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Effective field Theories and the Standard Model extensions

Oleg Antipin[a] **and Jahmall Bersini**[b]

[a] *Institute Ruđer Bošković, Bijenička cesta 54, Zagreb, Croatia*
[b] *Kavli IPMU (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan*

E-mail: oantipin@irb.hr, jahmall.bersini@ipmu.jp

These lecture notes on Effective Field Theories (EFTs) cover the basic notions of EFTs from the renormalization group point of view. After a detailed discussion of a simple 1D Ising theory of spins on a circle we construct the Standard Model (SM) Lagrangian using only the known symmetries and the fields content of this model. Various problems of the SM are classified by the dimension of the corresponding operators built from the SM fields. Solutions to these problems are sketched leading to the various extensions of the SM.

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1. Introduction

The journey we are going to take in these very short lectures is a journey through physical phenomena as we change the distance (or, equivalently, energy) scales probed by our experiments. The history of physics itself is such a journey. Starting from classical mechanics describing the movement of objects of human size we learned that this theory needs to be generalized to quantum mechanics to deal with movements of particles at subatomic level. In this sub-atomic world, Galelian kinematics of slow sub-relativistic speeds of classical mechanics needed to be generalised to special relativity. Marriage of quantum mechanics and special relativity led us to quantum field theory (QFT), while marriage of special relativity with Newtonian gravity led us to general relativity. Moving into even smaller scales we hope to marry QFT with general relativity into quantum gravity.

On this journey, to make a stop at some fixed scale and describe the physical system at that scale we need to determine:

1. Relevant degrees of freedom (d.o.f.). As we change our "microscope", relevant d.o.f. may change. For example, new collective excitations or composite particles may appear
2. Symmetries (types of interactions between d.o.f.)
3. Expansion parameters (power counting)

Since the d.o.f. may change we need to learn how to identify the relevant ones and remove the irrelevant ones. This can be done by a procedure of "integrating out" d.o.f. as we change the energy scale. Since this logic will be the central part of the effective field theories (EFT) examples below, let me illustrate the procedure in a very simple model.

2. Invitation

2a 1D Ising model of spins on a circle

To learn how to "integrate out" d.o.f. let us consider a simple 1d model of spins on a circle (see Fig.1) with Hamiltonian:

$$H = -J \sum_{i=1}^{N} S_i S_{i+1} = -J(S_1 S_2 + S_2 S_3 + \ldots + S_N S_1) \tag{1}$$

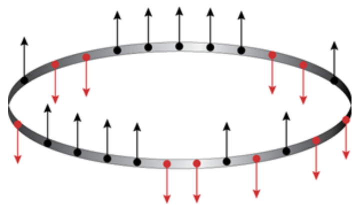

Figure 1: Spins on a circle.

In general, this system of spins is placed in an environment at a given fixed temperature T. The laws of thermodynamics teach us that the system prefers to arrange spins to be in a state with minimum free energy $F = H - TS$, where S is the entropy of the system. Consider the $T = 0$ case: we need to find a state where $F = H$ has a minimum. Clearly this is the state where all spins point up or all point down because the total interaction energy is minimized, $H = -NJ$.

Consider now the $T > 0$ case. We have to account for all possible configurations of spins and weight them according to their energies. For a canonical ensemble that is classical and discrete, this defines the canonical partition function

$$Z(K,N) = \sum_{states} e^{-H/T} = \sum_{S_1=-1}^{+1}\sum_{S_2=-1}^{+1}\dots\sum_{S_N=-1}^{+1} e^{K(S_1S_2+S_2S_3+\ldots+S_NS_1)} \tag{2}$$

where $K \equiv J/T$ and $F = -T\log Z(K,N)$.

Let us now sum over the two possibilities $S_2 = \pm1$ for spin S_2:

$$Z(K,N) = \sum_{S_1=-1}^{+1}\sum_{S_3=-1}^{+1}\dots\sum_{S_N=-1}^{+1}\left[e^{K(S_1+S_3)} + e^{-K(S_1+S_3)}\right]e^{K(S_3S_4+S_4S_5+\ldots+S_NS_1)}\,. \tag{3}$$

In the same fashion let us sum over the two possibilities $S_4 = \pm1$ for spin S_4:

$$Z(K,N) = \sum_{S_1=-1}^{+1}\sum_{S_3=-1}^{+1}\dots\sum_{S_N=-1}^{+1}\left[e^{K(S_1+S_3)}+e^{-K(S_1+S_3)}\right]\left[e^{K(S_3+S_5)}+e^{-K(S_3+S_5)}\right]e^{K(S_5S_6+S_6S_7+\ldots+S_NS_1)}\,. \tag{4}$$

We can repeat the exercise to sum over all even numbered spins:

$$Z(K,N) = \sum_{S_1=-1}^{+1}\sum_{S_3=-1}^{+1}\dots\sum_{S_{N-1}=-1}^{+1}\left[e^{K(S_1+S_3)}+e^{-K(S_1+S_3)}\right]\left[e^{K(S_3+S_5)}+e^{-K(S_3+S_5)}\right]\left[e^{K(S_5+S_7)}+e^{-K(S_5+S_7)}\right]\dots\,. \tag{5}$$

Rewrite the remaining sums defining:

$$e^{K(S+S')} + e^{-K(S+S')} \equiv f(K)e^{-K'SS'} \tag{6}$$

where both $f(K)$ and K' are functions of K. Now we have:

$$\begin{aligned} Z(K,N) &= f(K)^{N/2}\sum_{S_1=-1}^{+1}\sum_{S_3=-1}^{+1}\dots\sum_{S_{N-1}=-1}^{+1} e^{-K'S_1S_3}e^{-K'S_3S_5}e^{-K'S_5S_7}\dots \qquad (7)\\ &= f(K)^{N/2}\sum_{S_1=-1}^{+1}\sum_{S_3=-1}^{+1}\dots\sum_{S_{N-1}=-1}^{+1} e^{-K'(S_1S_3+S_3S_5+S_5S_7)}\dots = f(K)^{N/2}Z(K',N/2)\,. \end{aligned}$$

Note that we rewrote the original $Z(K,N)$ in terms of a new function $Z(K',N/2)$, i.e., a function with parameters that describe the model with half the number of spins and a different coupling parameter $K' = J'/T$. Let us find the functions $f(K)$ and K' implied by the transformation Eq.6. It is easy to show that they are given by:

$$K' = \frac{1}{2}\log(\cosh(2K)) \quad f(K) = 2\cosh^{1/2}(2K) \tag{8}$$

Exercise: Derive the result Eq.8.

Notice that $K' < K$. Eq.8 represents the renormalization group (RG) functions that describe how the interaction among the relevant d.o.f changes as we zoom in/out the system. Now, repeat the procedure to integrate out another half of spins to arrive at $Z(K'', N/4)$ then $Z(K''', N/8)$ and so on. Since $K > K' > K'' > K'''$, after many iterations the coupling parameter becomes negligibly small. Also notice that with each iteration the distance between the neighboring spins doubles in size.

We found that $K = 0$ ($J = 0$) is an attractive *fixed point* of the RG transformation in a 1d model of spins. At this fixed point the interaction between spins vanishes. Therefore the temperature effects will determine the emergent behavior at large (macroscopic) distances. These thermal fluctuations will tend to align spins randomly and at long distances the system is disordered.

Going to higher dimensions, the Ising model undergoes a phase transition between an ordered and a disordered phase in two dimensions or more. In contrast to 1d, there is a nontrivial fixed point between the two phases at a finite value of the coupling $K = K_c$. At this point, changing the scale does not the change the physics because the system is in a critical fractal state.

2b Relevant, marginal and irrelevant operators

We saw in the 1d Ising model that the coupling strength K between the spins decreases as we perform the RG transformations towards larger distances. We call such interactions *irrelevant*. Oppositely, interactions whose strength increases as we "integrate out" d.o.f. are called *relevant* and, finally, *marginal* interactions are those whose strength does not change. In QFT interactions are written in terms of operators and in d dimensions and at the classical level, operators with dimension$<$ $(>)d$ are relevant (irrelevant) while operators with dimension$=$ d are marginal. Quantum corrections will change classical (also known as engineering) dimensions of operators so that, for example, classically marginal operators can become relevant or irrelevant at the quantum level.

3. Constructing the SM

The main example of an EFT in these lectures will be the Standard Model (SM) of particle interactions. Let us try to understand how far we can go in building this model just using the notions of relevant, marginal and irrelevant operators above.

As we discussed above, to describe the physical system at some energy scale we need to:

1. Determine the relevant d.o.f. (fields). For the SM, the fermionic fields will be quarks and leptons, the vector ones are the gauge bosons $W^{\pm}, Z, \gamma$ and gluons, and finally the only scalar particle will be the famous Higgs.

2. Symmetries. These are given by the semi-simple product of three gauge groups $SU(3) \times SU(2) \times U(1)_Y$. Due to spontaneous symmetry breaking of the electroweak symmetry $SU(2) \times U(1)_Y \to U(1)_{EM}$, the photon γ, which mediates electromagnetism (EM), is a massless linear combination of the gauge bosons associated to the $U(1)_Y$ hypercharge group and the third generator of $SU(2)$ group W_3. The other linear combination is called Z and together with the gauge bosons $W^{\pm}$, these three massive gauge bosons mediate weak $SU(2)$ interactions. Finally, massless gluons mediate the strong (QCD) $SU(3)$ interactions.

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3. The expansion parameter is given by the scale at which electroweak symmetry is broken (which is experimentally observed to be ≈ 246 GeV) divided by the cutoff of the SM theory. This cutoff may be Planck scale or grand unified scale or any other high scale up to which the SM EFT is valid. Later we will see numerous examples where this cutoff scale will appear associated with the new interactions beyond the SM.

Of course, here I just stated the results for the d.o.f. and symmetries of the SM. It took incredible amount of research to arrive at this construction through deep theoretical ideas and experimental efforts.

Now, let us try to stay agnostic and imagine we do not know the Lagrangian of the SM but only know the d.o.f. and symmetries stated above. We will now build all possible operators out of the SM fields consistent with the SM gauge symmetry classifying these operators according to their dimension. Remember that we are working in four dimensions (three space and one time) so that dimension= 4 operators will be marginal. The classical scaling dimension of the fermion field is given by 3/2, while for the Higgs and gauge bosons fields it is 1. Remember that the action S, which is dimensionless, is given by $S = \int \mathcal{L}\ d^4x$, so that Lagrangian density $\mathcal{L}$ (which I will refer to as simply "Lagrangian" for brevity) has dimension=4 to balance dimension=-4 from the measure. Terms in the Lagrangian have the schematic structure "coupling constant $\times$ operator" so depending on the dimension d_O of the operator O built from the fermion, Higgs and vector fields in the SM, the coupling constant for such operator will have to have dimension $4 - d_O$. For illustration and in the spirit of EFT, the dimensionful parameter entering the coupling constant to balance the dimension will be the cutoff scale of the SM theory Λ.

- Dimension-0 operator: this is just identity operator $\mathbf{1}$. We will compute (the dimension=4) coefficient of this operator in the SM later.

- Dimension-2: These are "mass-terms" for the scalar fields and in the SM this will be the Higgs mass. Dim-2 mass terms for the gauge fields are forbidden since they are not gauge-invariant.

- Dimension-4: marginal operators and these we will present now. They are quartic, Yukawa and gauge interactions between the SM fields.

Summarizing, our SM Lagrangian so far schematically looks like:

$$\mathcal{L} = c \cdot \mathbf{1} + m^2 \phi^\dagger \phi + \mathcal{L}_4 + \dots \tag{9}$$

where $\phi(x)$ is the Higgs field.

Exercise: I purposefully omitted operators with dimension=1 and dimension=3. What is their role?

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3a SM Lagrangian

So now we present the SM Lagrangian writing only the dim-2 and dim-4 operators:

$$
\begin{aligned}
\mathcal{L}_{2,4} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}tr\left(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right) - \frac{1}{2}tr\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right) && (U(1)_Y, \text{ SU(2) and SU(3) gauge terms})\\
& + (\bar{\nu}_L, \bar{e}_L)\,\tilde{\sigma}^{\mu} iD_{\mu} \left(\begin{smallmatrix}\nu_L\\ e_L\end{smallmatrix}\right) + \bar{e}_R\sigma^{\mu} iD_{\mu} e_R + \bar{\nu}_R\sigma^{\mu} iD_{\mu}\nu_R + \text{h.c.} && (\text{lepton kinetic terms})\\
& - \frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\,\phi M^e e_R + \bar{e}_R\bar{M}^e\bar{\phi}\left(\begin{smallmatrix}\nu_L\\ e_L\end{smallmatrix}\right)\right] && (\text{electron, muon, tauon mass terms})\\
& - \frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\,\phi^* M^{\nu}\nu_R + \bar{\nu}_R\bar{M}^{\nu}\phi^T\left(\begin{smallmatrix}-e_L\\ \nu_L\end{smallmatrix}\right)\right] && (\text{neutrino mass terms})\\
& + \left(\bar{u}_L, \bar{d}_L\right)\tilde{\sigma}^{\mu} iD_{\mu}\left(\begin{smallmatrix}u_L\\ d_L\end{smallmatrix}\right) + \bar{u}_R\sigma^{\mu} iD_{\mu}u_R + \bar{d}_R\sigma^{\mu} iD_{\mu}d_R + \text{h.c.} && (\text{quarks kinetic terms})\\
& - \frac{\sqrt{2}}{v}\left[\left(\bar{u}_L, \bar{d}_L\right)\phi M^d d_R + \bar{d}_R\bar{M}^d\bar{\phi}\left(\begin{smallmatrix}u_L\\ d_L\end{smallmatrix}\right)\right] && (\text{down, strange, bottom mass terms})\\
& - \frac{\sqrt{2}}{v}\left[\left(-\bar{d}_L, \bar{u}_L\right)\phi^* M^u u_R + \bar{u}_R\bar{M}^u\phi^T\left(\begin{smallmatrix}-d_L\\ u_L\end{smallmatrix}\right)\right] && (\text{up, charm, top mass terms})\\
& + \overline{(D_{\mu}\phi)}D^{\mu}\phi - \frac{m^2\left[\bar{\phi}\phi - \frac{v^2}{2}\right]^2}{v^2} && (\text{Higgs kinetic and mass terms})
\end{aligned}
$$

where (h.c.) means Hermitian conjugate of preceding terms, $\bar{\phi} = (\text{h.c.})\phi = \phi^{\dagger} = \phi^{*T}$, and the covariant derivative operators are:

$$D_{\mu}\left(\begin{smallmatrix}\nu_L\\ e_L\end{smallmatrix}\right) = \left[\partial_{\mu} - \frac{ig_1}{2}B_{\mu} + \frac{ig_2}{2}\mathbf{W}_{\mu}\right]\left(\begin{smallmatrix}\nu_L\\ e_L\end{smallmatrix}\right)$$

$$D_{\mu}\left(\begin{smallmatrix}u_L\\ d_L\end{smallmatrix}\right) = \left[\partial_{\mu} - \frac{ig_1}{6}B_{\mu} + \frac{ig_2}{2}\mathbf{W}_{\mu} + ig\mathbf{G}_{\mu}\right]\left(\begin{smallmatrix}u_L\\ d_L\end{smallmatrix}\right)$$

$$D_{\mu}\nu_R = \partial_{\mu}\nu_R \qquad D_{\mu}e_R = \left[\partial_{\mu} - ig_1B_{\mu}\right]e_R$$

$$D_{\mu}u_R = \left[\partial_{\mu} + \frac{i2g_1}{3}B_{\mu} + ig\mathbf{G}_{\mu}\right]u_r \qquad D_{\mu}d_R = \left[\partial_{\mu} - \frac{ig_1}{3}B_{\mu} + ig\mathbf{G}_{\mu}\right]d_R$$

$$D_{\mu}\phi = \left[\partial_{\mu} + \frac{ig_1}{2}B_{\mu} + \frac{ig_2}{2}\mathbf{W}_{\mu}\right]\phi$$

where ϕ is a 2-component complex Higgs field. Since $\mathcal{L}$ is SU(2) gauge invariant, we can choose a gauge in which ϕ has the form:

$$\phi^T = \frac{(0, v+h)}{\sqrt{2}} \qquad <\phi>_0^T = (\text{expectation value of } \phi) = \frac{(0, v)}{\sqrt{2}}$$

where v is a real constant such that the Higgs potential $\mathcal{V}_{\phi} = \frac{m^2\left[\bar{\phi}\phi - \frac{v^2}{2}\right]^2}{v^2}$ is minimized, and h is a residual Higgs field. B_{μ}, $\mathbf{W}_{\mu}$, and $\mathbf{G}_{\mu}$ are the gauge boson vector potentials, and $\mathbf{W}_{\mu}$ and $\mathbf{G}_{\mu}$ are composed of 2×3 and 3×3 traceless Hermitian matrices respectively. Their associated field strength tensors are:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \qquad \mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + ig_2 \frac{(\mathbf{W}_\mu \mathbf{W}_\nu - \mathbf{W}_\nu \mathbf{W}_\mu)}{2}$$

$$\mathbf{G}_{\mu\nu} = \partial_\mu \mathbf{G}_\nu - \partial_\nu \mathbf{G}_\mu + (\mathbf{G}_\mu \mathbf{G}_\nu - \mathbf{G}_\nu \mathbf{G}_\mu)$$

The fermions include the leptons e_R, e_L, ν_R, ν_L and quarks u_R, u_L, d_R, d_L. They all have implicit generation indices, $e_i = (e, \mu, \tau)$, $\nu_i = (\nu_e, \nu_\mu, \nu_\tau)$, $u_i = (u, c, t)$, $d_i = (d, s, b)$, which contract into the fermion mass matrices $M^e_{ij}, M^\nu_{ij}, M^u_{ij}, M^d_{ij}$, and implicit indices which contract the Pauli matrices:

$$\sigma^\mu = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

$$\tilde{\sigma}^\mu = \left[\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3 \right] \qquad tr\left(\sigma^i\right) = 0 \qquad \sigma^{\mu\dagger} = \sigma^\mu \qquad tr\left(\sigma^\mu \sigma^\nu\right) = 2\delta^{\mu\nu}$$

We included right-handed neutrino and thus wrote the Dirac mass term for neutrino. The minimal formulation of the SM does not have right-handed neutrino so that neutrinos are massless. We will discuss the issue of generating neutrino mass later.

The quarks also have implicit 3-component color indices which contract into $\mathbf{G}_\mu$. So altogether the Lagrangian really has implicit sums over 3-component generation indices, 2-component Pauli indices, 3-component color indices in the quark terms, and 2-component SU(2) indices.

3b SM: $d > 4$ operators

According to our classification in sec.2b, $d > 4$ operators are irrelevant and we briefly discuss them now:

- Dimension-5: There is a unique dimension=5 operator in the SM. This is called the *Weinberg operator* and we will encounter it later while discussing the neutrino mass. According to dimensional analysis, the coefficient of this operator scales as $1/\Lambda$.

- Dimension-6: There are many dim-6 operators one can build using SM fields. These operators are again generated at a new physics scale Λ, and scale as $1/\Lambda^2$. There are eight different classes of operators: X^3, H^6, H^4D^2, X^2H^2, ψ^2H^3, ψ^2XH, ψ^2H^2D and ψ^4 in terms of their field content, where X,H,D and ψ stand for gauge field strength, Higgs field, covariant derivative and fermion field, respectively. The SM EFT classifying these operators is called SMEFT in the literature.

Let us stop here even though we could continue the list but clearly higher dimensional operators will be suppressed by more powers of the cutoff scale and so their effects will be smaller.

4. BSM: Problems with the SM addressed from EFT point of view

In the previous section we built all possible operators out of SM fields classifying them according to their dimension:

$$\mathcal{L} = c \cdot \mathbf{1} + m^2 \phi^\dagger \phi + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6 + \dots \tag{10}$$

Let us now discuss some of the problems related to these operators, going again in the direction of increasing operator dimensions.

- Dark energy: Let us calculate the sample contribution to the cosmological constant (C.C) c, the coefficient of the unit operator, from, say, the Higgs condensate. For example, if ϕ_{vac} is the value of the Higgs field $\phi(x)$ which minimizes the potential $V(\phi)$, then the lowest state has stress-energy momentum tensor $T_{\mu\nu} = g_{\mu\nu} V(\phi_{vac})$, which is the classical scalar field contribution to the vacuum energy. Concretely, minimizing Higgs potential

  $$V(\phi) = -m^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2 , \tag{11}$$

  The Higgs condensate contribution (at the classical level) to the cosmological constant is

  $$c_{Higgs} = V(\phi_{vac}) = \rho_{Higgs} = -\frac{m^4}{2\lambda} . \tag{12}$$

  Exercise: Derive Eq.12.

  Besides the Higgs condensate, there are other contributions to the C.C., for example from the QCD vacuum, potential grand unified theory (GUT) scale physics, etc. The experimentally measured physical value of the C.C. ρ_{phys} is given by

  $$\rho_{phys} = 10^{-47} \text{GeV}^4 . \tag{13}$$

  The problem now is that if we use Higgs mass $M_H \sim m = 125\,\text{GeV}$ then the corresponding value is $|\rho_{Higgs}| \simeq 10^8\, GeV^4$. In order to keep the QFT consistent with the observations, one has to demand that the contributions to the ρ_{phys} should cancel with great accuracy. For example, adding the vacuum contribution to the C.C. ρ_{vac}, which we can always add to the Lagrangian, the ρ_{vac} and ρ_{Higgs} should cancel with the precision of 55 decimal orders. This is the C.C. fine-tuning problem.

- Hierarchy problem: Thinking of SM as an EFT with the cutoff scale Λ, the Higgs mass term (dim-2 operator) is naturally expected to have a form $m^2 \phi^\dagger \phi \sim \Lambda^2 \phi^\dagger \phi$. The expected quadratic Λ^2 dependence of the coefficient leads to the so-called "hierarchy problem": the Higgs mass gets a correction of order $\Lambda \gg$ electroweak scale v. Indeed, from the sample diagram in Fig.2 coming from some hypothetical Yukawa interaction of some fermion f with the Higgs $y \bar{f} f \phi$, dimensional analysis suggests that

  $$m^2 \sim y^2 \int_0^\Lambda d^4k \times \frac{1}{k} \times \frac{1}{k} \sim \Lambda^2 . \tag{14}$$

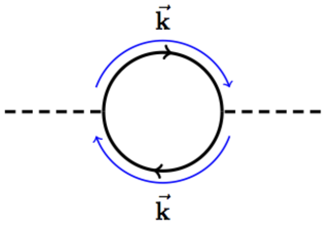

Figure 2: Higgs mass correction

where we used that the fermion propagator scales as $1/k$. Notice however that the sensitivity of m on Λ follows because we are computing low-energy observable (Higgs mass) in terms of high-energy parameters represented by Λ which is not in the spirit of the EFT where we are suppose to first integrate out high energy physics and then compute the Higgs mass within the low energy EFT. Nevertheless, many extensions of the SM were motivated by this problem among which are supersymmetry (SUSY), extra-dimensional models, technicolor and composite Higgs. In SUSY there is a cancellation between fermions and bosons protecting corrections to Higgs mass. In technicolor and composite Higgs models, Higgs is a composite particle (similar to mesons in QCD) and so as we reach the compositeness scale we have to change description. Extra-dimensional models are conceptually similar to technicolor and composite Higgs models via holographic AdS/CFT correspondence.

- Vacuum instability (dimension-4 operator): The analysis of the vacuum stability requires the knowledge of the effective potential of the model at hand. The standard model effective potential is known up to two loops. For large field values $\phi \gg v = 246$ GeV, the potential is very well approximated by its RG-improved tree-level expression,

$$V_{eff}^{tree} \approx \lambda(\mu)(\phi^\dagger \phi)^2 \ . \tag{15}$$

with RG scale $\mu = \mathcal{O}(\phi)$. Therefore if one is simply interested in the condition of absolute stability of the potential, it is possible to study the RG evolution of λ and determine the largest scale $\Lambda < M_{pl}$, with M_{pl} the Planck scale, above which the coupling becomes negative. The RG evolution of the Higgs quartic coupling λ in the SM is shown on the right panel in Fig.3 and we observe that the coupling becomes negative around 10^{10} GeV. The RG running of the Higgs quartic coupling in the SM is very sensitive to the value of the top quark mass. This is illustrated by the left plot in Fig.3 where we see that changing the value of the top mass by $\mathcal{O}(1\text{GeV})$ may bring us back to the (green) stability region.

- Neutrino masses (dim-5 Weinberg operator): Neutrinos are electrically neutral, and so can have either Majorana type or Dirac type mass terms. The existence of a Dirac mass term would necessitate the existence of right-handed neutrinos. In the minimal standard model, there is an effective dimension-five operator which generates Majorana neutrino masses

$$\Lambda^{-1}\phi^0\phi^0\nu_i\nu_j, \tag{16}$$

All models of neutrino mass and mixing can be summarised by this operator. Different models are merely different realizations of this operator. In the following I will show that

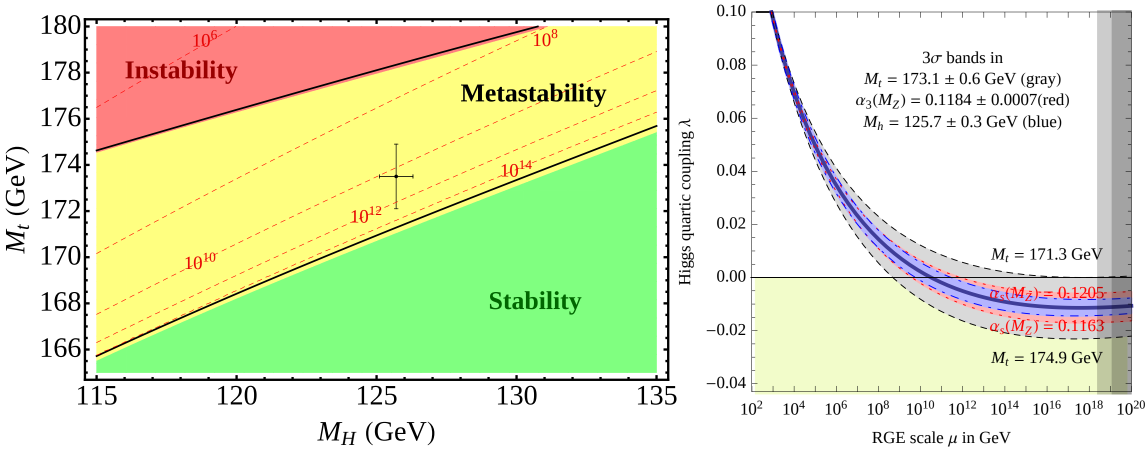

Figure 3: (Left) Standard model stability analysis based on the effective standard model Higgs quartic coupling. The red region indicates instability, the yellow metastability and the green absolute stability. The point with error bars shows the experimental values of the top and the Higgs masses. The red dashed lines show the value in GeV at which λ crosses zero. Figure is taken from arxiv:1306.3234. (Right) Running of the Higgs quartic coupling in the SM. Figure is taken from arxiv:1205.6497.

it has only three *tree-level* realisations, meaning that the leading Feynman diagrams are generated starting at tree-level. In addition, it also has three 1-loop realizations in *radiative* neutrino mass models.

To obtain the effective operator Eq.16 at tree-level we demonstrate now that there are only three possibilities. To start with, recall that in the SM, the left-handed neutrino and neutral component of the Higgs are parts of the left-handed doublets $\psi = \binom{\nu_L}{e_L}$ and $\Phi = (\phi^+\phi^0)$. So using the group theory multiplication $2 \otimes 2 = 3 \oplus 1$ we have the following options:

(I) $\psi \times \Phi \sim (\phi^0\nu_L - \phi^+ e_L)$ forms a fermion singlet,
(II) $\psi \times \psi$ forms a scalar triplet, with one of the components $\nu_L\nu_L$
(III) $\psi \times \Phi$ forms a fermion triplet with one of the components $(\phi^0\nu_L + \phi^+ e_L)$.

while the singlet combination of ψ^i and ψ^j is $\nu_L^i e_L^j - e_L^i \nu_L^j$ which does not generate Eq.16. In each of the three cases, we generate the operator Eq.16 among with the other interactions which altogether schematically look like:

$$(I) \qquad \Lambda^{-1}(\phi^0\nu_L - \phi^+ e_L)(\phi^0\nu_L - \phi^+ e_L), \tag{17}$$

$$(II) \qquad \Lambda^{-1}[\phi^0\phi^0\nu_L\nu_L - \phi^+\phi^0(\nu_L e_L + e_L\nu_L) + \phi^+\phi^+ e_L e_L], \tag{18}$$

$$(III) \qquad \Lambda^{-1}[(\phi^0\nu_L + \phi^+ e_L)(\phi^0\nu_L + \phi^+ e_L) - 2\phi^+\nu_L\phi^0 e_L - 2\phi^0 e_L\phi^+\nu_L]. \tag{19}$$

The intermediate heavy particle in the first case is clearly a fermion singlet (right-handed neutrino) and this is the well-known type-I seesaw mechanism. In the second case the intermediate heavy particle is a heavy scalar triplet $\xi = (\xi^{++}, \xi^+, \xi^0)$ realizing type-II seesaw. Finally, we replace the right-handed neutrino of the first model with a heavy Majorana fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ and obtain type-III seesaw mechanism. Each seesaw mechanism has its own unique implications about physics beyond the Standard Model.

- Dark matter (DM): Particle physics suggests an effective solution to this problem in terms of an electrically neutral and weakly interacting massive particle that is stable at cosmological scales. DM particles are predicted by many extensions of the Standard Model, including the well motivated ones that address other important theoretical or experimental issues such as SUSY and Composite Higgs models we discussed above. Because of the many possible choices for DM candidates, it has become customary and quite useful to consider EFT approaches, which allow to study in a model-independent manner the phenomenology of these particles. It is typically assumed that the new state is either a scalar, a vector or a fermion. In order to work with a manageable theory some restrictions on the DM sector need to be imposed. One popular direction can be summarised as follows:

  The field content of the theory is given by the SM extended by an extra multiplet X that belongs to an irreducible representation of the SM gauge group $G_{SM} = SU(3)\times SU(2)\times U(1)_Y$. Under the Lorentz group, X transforms either as a scalar, a spinor or a vector. All SM fields are even under a postulated extra Z_2 parity symmetry, while X is odd. This discrete symmetry stabilises the DM candidate which is the lightest member of the multiplet X. The DM-EFT Lagrangian can be schematically written as:

$$L = L_{SMEFT} + \sum c_i O_i \tag{20}$$

  where operators O_i include the SM and DM field X.

- Flavour problem (dim-6 operators): The SM does not explain the fermion masses and their mixing angles. These parameters are very different. Also, why are there 3 generations of quarks and leptons? From the Lagrangian point of view this problem is connected to the Yukawa couplings of the fermions to the Higgs and the Weinberg operator in neutrino sector:

$$Y_{ij}\psi^i_L\psi^j_R\phi + \Lambda^{-1}\phi^0\phi^0\nu_i\nu_j \tag{21}$$

  The main goal of the flavor physics model building is to identify the symmetries and symmetry-breaking patterns beyond those present in the SM which would explain fermion masses and mixing angles. The dim-6 operators built from the SM fields are very important as they typically give the leading effect, for example to B-$\bar{B}$ mesons mixing: $(\bar{d}_L\gamma_\mu b_L)(\bar{d}_L\gamma^\mu b_L)$.

- Proton decay (dim-6 operators): In the SM, the proton is stable because it is the lightest baryon and baryon number (quark number) is conserved. Many BSM models explicitly break the baryon number symmetry, allowing protons to decay. For example, in grand unified theories (GUTs) it can decay via the new X bosons (see fig.4) . Integrating out the heavy X bosons (just as we will do later, integrating out the W and Z bosons to obtain the Fermi theory) we are left with dimension-6 operators $\frac{\bar{e^c}\bar{u^c}qq}{\Lambda^2_{GUT}}$ and $\frac{\bar{u^c}\bar{d^c}ql}{\Lambda^2_{GUT}}$. All of these operators violate both baryon number (B) and lepton number (L) conservation but not the combination $B - L$. These operators mediate the decay of the proton to a positron and a neutral pion: $p \to e^+ + \pi^0$. Breaking of the baryon number symmetry is also important to explain the matter-antimatter asymmetry as we observe in our Universe.

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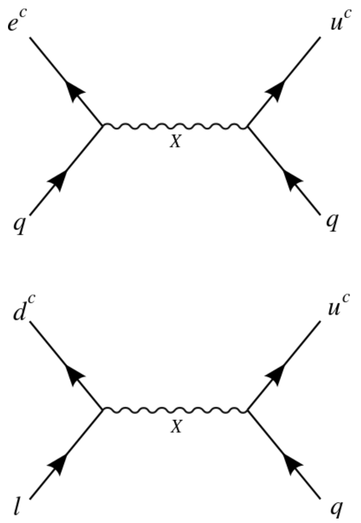

Figure 4: Proton decay mediated by the X boson $(3,2)_{-5/6}$ in $SU(5)$ GUT.

There are other puzzles in the SM which we do not have time to go to. Let me mention a few: Strong-CP problem, muon $g-2$, matter-antimatter asymmetry.

Notice that puzzles related to relevant dim<4 operators lead to "hierachy" problems due to the fact that the related coupling constants are expected to pick up contributions proportional to the large cutoff scale Λ whereas experimentally they are at the low (electroweak) scale. Problems related to irrelevant dim>4 have opposite, "decoupling" nature. If we take a cutoff scale infinitely large, the effects of these operators will be unobservable since the corresponding couplings scale inversely with the cutoff.

5. EFTs in general

Having discussed the SM EFT and its problems we now discuss how to construct EFT in general. There are two ways to do it:

1. Top-down: in this approach we integrate out heavy particles and match onto a low energy theory. The matching procedure will be illustrated in the example below. In the low energy EFT, we find new operators and new low energy constants.

2. Bottom-up: here you write down the most general possible operators/interactions consistent with symmetries. Couplings of your EFT will be unknown but can be obtained from experiment.

5a Examples of EFTs

Let me give some examples of EFTs keeping in mind that the list is not exhaustive. The first three examples will be examples of top-down approach while the last two will be bottom-up.

- HQET: describes the low-energy dynamics of hadrons (composite particles built from quarks and thus interacting via QCD interactions) containing a heavy quark. The theory is applied to hadrons containing b and c quarks. The expansion parameter is Λ_{QCD}/m_Q, where $m_Q = m_b, m_c$ is the mass of the heavy quark. The theory also has an expansion in powers of

$\alpha_s(m_Q)/(4\pi)$. The matching from QCD to HQET can be done in perturbation theory, since $\alpha_s(m_Q)/(4\pi)$ is small, $\alpha_s(m_b) \sim 0.22$, $\alpha_s(m_b)/(4\pi) \sim 0.02$.

- Fermi theory of weak interactions: this is an EFT for weak interactions at energies below the W and Z masses. The expansion parameter is p/M_W, where p is the momenta of a particle in the weak decay (which is related to b-quark mass in a b-decay, for example). We start with the tree-level amplitude for the $b \to c$ decay as our simple example:

$$A = \left(\frac{-ig}{2\sqrt{2}}\right)^2 V_{cb}\bar{c}\gamma_\mu(1-\gamma_5)b \quad \bar{\ell}\gamma_\nu(1-\gamma_5)\nu_\ell \quad \left(\frac{-ig^{\mu\nu}}{p^2 - M_W^2}\right) \tag{22}$$

where we have two charged weak currents $\bar{c}\gamma_\mu(1-\gamma_5)b$ (with the corresponding CKM matrix element V_{cb}) and $\bar{\ell}\gamma_\nu(1-\gamma_5)\nu_\ell$ coupled to the W boson, the last factor is the W propagator, and finally g is the gauge coupling of the $SU(2)$ interactions. For low momentum transfers, $p \ll M_W$, we can expand the W propagator:

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \dots\right). \tag{23}$$

Keeping only the first term we obtain the local Lagrangian:

$$A = \left(\frac{-ig}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} V_{cb}\bar{c}\gamma_\mu(1-\gamma_5)b \quad \bar{\ell}\gamma_\nu(1-\gamma_5)\nu_\ell + O\left(\frac{1}{M_W^4}\right). \tag{24}$$

This EFT no longer has dynamical W bosons, and the effect of W exchange in the SM has been included via dimension=6 four-fermion operators.

- SM below EW scale: Below $v \approx 246$ GeV scale, the electroweak symmetry is broken, so that one can write a low energy effective theory with quark and lepton fields, and only $SU(3)$ and $U(1)_{EM}$ gauge fields. Since SU(2) gauge invariance is no longer a requirement, there are several new types of operators beyond those in SMEFT.
  - There are dimension-three $\nu_L\nu_L$ operators which give a Majorana neutrino mass for left-handed neutrinos as we discussed in the previous section.
  - There are dimension-five dipole operators e.g. $\psi\sigma_{\mu\nu}\psi F^{\mu\nu}$.
  - There are X^3 and ψ^4 operators as in SMEFT classification, but operators containing the Higgs field ϕ are no longer present.
  - There are many dimension-six four-fermion operators e.g. $\bar{\psi}\psi\nu_L\nu_L$.

- Chiral perturbation theory (χPT): our first example of bottom-up approach describes the interactions of pions and nucleons at low momentum transfer p. It is not possible to analytically compute the matching onto the EFT, since the matching is non-perturbative. The two theories, QCD and χPT, are not written in terms of the same fields. The QCD Lagrangian has quark and gluon fields, whereas χPT has meson and baryon fields. The parameters of the chiral Lagrangian are usually fit to experiment. The expansion parameter of χPT is p/Λ_χ, where $\Lambda_\chi \sim 1$ GeV is referred to as the scale of chiral symmetry breaking.

- General relativity

  The field relevant for gravity is the metric, $g_{\mu\nu}$ (whose matrix inverse is denoted $g^{\mu\nu}$). For applications on macroscopic scales we use the most general effective Lagrangian consistent with general covariance:

$$L_{grav} = \sqrt{-g}\left(\frac{1}{2}M_p R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \frac{c_4}{m^2}R^3 + \ldots\right) \tag{25}$$

  The term linear in R is the usual Einstein-Hilbert action, with M_p denoting the usual Planck mass. The remaining effective couplings c_k are dimensionless and unknown a priori. The scale m stands for the lightest particle that have been integrated out to obtain this EFT (say, the electron) since (being in denominator) it gives the largest coupling constant.

5b Top-down approach: Example of tree-level matching

Before we conclude these lectures let me give an example of tree-level matching technique from the full theory to the effective one in the top-down approach. We consider the theory with $U(1)$ global symmetry:

$$L = \partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda\left[\bar{\phi}\phi - v^2\right]^2}{4}\,. \tag{26}$$

We have a symmetry $\phi \longrightarrow e^{i\omega}\phi$ for $\partial_\mu \omega = 0$. Redefine $\phi \equiv \chi e^{i\theta}$ to obtain:

$$L = \partial_\mu \chi \partial^\mu \chi + \chi^2 \partial_\mu \theta \partial^\mu \theta - \frac{\lambda\left[\chi^2 - v^2\right]^2}{4}\,. \tag{27}$$

The structure of the theory is now transparent. We see that we have two fields: θ, which is massless, and χ with mass $M = \sqrt{\lambda}v$. As usual, we have to shift the χ field and so we define new fields: $\chi = v + \frac{\psi}{\sqrt{2}}$ and $\theta = \frac{\xi}{\sqrt{2}v}$. Our Lagrangian becomes:

$$L = \frac{1}{2}\partial_\mu \psi \partial^\mu \psi + \frac{1}{2}(1 + \frac{\psi}{\sqrt{2}v})^2 \partial_\mu \xi \partial^\mu \xi - \frac{\lambda(\sqrt{2}v\psi + \psi^2/2)^2}{4}\,. \tag{28}$$

To construct our EFT we will need to choose some observable to calculate. Let us use $\xi\xi \longrightarrow \xi\xi$ scattering, which occurs at tree-level in the full theory through the s, t and u channel processes, all formed from the $\psi\partial_\mu \xi \partial^\mu \xi$ vertex. We will assign momenta to the external lines as follows: p and q to incoming lines, p' and q' to outgoing. Then the amplitude in the full theory is:

$$A_{full} = \frac{2}{v^2}\left(\frac{(p\cdot q)^2}{(p+q)^2 + M^2} + \frac{(p\cdot p')^2}{(p-p')^2 + M^2} + \frac{(p\cdot q')^2}{(p-q')^2 + M^2}\right). \tag{29}$$

Exercise: Derive the result Eq.29.

To order $O(1/M^2)$ we simply have:

$$A_{LO} = \frac{2}{v^2 M^2}\left((p\cdot q)^2 + (p\cdot p')^2 + (p\cdot q')^2\right). \tag{30}$$

We now need to construct the effective Lagrangian for ξ, and calculate the same amplitude using this EFT. We have:

$$L_{eff} = \frac{1}{2}\partial_\mu \xi \partial^\mu \xi - a(\partial_\mu \xi \partial^\mu \xi)^2 + \ldots \tag{31}$$

where a is unknown coefficient. Using this effective Lagrangian we obtain for the amplitude:

$$A_{eff} = 8a\Big((p\cdot q)^2 + (p\cdot p')^2 + (p\cdot q')^2\Big)\,, \tag{32}$$

so that comparing we obtain $a = \frac{1}{4v^2M^2}$. By matching the coefficient in the effective theory to that produced (approximately) by the full theory we embeded information about the heavy field ψ, which is not itself part of the EFT, into our results.

References

[1] A. V. Manohar, [arXiv:1804.05863 [hep-ph]].

[2] D. B. Kaplan, [arXiv:nucl-th/9506035 [nucl-th]].

[3] Youtube lectures series by Iain Stewart: https://www.youtube.com/watch?v=WB8r7CU7clk