_0 is the mean signal strength. The LLR methodology we adopted for defining $\mathrm{P}_{50}^{3\sigma}$ sensitivity is detailed in Reference [4].

3. Sensitivity projections

A case study was performed to make sensitivity projections on future $0\nu\beta\beta$ experiments considering ^{136}Xe isotope as an example using our developed LLR analysis. The half-life ($\tau_{1/2}^{0\nu}$)

and effective Majorana neutrino mass $\langle m_{\beta\beta} \rangle$ versus $\Delta_{Q_{\beta\beta}}$ (FWHM energy resolution) for ambient background index BI_0=10^{-6} counts/(FWHM-ton-yr) at different Σ contours (1,10,100,1000 ton-yr) are depicted in Fig. 2(a), along with predicted ranges of IO and NO (inverted and normal mass ordering). Solid and dotted lines diverge depending on Σ and BI_0, and these divergent points specify the $\Delta_{Q_{\beta\beta}}$-values above which the irreducible $2\nu\beta\beta$ background dominates.



(a) (b)

Figure 2: (a) Combined background LLR analysis for ^{136}Xe in $(\Delta_{Q_{\beta\beta}}, \tau^{0\nu}_{1/2})$ space at different Σ contours (1,10,100,1000 ton-yr) taking BI_0= 10^{-6}counts/(FWHM-ton-yr). (b) Requirements in $(BI_0, \Delta_{Q_{\beta\beta}})$ space for $0\nu\beta\beta$ experiments [4] with ^{136}Xe to achieve $P^{3\sigma}_{50}$, for Σ at 1000 ton-yr.

As shown in Fig. 2(b), the region where Σ=1000 ton-yr to achieve $P^{3\sigma}_{50}$ in $(\Delta_{Q_{\beta\beta}}, BI_0)$ space is compared with the performance specifications of the next generation of ^{136}Xe-projects. At a chosen Σ level, the ambient and $2\nu\beta\beta$ background are solely contingent on BI_0 and $\Delta_{Q_{\beta\beta}}$, respectively. As well, we have shown the contours of the 1st event from the $B_{2\nu\beta\beta}$ & B_{ambient} within the RoI=$Q_{\beta\beta}\pm4\sigma_{E_0}$. The sensitivity projection in Reference [4] is explained in detail.

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