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Classical vs. quantum corrections to jet broadening in a weakly coupled QGP

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We compute double-logarithmically enhanced corrections to $\hat{q}$ at relative order $O(g^2)$ in the setting of a weakly coupled quark-gluon plasma, observing how the thermal scale affects the region of phase space, which gives rise to these corrections. We furthermore clarify how the region of phase from which these corrections are borne is situated with respect to that from which the classical corrections arise at relative order $O(g)$. This represents a significant step towards our eventual goal of understanding which class of corrections dominate, thereby pushing forward our quantitative grasp on the phenomenon of jet quenching in heavy-ion collisions.

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1. Introduction

In the context of heavy-ion collisions, jets provide an ideal *hard probe* of the quark-gluon plasma (QGP). Through interacting with the QGP, they receive momentum kicks in the directions transverse to their propagation – *transverse momentum broadening*. This broadening can be captured by the *transverse momentum broadening coefficient*, $\hat{q} = \langle k_\perp^2 \rangle / L$, which specifies the transverse momentum picked up per unit length, L by a hard parton propagating through the QGP. See the recent reviews on jets in heavy-ion collisions [1] or extractions of $\hat{q}$ from data [2] for more information.

For a weakly coupled QGP, $\hat{q}$ can be expressed in terms of the transverse scattering kernel

$$\hat{q}(\mu) = \int^{\mu} \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 C(k_\perp), \qquad C(k_\perp) \equiv (2\pi)^2 \frac{d\Gamma}{dk_\perp^2}, \tag{1}$$

where $\frac{d\Gamma}{dk_\perp^2}$ is the rate for a hard parton with energy, $E \gg T$, the temperature of the plasma, propagating along the z direction to pick up $k_\perp$. The cutoff, μ is installed so as not to include larger momentum scatterings, which include two hard partons in the final state. See App. A of [3] for our conventions.

At leading order (LO) in g, $\hat{q}$ receives contributions from the hard (T) [4] and soft (gT) [5] scales, which give rise to the parametric form (up to logarithms) $\hat{q} \sim g^4 T^3$. The soft contribution is cut off in the IR by dynamical screening, implemented through Hard Thermal Loop Effective Theory (HTL) resummation [6]. NLO corrections also come from the soft scale [7]. Ultrasoft ($g^2 T$) modes contribute at $\mathcal{O}(g^2)$, for which the perturbative expansion breaks down. We refer to these NLO and NNLO contributions as *classical corrections*: they are distributed on the T/ω IR tail of the Bose-Einstein distribution, $n_{\rm B}(\omega)$ and are therefore sourced by the Matsubara zero-mode.

Caron-Huot [7] demonstrated that one may compute the zero-mode contribution to $C(k_\perp)$ in Electrostatic QCD (EQCD) [8], meaning that one can bypass the somewhat cumbersome HTL computation. More importantly, as a theory of static modes, EQCD is amenable to study using three-dimensional lattice simulations, which can thus provide a *non-perturbative* evaluation of $C(k_\perp)$, summing contributions from the soft and ultrasoft scales to all orders [9, 10]. Recently, the impact of these classical corrections on the in-medium splitting rate was assessed [11, 12] and found to be very relevant. A similar program is well underway for the non-perturbative determination of classical corrections to the *asymptotic mass* [13–15].

These classical corrections are at odds with doubly-logarithmically enhanced radiative, *quantum corrections*, appearing at $\mathcal{O}(g^2)$, first identified in [16, 17]. There, the leading enhancement is $\sim \ln^2 L_{\rm med}/\tau_{\rm min}$, with $L_{\rm med}$ the length of the medium and $\tau_{\rm min} \sim 1/T$ the minimum formation time of the associated radiation. This potentially large double-logarithm can be resummed [18], with the evolution equations solved numerically in [19, 20]. Interestingly, they also arise in the context of double gluon emission [18, 21], implying that these logarithms are subject to a certain universality.

These corrections come from the *single scattering regime* where bremsstrahlung is sourced by a single scattering with the medium. This is in contrast to the *multiple scattering regime*, where the bremsstrahlung's formation time, τ is long enough so that it is coherently triggered by multiple collisions, accounted for through LPM resummation. In [3], we compute these doubly-logarithmically enhanced corrections in the context of a weakly coupled QGP, carefully analysing

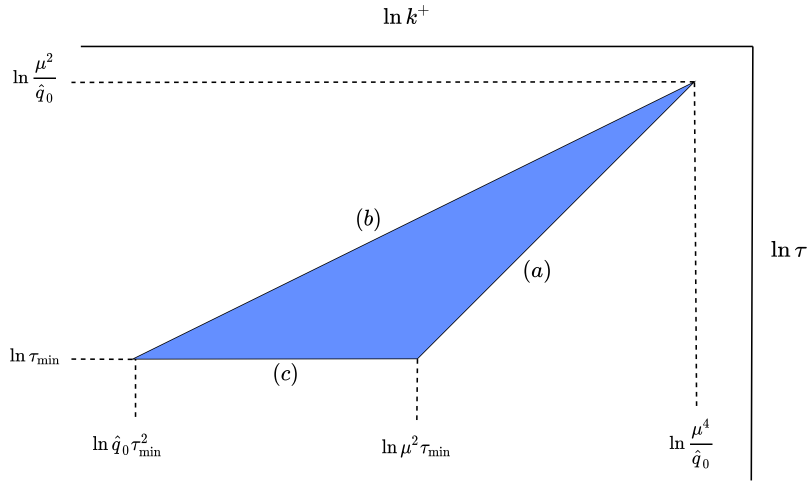

Figure 1: Depiction of bounds from the integration in Eq. (3). The (b) boundary is defined by $\tau = \sqrt{k^+/\hat{q}_0}$ and the (a) boundary by $\tau = k^+/\mu^2$. Figure taken from [3].

how the thermal scale deforms the region of phase space from which the double-logarithms emerge.

2. Double Logarithmic Corrections and the Thermal Scale

The correction from [16] emerges in the standard dipole picture

$$\delta\hat{q}_{[16,\,17]}(\mu) = 4\alpha_s C_R \hat{q}_0 \int^{\mu} \frac{d^2k_\perp}{k_\perp^2} \int \frac{dk^+}{k^+}, \tag{2}$$

where $k^+ \equiv k_\perp^2 \tau$ is the energy of the bremsstrahlung and $\hat{q}_0$ is the LO transverse momentum broadening coefficient, stripped of the Coulomb logarithm as is done in the *harmonic oscillator approximation* (HOA). Here we can explicitly see that one of the logarithms comes from a soft, dk^+/k^+ divergence, with the other coming from a collinear, $d^2k_\perp/k_\perp^2$ divergence. For what follows, it turns out to be more convenient to work with τ and k^+

$$\delta\hat{q}_{[16,\,17]}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\rm min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{dk^+}{k^+} = \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\rm min}}. \tag{3}$$

The limits above come from integrating over the triangle presented in Fig. 1. Boundary (a) arises from the need to cut off non-diffusive momentum exchanges above the scale μ. The line (b) is then defined by $k_\perp^2 \equiv \hat{q}_0\tau$, marking the boundary with the *deep LPM regime* in which multiple scatterings occur. Above boundary (b) there is no longer a double-logarithmic enhancement as the k^+ integrand changes as $1/k^+ \to 1/\sqrt{k^+}$. Finally, boundary (c) is an artefact of the *instantaneous approximation*: scatterings between the jet and medium are assumed to take place instantaneously compared to the formation time associated with the radiation. The result from [16] is recovered upon identifying μ with the *saturation scale*, $Q_s \equiv \hat{q}L_{\rm med}$.

In a weakly coupled QGP, as soon as the energy overlaps with the temperature scale, one needs to account for more medium effects than those captured by these instantaneous, spacelike

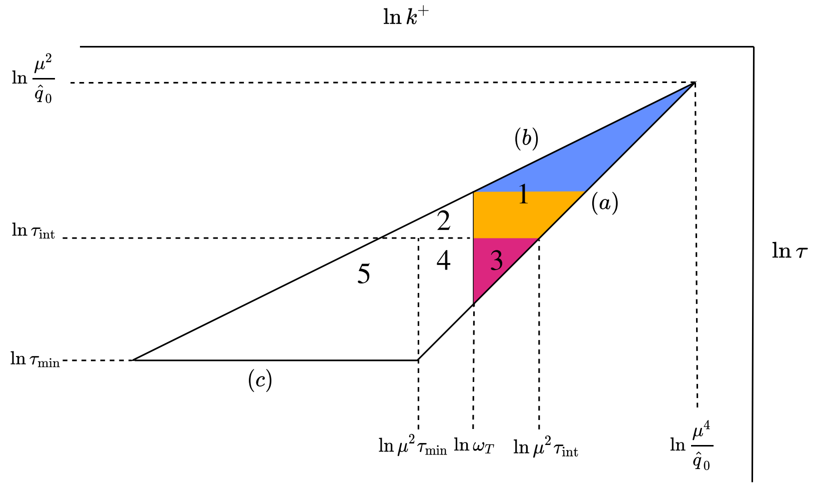

Figure 2: Deformation of the double-logarithmic phase space with the inclusion of thermal effects. Regions 1 and 2 form the "few scattering" regime, over which we integrate to get Eq. (4) whereas we integrate over regions 3 and 4, the "strict single scattering" regime to get Eq. (5). Region 5 then gives rise to the $O(g)$ corrections to $\hat{q}$, calculated in [7]. Figure taken from [3].

interactions. Specifically, by taking $T > \mu \gg \sqrt{g}T$[1] and replacing $1 \longrightarrow (1 + 2n_{\rm B}(k^+))$ in the k^+ integrand of Eq. (3), we find

$$\delta\hat{q}(\mu)^{\rm few} = \frac{\alpha_s C_R}{2\pi}\hat{q}_0\left\{\ln^2\frac{\mu^2}{\hat{q}_0\tau_{\rm int}} - \frac{1}{2}\ln^2\frac{\omega_{\rm T}}{\hat{q}_0\tau_{\rm int}^2}\right\} \quad \text{with } \omega_{\rm T} = \frac{2\pi T}{e^{\gamma_E}} \quad \text{for } \frac{\omega_{\rm T}}{\mu^2} \ll \tau_{\rm int} \ll \sqrt{\frac{\omega_{\rm T}}{\hat{q}_0}}. \tag{4}$$

In doing so, we account for the Bose-Einstein stimulated emission of the radiated gluon as well as the absorption of a gluon from the medium. We will comment shortly on the purpose of $\tau_{\rm int}$. But can these additional effects be neglected in a way that is consistent with single scattering, for instance, by demanding that $\hat{q}_0\tau_{\rm min}^2 \gg T$ in Eq. (3)[2]? It turns out that the answer is no [3]: such a choice of $\tau_{\rm min}$ would necessarily allow for formation times associated with the deep LPM regime, where $\tau_{\rm LPM} \gtrsim 1/g^2T$. Note that the correction in Eq. (4) corresponds to integrating over the 1 and 2 regions in Fig. 2.

The requirement $1/g^2T \gg \tau_{\rm int} \gg 1/gT$ means that processes where a *few scatterings* occur are included in Eq. (4). Indeed, $\tau_{\rm int}$ defines a border with what we have identified as a *strict single scattering* regime, where the formation time is *a priori* consistent with single scattering, i.e $\tau \ll 1/g^2T$. This region is characterised by so-called *semi-collinear* processes [22], where timelike as well as spacelike exchanges are allowed to occur. The leading contribution from this region is given by integrating over the 3 and 4 regions in Fig. 2 and yields

$$\delta\hat{q}_{\rm semi}(\mu) = \frac{\alpha_s C_R}{2\pi}\hat{q}_0\ln^2\frac{\mu^2\tau_{\rm int}}{\omega_{\rm T}}, \tag{5}$$

where we have taken the HOA. Adding Eqs. (4), (5), we then find

$$\delta\hat{q}(\mu_\perp)_{\rm dlog} = \frac{\alpha_s C_R}{4\pi}\hat{q}_0\ln^2\frac{\mu^4}{\hat{q}_0\omega_{\rm T}}. \tag{6}$$

[1]The $\mu > T$ case is studied in [3].
[2]This demand is motivated by the fact that $n_{\rm B}(k^+)$ is exponentially suppressed for $k^+ \gg T$.

As well as the disappearance of τ_{int}, we note the absence of an IR cutoff, τ_{min}; looking to Fig. 2, the double-logarithm is instead cut off by the scale ω_{T}. Thus, the thermal scale plays an extremely important role in this context.

3. Relation to Classical Corrections

As well as double-logarithmic corrections at $O(g^2)$, we also find *power law corrections* when integrating over regions 3 and 4

$$\delta q_{\text{PL}} = \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \frac{4T \ln\left(\frac{\mu^2 \tau_{\text{int}}}{k^+_{\text{IR}} e}\right)}{k^+_{\text{IR}}}, \tag{7}$$

where k^+_{IR} is an IR cutoff on the energy. Power law corrections of this kind are usually discarded as they are unphysical – they always cancel against other power law corrections coming from adjacent regions of phase space. Here, we use this fact to our advantage; in the calculation of the $O(g)$ corrections, power law corrections should appear, with k^+_{IR} instead acting as a UV cutoff there.

In more detail, one can use causality properties of $C(k_\perp)$, also revealed in [7], to carry out the k^+ integral by analytically continuing into the k^+ complex plane. k^+_{IR} then appears as the radius of the arc of the deformed contour, with the arc lying between the zeroth and first Matsubara modes. There is no dependence on k^+_{IR} in [7] as the $1/k^+_{\text{IR}}$ terms go to zero and can thus be safely neglected. Nevertheless, we have indeed computed these arc contributions explicitly and shown that they cancel exactly against the result from Eq. (7), further confirming how the region from which the classical corrections emerge is connected to that associated with the logarithmically-enhanced quantum corrections.

4. Conclusion and Outlook

We have studied how, in the setting of a weakly coupled QGP, the thermal scale affects the double-logarithmic phase space, originally identified in [16, 17]. In more detail, we showed how the scale, ω_{T} cuts off this region of phase space and furthermore, how the region, which gives rise to the classical corrections, computed in [7] fits in comparison.

In obtaining Eq. (6), we have taken the HOA, neglecting a neighbouring region phase space, which permits both single and multiple scattering processes. To properly deal with such a region, we would need to solve an LPM resummation equation, derived in [3] (see also [23]), differential in the transverse momentum picked up by the parton. We foresee that the use of the *improved opacity expansion* [24] could allow us to arrive at an approximate solution of this equation but we leave such an endeavour to future work.

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