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Starobinsky-Type $B-L$ Higgs Inflation Leading Beyond MSSM

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Models of induced-gravity inflation are formulated within Supergravity employing as inflaton the Higgs field which leads to a spontaneous breaking of a $U(1)_{B-L}$ symmetry at $M_{\rm GUT} = 2\cdot 10^{16}$ GeV. We use a renormalizable superpotential, fixed by a $U(1)$ R symmetry, and logarithmic or semi-logarithmic Kähler potentials with integer prefactors which exhibit a quadratic non-minimal coupling to gravity. We find inflationary solutions of Starobinsky type in accordance with the observations. The inflaton mass is predicted to be of the order of 10^{13} GeV. The model can be nicely linked to MSSM offering an explanation of the magnitude of the μ parameter consistently with phenomenological data. Also it allows for baryogenesis via non-thermal leptogenesis, provided that the gravitino is heavier than about 10 TeV.

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1. Introduction

It is well-known [1–3] that one of the possible incarnations of Starobisky-type inflation [4] in *Supergravity* (SUGRA) can be relied on the hypothesis of induced gravity [5–7]. According to this, inflation is driven in the presence of a non-minimal coupling among the inflaton field and the Ricci scalar curvature, f_R, such that the reduced Planck mass $m_{\rm P}$ is determined by a large (close to Planckian scale $m_{\rm P}$) *vacuum expectation value* (v.e.v) of the inflaton at the end of the slow roll. This is to be contrasted to the case of non-minimal [8–10] or pole-induced [11] Higgs inflation where the v.e.v of inflaton is negligible. In this talk we focus on the implementation of this scenario employing as inflaton a Higgs field within an "elementary" *Grand Unified Theory* (GUT) which extends the gauge symmetry of the *Standard Model* (SM) by a $U(1)_{B-L}$ factor [12]. In a such case, the unification condition within *Minimal Supersymmetric SM* (MSSM) may be employed to uniquely determined the strength of f_R giving rise to an economical, predictive and well-motivated setting, thereby called *Induced-gravity Higgs inflation* (IHI) – cf. Ref. [13].

Here, we concentrate on the simplest models of IHI introduced in Ref. [12] considering exclusively integer prefactors for the logarithms included in the Kähler potentials. The particle physics framework of our presentation is described in Sec. 2 whereas the engineering of induced-gravity hypothesis is outlined in Sec. 3. The inflationary part of this context is investigated in Sec. 4. Then, in Sec. 5, we explain how the MSSM is obtained as a low energy theory and, in Sec. 6, we outline how the observed *baryon asymmetry of the universe* (BAU) is generated via *non-thermal leptogenesis* (nTL). Our conclusions are summarized in Sec. 7. Throughout the text, the subscript of type $,z$ denotes derivation *with respect to* (w.r.t) the field z and charge conjugation is denoted by a star. Unless otherwise stated, we use units where $m_{\rm P}=2.433\cdot 10^{18}$ GeV is taken unity.

2. Particle Physics Embedding

We focus on a "GUT" based on $G_{B-L}=G_{\rm SM}\times U(1)_{B-L}$, where $G_{\rm SM}=SU(3)_{\rm C}\times SU(2)_{\rm L}\times U(1)_Y$ is the gauge group of the SM and B and L denote the baryon and lepton number respectively. We below – see Secs. 2.1 and 2.2 – present the basic ingredients of our proposal.

2.1 Superpotential

The superpotential of our model naturally splits into four parts:

$$W=W_{\rm MSSM}+W_{\rm HI}+W_{\mu}+W_{\rm RHN},\quad \text{where} \tag{1}$$

(a) $W_{\rm MSSM}$ is the part of W which contains the usual terms – except for the μ term – of MSSM, supplemented by Yukawa interactions among the left-handed leptons (L_i) and N_i^c:

$$W_{\rm MSSM}=h_{ijD}d_i^cQ_jH_d+h_{ijU}u_i^cQ_jH_u+h_{ijE}e_i^cL_jH_d+h_{ijN}N_i^cL_jH_u. \tag{2.2a}$$

Here the ith generation $SU(2)_{\rm L}$ doublet left-handed quark and lepton superfields are denoted by Q_i and L_i respectively, whereas the $SU(2)_{\rm L}$ singlet antiquark [antilepton] superfields by u_i^c and $d_i{}^c$ [e_i^c and N_i^c] respectively. The electroweak Higgs superfields which couple to the up [down] quark superfields are denoted by H_u [H_d]. Note that the introduction of three right-handed neutrinos, N_i^c, is necessary to cancel the $B-L$ gauge anomaly.

SUPERFIELDS	REPRESENTATIONS UNDER G_{B-L}	GLOBAL SYMMETRIES R	B	L
	MATTER FIELDS			
e_i^c	$(\mathbf{1},\mathbf{1},1,1)$	1	0	−1
N_i^c	$(\mathbf{1},\mathbf{1},0,1)$	1	0	−1
L_i	$(\mathbf{1},\mathbf{2},-1/2,-1)$	1	0	1
u_i^c	$(\mathbf{3},\mathbf{1},-2/3,-1/3)$	1	−1/3	0
d_i^c	$(\mathbf{3},\mathbf{1},1/3,-1/3)$	1	−1/3	0
Q_i	$(\bar{\mathbf{3}},\mathbf{2},1/6,1/3)$	1	1/3	0
	HIGGS FIELDS			
H_d	$(\mathbf{1},\mathbf{2},-1/2,0)$	0	0	0
H_u	$(\mathbf{1},\mathbf{2},1/2,0)$	0	0	0
S	$(\mathbf{1},\mathbf{1},0,0)$	2	0	0
Φ	$(\mathbf{1},\mathbf{1},0,2)$	0	0	−2
$\bar{\Phi}$	$(\mathbf{1},\mathbf{1},0,-2)$	0	0	2

Table 1: *Representations under G_{B-L} and extra global charges of the superfields of our model.*

(b) $W_{\rm HI}$ is the part of W which is relevant for IHI and takes the form

$$W_{\rm HI}=\lambda S\left(\bar\Phi\Phi-M^2/4\right). \tag{2.2b}$$

The imposed $U(1)_R$ symmetry ensures the linearity of $W_{\rm HI}$ w.r.t S. This fact allows us to isolate easily via its derivative the contribution of the inflaton into the F-term SUGRA potential, placing S at the origin – see Sec. 4.1. The inflaton is contained in the system $\bar\Phi-\Phi$. We are obliged to restrict ourselves to subplanckian values of $\bar\Phi\Phi$ since the imposed symmetries do not forbid non-renormalizable terms of the form $(\bar\Phi\Phi)^p$ with $p>1$ – see Sec. 4.2.

(c) W_μ is the part of W which is responsible for the generation of the μ term of MSSM and takes the form

$$W_\mu=\lambda_\mu SH_uH_d. \tag{2.2c}$$

As $W_{\rm HI}$, W_μ is also linear to S and so, the imposed $U(1)_R$ plays also a key role in the resolution of the μ problem of MSSM – see Sec. 5.

(d) $W_{\rm RHN}$ is the part of W which provides Majorana masses for netrinos and reads

$$W_{\rm RHN}=\lambda_{iN^c}\bar\Phi N_i^{c2}. \tag{2.2d}$$

The same term assures the decay of the inflaton to $\widetilde{N}_i^c$, whose subsequent decay can activate nTL [14]. Here, we work in the so-called N_i^c*-basis*, where M_{iN^c} is diagonal, real and positive. These masses, together with the Dirac neutrino masses of the forth term in Eq. (2.2a), lead to the light neutrino masses via the seesaw mechanism – see Sec. 6.2.

2.2 Kähler Potentials

The objectives of our model are feasible if W in Eq. (1) cooperates with *one* of the following Kähler potentials:

$$K_1 = -3\ln\left(c_R(F_R+F_R^*)-\frac{|\Phi|^2+|\bar\Phi|^2}{3}+F_{1X}(|X|^2)\right) \quad\text{with}\quad F_{1X}=-\ln\left(1+\frac{|X|^2}{3}\right) \tag{2.3a}$$

$$K_2 = -2\ln\left(c_R(F_R+F_R^*)-\frac{|\Phi|^2+|\bar\Phi|^2}{2}\right)+F_{2X}(|X|^2) \quad\text{with}\quad F_{2X}=N_X\ln\left(1+\frac{|X|^2}{N_X}\right) \tag{2.3b}$$

where $F_R=\Phi\bar\Phi$, $0<N_X<6$, $X^\gamma=S,H_u,H_d,\widetilde{N}^c_i$ and the complex scalar components of the superfields $\Phi,\bar\Phi,S,H_u$ and H_d are denoted by the same symbol whereas this of N_i^c by $\widetilde{N}_i^c$. We assume that X^γ have identical kinetic terms expressed by the functions F_{lX} with $l=1,2$. These functions ensures the stability and the heaviness of these modes [15] employing *exclusively* quadratic terms. Both K's reduce to the same K_0 for $X^\alpha=0$ with the aid of the frame function Ω defined as

$$K_0=-N\ln\left(-\frac{\Omega}{N}\right) \quad\text{with}\quad \frac{\Omega}{N}=-c_R(F_R+F_R^*)+\frac{|\Phi|^2+|\bar\Phi|^2}{N} \quad\text{and}\quad N=\begin{cases}3 & \text{for}\quad K=K_1,\\ 2 & \text{for}\quad K=K_2.\end{cases} \tag{4}$$

Henceforth, N assists us to unify somehow the two K's considered in Eqs. (2.3a) and (2.3b).

3. SUGRA Version of Induced-Gravity Conjecture

The scale M and the function F_R involved in Eqs. (2.2b), (2.3a) and (2.3b) assist us in the implementation of the idea of induced gravity. To explain how it works, we introduce our notation in the two relevant frames in Sec. 3.1 and then, in Sec. 3.2, we derive the SUSY vacuum which plays a key role imposing the induced-gravity condition – see Sec. 3.3.

3.1 From Einstein to Jordan Frame

We concentrate on $W_{\rm HI}$ and extract the part of the *Einstein frame* (EF) action within SUGRA related to the complex scalars $z^\alpha=S,\Phi,\bar\Phi$. This has the form [12]

$$\mathsf{S}=\int d^4x\sqrt{-\widehat{\mathfrak{g}}}\left(-\frac{1}{2}\widehat{R}+K_{\alpha\bar\beta}\widehat{g}^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-\widehat{V}_{\rm SUGRA}\right), \tag{5}$$

where $\widehat{R}$ is the EF Ricci scalar curvature, D_μ is the gauge covariant derivative, $K_{\alpha\bar\beta}=K_{,z^\alpha z^{*\bar\beta}}$, and $K^{\alpha\bar\beta}K_{\bar\beta\gamma}=\delta^\alpha_\gamma$ and $\mathfrak{g}$ is the determinant of the EF metric $\widehat{g}^{\mu\nu}$. Also, $\widehat{V}$ is the EF SUGRA potential which can be found in terms of $W_{\rm HI}$ in Eq. (2.2b) and the K's in Eqs. (2.3a) – (2.3b) via the formula

$$\widehat{V}_{\rm SUGRA}=\widehat{V}_{\rm F}+\widehat{V}_{\rm D} \quad\text{with}\quad \widehat{V}_{\rm F}=e^K\left(K^{\alpha\bar\beta}(D_\alpha W_{\rm HI})D^*_{\bar\beta}W^*_{\rm HI}-3|W_{\rm HI}|^2\right) \quad\text{and}\quad \widehat{V}_{\rm D}=\frac{g_{BL}^2}{2}{\rm D}_{BL}^2. \tag{3.6a}$$

Here the Kähler covariant derivative reads $D_\alpha W_{\rm HI}=W_{{\rm HI},z^\alpha}+K_{,z^\alpha}W_{\rm HI}$ whereas the D term due to $B-L$ symmetry is found to be

$${\rm D}_{BL}=\left(|\Phi|^2-|\bar\Phi|^2\right)/(-\Omega/N). \tag{3.6b}$$

As induced by Eqs. (4) and (3.6b), the field configuration

$$\langle\Phi\rangle_{\rm I}=\langle\bar\Phi\rangle_{\rm I}\quad\text{and}\quad\langle X^\alpha\rangle_{\rm I}=0, \tag{7}$$

assures $\langle\widehat V_{\rm D}\rangle_{\rm I}=0$ where the symbol $\langle Q\rangle_{\rm I}$ denotes values of a quantity Q along the path of Eq. (7). Henceforth, we confine ourselves to this path – assuming in addition that $\arg(\Phi)=\arg(\bar\Phi)$ – which is a honest inflationary trajectory, supporting IHI driven exclusively by $\widehat V_{\rm F}$.

The performance of a conformal transformation after defining the *Jordan Frame* (JF) metric as

$$g^{\mu\nu}=-\frac{\Omega}{N}\widehat g^{\mu\nu}\quad\text{yields [12] via Eq. (5)}\quad \mathsf{S}=\int d^4x\sqrt{-g}\left(\frac{\Omega}{2N}R-\cdots\right) \tag{8}$$

which reveals that $-\Omega/N$ plays the role of a (dimensionless) non-minimal coupling to gravity – here we use unhatted symbols for the JF quantities and the ellipsis includes terms irrelevant for our discussion. Comparing Eq. (4) with the K's in Eqs. (2.3a) and (2.3b) we can infer that the emergence of Einstein gravity at the vacuum dictates

$$-\langle\Omega/N\rangle=2(Nc_R+1)\langle\Phi\rangle^2/N=1, \tag{9}$$

where we assume that $\langle\Phi\rangle$ is included in the inflationary trough of Eq. (7). Its value as a function of the model parameters is calculated in the next section.

3.2 SUSY Potential

The implementation of the IHI requires the generation of $m_{\rm P}$ at the vacuum of the theory. It can be determined expanding $V_{\rm SUGRA}$ in powers of $1/m_{\rm P}$. Namely, we obtain the following low-energy effective potential which plays the role of SUSY one

$$V_{\rm SUSY}=\left\langle\widetilde K^{\alpha\bar\beta}W_{{\rm HI}\alpha}W^*_{{\rm HI}\bar\beta}\right\rangle_{\rm I}+\cdots, \tag{3.10a}$$

where the ellipsis represents terms proportional to $W_{\rm HI}$ or $|W_{\rm HI}|^2$ which obviously vanish along the path in Eq. (7). Also, $\widetilde K$ is the limit of the K's in Eqs. (2.3a) and (2.3b) for $m_{\rm P}\to\infty$. The absence of unity in the arguments of the logarithms multiplied by N in these K's prevents the drastic simplification of $\widetilde K$ – cf. Ref. [10]. As a consequence, the expression of the resulting $V_{\rm SUSY}$ is rather lengthy. For this reason we confine ourselves below to $K=K_2$ where F_{2S} is placed outside the first logarithm in Eq. (2.3a) and so $\widetilde K$ can be somehow simplified. Namely, we get

$$\widetilde K=-N\ln\left(-\Omega/N\right)+|S|^2\,, \tag{3.10b}$$

from which we can then compute

$$\left(\langle\widetilde K_{\alpha\bar\beta}\rangle_{\rm I}\right)=\mathsf{diag}\left(\widetilde M_{\bar\Phi\Phi},1\right)\quad\text{with}\quad \widetilde M_{\bar\Phi\Phi}=\frac{2}{\langle\Omega\rangle_{\rm I}^2}\begin{pmatrix}(4c_R-1)|\Phi|^2 & |2c_R\Phi-\Phi^*|^2\\ |2c_R\Phi-\Phi^*|^2 & (4c_R-1)|\Phi|^2\end{pmatrix}. \tag{3.11a}$$

To compute $V_{\rm SUSY}$ we need to know

$$\langle\widetilde K^{\alpha\bar\beta}\rangle_{\rm I}=\mathsf{diag}\left(\widetilde M^{-1}_{\bar\Phi\Phi},1\right),\quad\text{where}\quad \widetilde M^{-1}_{\bar\Phi\Phi}=-\frac{\langle\Omega\rangle_{\rm I}^2}{2\det\widetilde M_{\bar\Phi\Phi}}\begin{pmatrix}-(4c_R-1)|\Phi|^2 & |2c_R\Phi-\Phi^*|^2\\ |2c_R\Phi-\Phi^*|^2 & -(4c_R-1)|\Phi|^2\end{pmatrix}, \tag{3.11b}$$

where the prefactor can be explicitly written as

$$\frac{\langle\Omega\rangle_{\rm I}^2}{\det\widetilde M_{\bar\Phi\Phi}}=\frac{|\Phi|^2-c_R(\Phi^2-\Phi^{*2})}{c_R(\Phi^2+\Phi^{*2}-4c_R|\Phi|^2)} \tag{3.11c}$$

Upon substitution of Eq. (3.11b) into Eq. (3.10a) we obtain

$$V_{\rm SUSY}\simeq\lambda^2\left|\bar\Phi\Phi-\frac14M^2\right|^2+\frac{\langle\Omega\rangle_{\rm I}^2}{\det\widetilde M_{\bar\Phi\Phi}}\lambda^2|S|^2|\Phi|^2\left((4c_R^2-1)|\Phi|^2-|\Phi-2c_R\Phi^*|^2\right). \tag{12}$$

We remark that the SUSY vacuum lies along the direction in Eq. (7) with

$$\langle S\rangle=0\ \text{ and }\ |\langle\Phi\rangle|=|\langle\bar\Phi\rangle|=M/2, \tag{13}$$

where $\langle S\rangle$ may slightly deviate from its value above after inclusion of soft SUSY breaking effects – see Sec. 5.1. The result in Eq. (13) holds also for $K=K_1$ as we can verify after a more tedious computation. From Eq. (13) it is clear that $\langle\Phi\rangle$ and $\langle\bar\Phi\rangle$ spontaneously break $U(1)_{B-L}$ down to $\mathbb{Z}_2^{B-L}$. Note that $U(1)_{B-L}$ is already broken during IHI and so no cosmic string are formed – see Sec. 4.2.

3.3 Induced-Gravity Requirement

Inserting Eq. (13) into Eq. (9) we deduce that the conventional Einstein gravity can be recovered at the vacuum if

$$M=\sqrt{2N/(Nc_R-1)}. \tag{14}$$

As we show in Sec. 4.3, the GUT requirement offers the prediction $c_R\sim10^4$. Therefore, the resulting M has a size comparable to $m_{\rm P}$ as expected from the establishment of the theory in Sec. 2.1.

4. Inflationary Scenario

The salient features of our inflationary scenario are studied at tree level in Sec. 4.1 and at one-loop level in Sec. 4.2. We then present its predictions in Sec. 4.3.

4.1 Inflationary Potential

If we express $\Phi,\bar\Phi$ and $X^\gamma=S,H_u,H_d,\widetilde N_i^c$ according to the parametrization

$$\Phi=\phi\,e^{i\theta}\cos\theta_\Phi/\sqrt2,\quad \bar\Phi=\phi\,e^{i\bar\theta}\sin\theta_\Phi/\sqrt2\ \text{ and }\ X^\gamma=(x^\gamma+i\bar x^\gamma)/\sqrt2,\ \text{ where }\ 0\le\theta_\Phi\le\pi/2, \tag{15}$$

the D-flat direction in Eq. (7) is now expressed as

$$x^\gamma=\bar x^\gamma=\theta=\bar\theta=H_u=H_d=\widetilde N_i^c=0\ \text{ and }\ \theta_\Phi=\pi/4\,. \tag{16}$$

Along this, the only surviving term of $\widehat V_{\rm SUGRA}$ in Eq. (3.6a) – extended to all fields above – can be written as

$$\widehat V_{\rm IHI}=e^KK^{SS^*}|W_{{\rm HI},S}|^2=\frac{\lambda^2(\phi^2-M^2)^2}{16f_R^N}\cdot\begin{cases}f_R & \text{for }K=K_1,\\ 1 & \text{for }K=K_2,\end{cases}\ \text{ where }\ f_R=-\left\langle\frac{\Omega}{N}\right\rangle_{\rm I}=\frac{(Nc_R-1)\phi^2}{2N} \tag{17}$$

Fields	Einge-states	Masses Squared		
			$K=K_1$	$K=K_2$
14 Real Scalars	$\widehat{\theta}_+$	$\widehat{m}^2_{\theta+}$	$4\widehat{H}^2_{\rm IHI}$	$6\widehat{H}^2_{\rm IHI}$
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta_\Phi}$	M^2_{BL}	M^2_{BL}
	$\widehat{s},\widehat{\bar{s}}$	$\widehat{m}^2_{s}$	$\widehat{H}^2_{\rm IHI}(c_R\phi^2-9)$	$6\widehat{H}^2_{\rm IHI}/N_X$
	$\widehat{h}_\pm,\widehat{\bar{h}}_\pm$	$\widehat{m}^2_{h\pm}$	$3\widehat{H}^2_{\rm IHI}c_R\left(\phi^2/6\pm2\lambda_\mu/\lambda\right)$	$3\widehat{H}^2_{\rm IHI}\left(1+1/N_X\pm4\lambda_\mu/\lambda\phi^2\right)$
	$\widehat{\widetilde{\nu}}^c_i,\widehat{\bar{\widetilde{\nu}}}^c_i$	$\widehat{m}^2_{i\tilde{\nu}^c}$	$3\widehat{H}^2_{\rm IHI}c_R\left(\phi^2/6+8\lambda^2_{iN^c}/\lambda^2\right)$	$3\widehat{H}^2_{\rm IHI}\left(1+1/N_X+16\lambda^2_{iN^c}/\lambda^2\phi^2\right)$
1 Gauge Boson	A_{BL}	M^2_{BL}	$2Ng^2/(Nc_R-1)$	
7 Weyl Spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$12\widehat{H}^2_{\rm IHI}/c^2_R\phi^4$	
	$\lambda_{BL},\widehat{\psi}_{\Phi-}$	M^2_{BL}	$2Ng^2/(Nc_R-1)$	
	$\widehat{N}^c_i$	$\widehat{m}^2_{iN^c}$	$48\widehat{H}^2_{\rm IHI}c_R\lambda^2_{iN^c}/\lambda^2\phi^2$	

Table 2: *The mass squared spectrum of our models along the path in Eq.* (4.2) *for* $\phi\ll1$ *and* N*'s defined in Eq.* (2.4).

– see Eq. (4). Clearly $\widehat{V}_{\rm IHI}$ develops an inflationary plateau as in the original Starobisky inflationary model [1, 16]. To specify the EF canonically normalized fields, we note that, for the K's in Eqs. (2.3a) and (2.3b), $K_{\alpha\bar\beta}$ along the configuration in Eq. (16) takes the form

$$\langle K_{\alpha\bar\beta}\rangle_{\rm I}=\mathsf{diag}\Big(M_{\bar\Phi\Phi},\underbrace{K_{\gamma\bar\gamma},...,K_{\gamma\bar\gamma}}_{8\ \text{elements}}\Big)\ \ \text{with}\ \ M_{\bar\Phi\Phi}=\begin{pmatrix}\kappa & \bar\kappa\\ \bar\kappa & \kappa\end{pmatrix}\ \ \text{and}\ \ K_{\gamma\bar\gamma}=\begin{cases}f_R^{-1} & \text{for}\ \ K=K_1\,,\\ 1 & \text{for}\ \ K=K_2\,,\end{cases}\tag{18}$$

where $\kappa=(1+Nc_R)/2f_R$ and $\bar\kappa=N/\phi^2$. Upon diagonalization of $M_{\bar\Phi\Phi}$ we find its eigenvalues which are

$$\kappa_+=Nc_R/f_R\ \ \text{and}\ \ \kappa_-=1/f_R.\tag{19}$$

Note that the existence of the real terms $|\Phi|^2+|\bar\Phi|^2$ in Eqs. (2.3a) and (2.3b) is vital for our models, since otherwise the off diagonal elements of $M_{\bar\Phi\Phi}$ would have been zero, one of the eigenvalues above would have been zero and so no $M^{-1}_{\bar\Phi\Phi}$ could have been defined.

Inserting Eqs. (15) and (18) into the kinetic term of S in Eq. (5) we can specify the canonically normalized (hatted) fields, as follows

$$\frac{d\widehat\phi}{d\phi}=J,\ \ \widehat\theta_+=\frac{J}{\sqrt2}\phi\theta_+,\ \ \widehat\theta_-=\sqrt{\frac{\kappa_-}{2}}\phi\theta_-,\ \ \widehat\theta_\Phi=\sqrt{\kappa_-}\phi\left(\theta_\Phi-\frac{\pi}{4}\right)\ \ \text{and}\ \ (\widehat{x}^\gamma,\widehat{\bar{x}}^\gamma)=\sqrt{K_{\gamma\bar\gamma}}(x^\gamma,\bar{x}^\gamma)\,,\tag{20}$$

where $J=\sqrt{\kappa_+}$ and $\theta_\pm=\left(\bar\theta\pm\theta\right)/\sqrt2$. As we show below, the masses of the scalars besides $\widehat\phi$ during IHI are heavy enough such that the dependence of the hatted fields on ϕ does not influence their dynamics.

4.2 Stability and one-Loop Radiative Corrections

We can verify that the inflationary direction in Eq. (16) is stable w.r.t the fluctuations of the non-inflaton fields. To this end, we construct the mass-squared spectrum of the scalars taking into account the canonical normalization of the various fields in Eq. (20). In the limit $c_R \gg 1$, we find the expressions of the masses squared $\widehat{m}^2_{z^\alpha}$ (with $z^\alpha = \theta_+, \theta_\Phi, x^\gamma$ and $\bar{x}^\gamma$) arranged in Table 2. These results approach rather well for $\phi = \phi_\star$ – see Sec. 4.2 – the quite lengthy, exact expressions taken into account in our numerical computation. The various unspecified there eigenstates are defined as follows

$$\widehat{h}_\pm = (\widehat{h}_u \pm \widehat{h}_d)/\sqrt{2}, \quad \widehat{\bar{h}}_\pm = (\widehat{\bar{h}}_u \pm \widehat{\bar{h}}_d)/\sqrt{2} \quad \text{and} \quad \widehat{\psi}_\pm = (\widehat{\psi}_{\Phi+} \pm \widehat{\psi}_S)/\sqrt{2}, \tag{4.21a}$$

where the (unhatted) spinors ψ_Φ and $\psi_{\bar{\Phi}}$ associated with the superfields Φ and $\bar{\Phi}$ are related to the normalized (hatted) ones in Table 2 as follows

$$\widehat{\psi}_{\Phi\pm} = \sqrt{\kappa_\pm}\psi_{\Phi\pm} \quad \text{with} \quad \psi_{\Phi\pm} = (\psi_\Phi \pm \psi_{\bar{\Phi}})/\sqrt{2}\,. \tag{4.21b}$$

From Table 2 it is evident that $0 < N_X \leq 6$ assists us to achieve $m_s^2 > \widehat{H}^2_{\rm IHI} = \widehat{V}_{\rm IHI}/3$ – in accordance with the results of Ref. [15] – and also enhances the ratios $m^2_{X^{\tilde{\gamma}}}/\widehat{H}^2_{\rm IHI}$ for $X^{\tilde{\gamma}} = h_+, \tilde{\nu}^c_i$ w.r.t the values that we would have obtained, if we had used just canonical terms in the K's. On the other hand, $\widehat{m}^2_{h-} > 0$ implies

$$\lambda_\mu \lesssim \lambda\phi^2/4N \quad \text{for} \quad K = K_1 \quad \text{and} \quad \lambda_\mu \lesssim \lambda\phi^2(1 + 1/N_X)/4 \quad \text{for} \quad K = K_2\,. \tag{22}$$

In both cases, the quantity in the right-hand side of the inequalities takes its minimal value at $\phi = \phi_{\rm f}$ – see Sec. 4.2 – and numerically equals to $2 \cdot 10^{-5} - 5 \cdot 10^{-6}$. In Table 2 we display also the mass M_{BL} of the gauge boson A_{BL} which becomes massive having 'eaten' the Goldstone boson θ_-. This signals the fact that G_{B-L} is broken during IHI and so no cosmological defects are produced. Also, we can verify [12] that radiative corrections á la Coleman-Weinberg can be kept under control provided that we conveniently select the relevant renormalization mass scale involved.

4.3 SUSY Gauge Coupling Unification

The value of M_{BL} in Table 2 computed at the vacuum of Eq. (13), $\langle M_{BL}\rangle$, may in principle, be unconstrained since $U(1)_{B-L}$ does not disturb the unification of the MSSM gauge coupling constants. To be more specific, though, we prefer to determine M_{BL} by requiring that it takes the value $M_{\rm GUT}$ dictated by this unification at the vacuum of Eq. (13). Namely, we impose

$$\langle M_{BL}\rangle = M_{\rm GUT} \simeq 2/2.43 \cdot 10^{-2} = 8.22 \cdot 10^{-3}\,. \tag{23}$$

This simple principle has an important consequence for IHI, since it implies via the findings of Table 2

$$c_R = \frac{1}{N} + \frac{2g^2_{BL}}{M^2_{\rm GUT}} \simeq 1.451 \cdot 10^4\,, \tag{24}$$

leading to $M \simeq 0.0117$ via Eq. (14). Here we take $g_{BL} \simeq 0.7$ which is the value of the unified coupling constant within MSSM.

Although c_R above is very large, there is no problem with the validity of the effective theory, in accordance with the results of earlier works [1, 3, 7]. To clarify further this point, we have to identify the ultraviolet cut-off scale $\Lambda_{\rm UV}$ of theory analyzing the small-field behavior of our models.

Indeed, expanding about $\langle\phi\rangle = M$ – see Eq. (14) – the second term in the r.h.s of Eq. (5) for $\mu = \nu = 0$ and $\widehat{V}_{\rm IHI}$ in Eq. (17) we obtain

$$J^2\dot\phi^2 \simeq \left(1-\sqrt{\frac{2}{N}}\widehat{\delta\phi}+\frac{3}{2N}\widehat{\delta\phi}^2-\sqrt{\frac{2}{N^3}}\widehat{\delta\phi}^3+\cdots\right)\dot{\widehat{\delta\phi}}^2, \tag{4.25a}$$

where $\widehat{\delta\phi}$ is the canonically normalized inflaton at the vacuum – see Sec. 6.1 – and

$$\widehat{V}_{\rm IHI} \simeq \frac{\lambda^2\widehat{\delta\phi}^2}{2Nc_R^2}\left(1-\frac{2N-1}{\sqrt{2N}}\widehat{\delta\phi}+\frac{8N^2-4N+1}{8N}\widehat{\delta\phi}^2+\cdots\right). \tag{4.25b}$$

These expressions indicate that $\Lambda_{\rm UV} = m_{\rm P}$, since c_R does not appear in any of their numerators.

4.4 Inflationary Observables

A period of slow-roll IHI is controlled by the strength of the slow-roll parameters

$$\widehat\epsilon = \frac{1}{2}\left(\frac{\widehat{V}_{{\rm IHI},\widehat\phi}}{\widehat{V}_{\rm IHI}}\right)^2 \simeq 16\frac{f_{\rm W}^2}{Nc_R^4\phi^8}\quad\text{and}\quad \widehat\eta = \frac{\widehat{V}_{{\rm IHI},\widehat\phi\widehat\phi}}{\widehat{V}_{\rm IHI}} \simeq 8\frac{2-f_{\rm W}}{Nf_{\rm W}^2}\quad\text{with}\quad f_{\rm W} = c_R\phi^2-2. \tag{26}$$

Expanding $\widehat\epsilon$ and $\widehat\eta$ for $\phi \ll 1$ we can find that IHI terminates for $\phi = \phi_{\rm f}$ such that

$$\max\{\widehat\epsilon(\phi_{\rm f}), |\widehat\eta(\phi_{\rm f})|\} = 1 \quad\Rightarrow\quad \phi_{\rm f} \simeq \max\left(\frac{2}{\sqrt{c_R\sqrt{N}}}, 2\sqrt{\frac{2}{Nc_R}}\right). \tag{27}$$

The number of e-foldings, $\widehat{N}_\star$, that the pivot scale $k_\star = 0.05/{\rm Mpc}$ suffers during IHI can be calculated through the relation

$$\widehat{N}_\star = \int_{\widehat\phi_{\rm f}}^{\widehat\phi_\star} d\widehat\phi\,\frac{\widehat{V}_{\rm IHI}}{\widehat{V}_{{\rm IHI},\widehat\phi}} \simeq \frac{Nc_R}{8}\phi_\star^2 \;\Rightarrow\; \phi_\star \simeq 2\left(\frac{2\widehat{N}_\star}{Nc_R}\right)^{1/2} \simeq \begin{cases}0.11, & K = K_1,\\ 0.13, & K = K_2,\end{cases} \tag{28}$$

where $\widehat\phi_\star$ $[\phi_\star]$ with $\phi_\star \gg \phi_{\rm f}$ is the value of $\widehat\phi$ $[\phi]$ when $k_\star$ crosses the inflationary horizon. Thanks to large c_R in Eq. (24), $\phi_\star \ll 1$ and therefore, our proposal is automatically well stabilized against corrections from higher order terms of the form $(\Phi\bar\Phi)^p$ with $p > 1$ in $W_{\rm HI}$ – see Eq. (2.2b).

The normalization of the amplitude, $A_{\rm s}$, of the power spectrum of the curvature perturbations generated by ϕ at the pivot scale $k_\star$ allows us to determine λ as follows

$$\sqrt{A_{\rm s}} = \frac{1}{2\sqrt{3}\pi}\frac{\widehat{V}_{\rm IHI}(\widehat\phi_\star)^{3/2}}{|\widehat{V}_{{\rm IHI},\widehat\phi}(\widehat\phi_\star)|} = 4.58\cdot10^{-5} \;\Rightarrow\; \lambda = 32\pi\sqrt{6NA_{\rm s}}c_R\frac{\widehat{N}_\star}{(4\widehat{N}_\star-N)^2} \simeq \begin{cases}0.29, & K = K_1,\\ 0.24, & K = K_2.\end{cases} \tag{29}$$

The resulting relation reveals that λ is proportional to c_R. For these λ values we display $\widehat{V}_{\rm IHI}$ as a function of ϕ in Fig. 1. We observe that $\widehat{V}_{\rm IHI}$ is a monotonically increasing function of ϕ. The inflationary scale, $\widehat{V}_{\rm IHI}^{1/4}$, approaches the SUSY GUT scale in Eq. (23) and lies well below $\Lambda_{\rm UV} = 1$, consistently with the classical approximation to the inflationary dynamics.

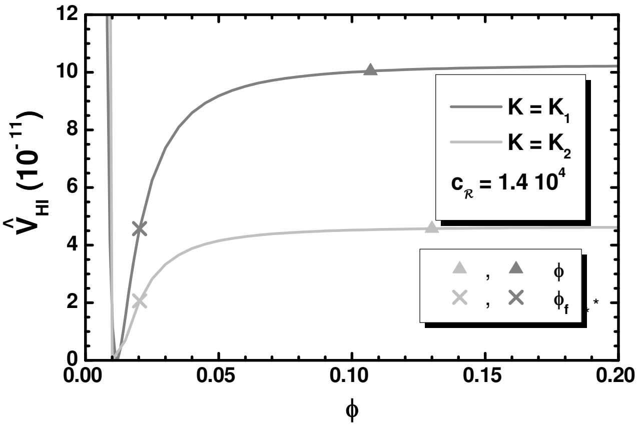


Figure 1: *Inflationary potential $\widehat{V}_{\rm IHI}$ as a function of ϕ for $\phi > 0$, $c_{\mathcal{R}}$ in Eq.* (4.10) *and $K = K_1$ (dark gray line) or $K = K_2$ (light gray line) – the values of $\phi_\star$ and $\phi_{\rm f}$ are also indicated.*

At the pivot scale, we can also calculate the scalar spectral index, $n_{\rm s}$, its running, $a_{\rm s}$, and the tensor-to-scalar ratio, r, via the relations

$$n_{\rm s} = 1-6\widehat{\epsilon}_\star + 2\widehat{\eta}_\star \simeq 1-\frac{2}{\widehat{N}_\star} = 0.963, \quad r = 16\widehat{\epsilon}_\star \simeq \frac{4N}{\widehat{N}^2_\star} = 0.0032\ [0.0022], \tag{4.30a}$$

$$a_{\rm s} = \frac{2}{3}\left(4\widehat{\eta}^2_\star-(n_{\rm s}-1)^2\right)-2\widehat{\xi}_\star \simeq -\frac{2}{\widehat{N}^2_\star}-\frac{7N}{2\widehat{N}^3_\star} = -0.005 \quad \text{for} \quad K=K_1 \quad [K_2] \tag{4.30b}$$

with $\widehat{\xi} = \widehat{V}_{{\rm IHI},\widehat{\phi}}\widehat{V}_{{\rm IHI},\widehat{\phi}\widehat{\phi}\widehat{\phi}}/\widehat{V}^2_{\rm IHI}$ and the variables with subscript $\star$ are being evaluated at $\phi = \phi_\star$. The numerical values are obtained employing $\widehat{N}_\star \simeq (57.5-60)$ which corresponds to a quartic potential. It is expected to approximate $\widehat{V}_{\rm IHI}$ rather well for $\phi \ll 1$ [12].

The results above turn out to be in nice agreement with the fitting of the *Planck* (release 4) [16], baryon acoustic oscillations, cosmic microwave background lensing and Bicep2/*Keck Array* data [18] with the ΛCDM+r model, i.e.,

$$\text{(a)}\ n_{\rm s} = 0.965 \pm 0.009 \quad \text{and} \quad \text{(b)}\ r \leq 0.032, \tag{31}$$

at 95% *confidence level* (c.l.) with $|a_{\rm s}| \ll 0.01$.

5. IHI and μ Term of MSSM

A byproduct of our setting is that it assists us to understand the origin of μ term of MSSM, as we show in Sec. 5.1, consistently with the low-energy phenomenology of MSSM – see Sec. 5.2. Hereafter we restore units, i.e., we take $m_{\rm P} = 2.433 \cdot 10^{18}$ GeV.

5.1 Generation of the μ Term of MSSM

The contributions from the soft SUSY breaking terms, although negligible during IHI, since these are much smaller than $\phi \sim m_{\rm P}$, may shift slightly $\langle S\rangle$ from zero in Eq. (13). Indeed, the relevant potential terms are

$$V_{\rm soft} = \left(\lambda A_\lambda S\bar\Phi\Phi + \lambda_\mu A_\mu S H_u H_d + \lambda_{iN^c} A_{iN^c}\Phi\widetilde N_i^{c2} - {\rm a}_S S\lambda M^2/4 + {\rm h.c.}\right) + m_\gamma^2 \left|X^\gamma\right|^2, \tag{32}$$

where $m_\gamma, A_\lambda, A_\mu, A_{iN^c}$ and ${\rm a}_S$ are soft SUSY breaking mass parameters. Rotating S in the real axis by an appropriate R-transformation, choosing conveniently the phases of A_λ and ${\rm a}_S$ so as the total low energy potential $V_{\rm tot} = V_{\rm SUSY} + V_{\rm soft}$ to be minimized – see Eq. (12) – and substituting in $V_{\rm soft}$ the Φ and $\bar\Phi$ values from Eq. (13) we get

$$\langle V_{\rm tot}(S)\rangle = \lambda^2\frac{(Nc_R-1)M^4S^2}{4N^2m_{\rm P}^2c_R} - \lambda M^2 S{\rm a}_{3/2}m_{3/2} \;\;\text{with}\;\; {\rm a}_{3/2} = \left(|A_\lambda| + |{\rm a}_S|\right)/2m_{3/2}, \tag{5.33a}$$

where $m_{3/2}$ is the gravitino ($\widetilde G$) mass and ${\rm a}_{3/2} > 0$ is a parameter of order unity which parameterizes our ignorance for the dependence of $|A_\lambda|$ and $|{\rm a}_S|$ on $m_{3/2}$. We also take into account that $m_S \ll M$. The extermination condition for $\langle V_{\rm tot}(S)\rangle$ w.r.t S leads to a non vanishing $\langle S\rangle$ as follows

$$d\langle V_{\rm tot}(S)\rangle/dS = 0 \;\;\Rightarrow\;\; \langle S\rangle \simeq Nc_R{\rm a}_{3/2}m_{3/2}/\lambda, \tag{5.33b}$$

where we make use of Eq. (14). The extremum above turns out to be a global minimum since $d^2\langle V_{\rm tot}(S)\rangle/dS^2 > 0$. The generated μ term from the term in Eq. (2.2c) is

$$\mu = \lambda_\mu\langle S\rangle \simeq \frac{\lambda_\mu}{32\pi}\sqrt{\frac{N}{6A_{\rm s}}}\frac{(4\widehat N_\star - N)^2}{\widehat N_\star}{\rm a}_{3/2}m_{3/2}, \tag{34}$$

where we make use of Eq. (29) which reveals that the resulting μ above does not depend on λ and c_R. Thanks to the presence of $\sqrt{A_{\rm s}} \sim 10^{-5}$ in the denominator any $\mu/m_{3/2} < 1$ value is accessible for $\lambda_\mu \sim 10^{-5}$ which is allowed by Eq. (22) without causing any ugly hierarchy between $m_{3/2}$ and μ. On the other hand, given that $m_{3/2}$ is currently constrained beyond the TeV region a mild hierarchy between μ and $m_{3/2}$ assists us to alleviate the little hierarchy problem ameliorating the naturalness of SUSY models after the LHC Higgs discovery [19].

5.2 Connection with the MSSM Phenomenology

The SUSY breaking effects, considered in Eq. (32), explicitly break $U(1)_R$ to a subgroup, $\mathbb{Z}_2^R$ which remains unbroken by $\langle S\rangle$ in Eq. (5.33b) and so no disastrous domain walls are formed. Combining $\mathbb{Z}_2^R$ with the $\mathbb{Z}_2^{\rm f}$ fermion parity, under which all fermions change sign, yields the well-known R-parity. This residual symmetry prevents rapid proton decay and guarantees the stability of the *lightest SUSY particle* (LSP), providing thereby a well-motivated *cold dark matter* (CDM) candidate.

The candidacy of LSP may be successful, if its abundance is consistent with the expectations for it from the ΛCDM model [17] within a concrete low energy framework. We here adopt the *Constrained MSSM* (CMSSM), which is relied on the following free parameters

$${\rm sign}\mu,\;\;\tan\beta = \langle H_u\rangle/\langle H_d\rangle,\;\; M_{1/2},\;\; m_0\;\;\text{and}\;\; A_0, \tag{35}$$

where signμ is the sign of μ, and the three last mass parameters denote the common gaugino mass, scalar mass and trilinear coupling constant, respectively, defined (normally) at $M_{\rm GUT}$. Imposing a

CMSSM Region		$\lvert A_0\rvert$ (TeV)	m_0 (TeV)	$\lvert\mu\rvert$ (TeV)	$a_{3/2}$	λ_μ $(\mathbf{10^{-6}})$	
						$K=K_1$	$K=K_2$
(I)	A/H Funnel	9.9244	9.136	1.409	1.086	**0.963**	**1.184**
(II)	$\tilde{\tau}_1-\chi$ Coannihilation	1.2271	1.476	2.62	0.831	***14.48***	**17.81**
(III)	$\tilde{t}_1-\chi$ Coannihilation	9.965	4.269	4.073	2.33	**2.91**	**3.41**
(IV)	$\tilde{\chi}_1^{\pm}-\chi$ Coannihilation	9.2061	9.000	0.983	1.023	**0.723**	**0.89**

Table 3: *The required λ_μ values which render our models compatible with the best-fit points in the CMSSM, as found in Ref.* [20], *for the assumptions of Eq.* (36), *$N_X=2$, and $K=K_1$ or $K=K_2$.*

number of cosmo-phenomenological constraints – from which the consistency of LSP relic density with observations plays a central role – the best-fit values of $|A_0|$, m_0 and $|\mu|$ can be determined as in Ref. [20]. Their results are listed in the first four lines of Table 3. We see that there are four allowed regions characterized by the specific mechanism for suppressing the relic density of the LSP which is the lightest neutralino (χ) – $\tilde{\tau}_1, \tilde{t}_1$ and $\tilde{\chi}_1^{\pm}$ stand for the lightest stau, stop and chargino eigenstate whereas A/H is the CP-odd and heavier CP-even Higgs bosons of MSSM. The proposed regions pass all the currently available LHC bounds [21] on the masses of the various sparticles.

Enforcing the conditions for the electroweak symmetry breaking a value for the parameter $|\mu|$ can be achieved in each of the regions in Table 3. Taking this $|\mu|$ value as input we can extract the λ_μ values, if we first derive $a_{3/2}$ setting, e.g.,

$$m_0 = m_{3/2} \quad \text{and} \quad |A_0| = |A_\lambda| = |a_S|. \tag{36}$$

Here we ignore possible renormalization group effects. The outputs of our computation is listed in the two rightmost columns of Table 3 for $K=K_1$ and K_2. From these we infer that the required λ_μ values, in all cases besides the one, written in italics, are comfortably compatible with Eq. (22) for $N_X=2$ which imply $\lambda_\mu \lesssim 2\cdot 10^{-5}$. Concluding, the whole inflationary scenario can be successfully combined with all the allowed regions CMSSM besides region (II) for $K=K_1$. On the other hand, regions (I) & (IV) are more favored from the point of view of the $\widetilde{G}$ constraint. Indeed, only for $m_{3/2}\gtrsim 9$ TeV the unstable $\widetilde{G}$ becomes cosmologically safe for the $T_{\rm rh}$ values, necessitated for satisfactory nTL – see Eqs. (47) and (6.48b) in Sec. 6.3 below.

6. Non-Thermal Leptogenesis and Neutrino Masses

We below specify how our inflationary scenario makes a transition to the radiation dominated era (Sec. 6.1) and offers an explanation of the observed BAU (Sec. 6.2) consistently with the $\widetilde{G}$ constraint and the low energy neutrino data. Our results are summarized in Sec. 6.3.

6.1 Inflaton Mass & Decay

Soon after the end of IHI, the (canonically normalized) inflaton

$$\widehat{\delta\phi} = \langle J\rangle\delta\phi \quad \text{with} \quad \delta\phi = \phi - M \quad \text{and} \quad \langle J\rangle = \sqrt{Nc_R} \tag{37}$$

with mass given by

$$\widehat{m}_{\delta\phi} = \left\langle \widehat{V}_{\mathrm{IHI},\widehat{\phi}\widehat{\phi}} \right\rangle^{1/2} = \left\langle \widehat{V}_{\mathrm{IHI},\phi\phi}/J^2 \right\rangle^{1/2} \simeq \frac{\lambda m_{\mathrm{P}}}{\sqrt{c_R\,(Nc_R-1)}} \simeq 2.8\cdot 10^{13}\ \mathrm{GeV}. \tag{38}$$

settles into a phase of damped oscillations abound the minimum in Eq. (13) reheating the universe at a temperature [12]

$$T_{\mathrm{rh}} = \left(72/5\pi^2 g_*\right)^{1/4}\left(\widehat{\Gamma}_{\delta\phi} m_{\mathrm{P}}\right)^{1/2} \quad \text{with} \quad \widehat{\Gamma}_{\delta\phi} = \widehat{\Gamma}_{\delta\phi\to N^c_i N^c_i} + \widehat{\Gamma}_{\delta\phi\to H_u H_d} + \widehat{\Gamma}_{\delta\phi\to XYZ}\,. \tag{39}$$

Also $g_* = 228.75$ counts the MSSM effective number of relativistic degrees of freedom and we take into account the following decay widths

$$\widehat{\Gamma}_{\delta\phi\to N^c_i N^c_i} = \frac{g^2_{iN^c}}{16\pi}\widehat{m}_{\delta\phi}\left(1-\frac{4M^2_{iN^c}}{\widehat{m}^2_{\delta\phi}}\right)^{3/2} \quad \text{with} \quad g_{iN^c} = (N-1)\frac{\lambda_{iN^c}}{\langle J\rangle}, \tag{6.40a}$$

$$\widehat{\Gamma}_{\delta\phi\to H_u H_d} = \frac{2}{8\pi} g^2_H \widehat{m}_{\delta\phi} \quad \text{with} \quad g_H = \frac{\lambda_\mu}{\sqrt{2}}, \tag{6.40b}$$

$$\widehat{\Gamma}_{\delta\phi\to XYZ} = g^2_y \frac{14}{512\pi^3}\frac{\widehat{m}^3_{\delta\phi}}{m^2_{\mathrm{P}}} \quad \text{with} \quad g_y = y_3\left(\frac{Nc_R-1}{2c_R}\right)^{1/2} \tag{6.40c}$$

and $y_3 = h_{t,b,\tau}(\widehat{m}_{\delta\phi}) \simeq 0.5$. Here h_t, h_b and h_τ are the Yukawa coupling constants h_{3U}, h_{2D} and h_{3E} in Eq. (2.2a) respectively – we assume that diagonalization has been performed in the generation space. They arise from the lagrangian terms

$$\mathcal{L}_{\widehat{\delta\phi}\to N^c_i N^c_i} = -\frac{1}{2}e^{K/2m^2_{\mathrm{P}}} W_{\mathrm{RHN},N^c_i N^c_i} N^c_i N^c_i \ + \text{h.c.} = g_{iN^c}\widehat{\delta\phi}\ \left(N^c_i N^c_i\ + \text{h.c.}\right) + \cdots \tag{6.41a}$$

$$\mathcal{L}_{\widehat{\delta\phi}\to H_u H_d} = -e^{K/m^2_{\mathrm{P}}} K^{SS^*}\left|W_{\mu,S}\right|^2 = -g_H \widehat{m}_{\delta\phi}\widehat{\delta\phi}\left(H^*_u H^*_d\ + \text{h.c.}\right) + \cdots, \tag{6.41b}$$

$$\mathcal{L}_{\widehat{\delta\phi}\to XYZ} = -\lambda_y(\widehat{\delta\phi}/m_{\mathrm{P}})\left(X\psi_Y\psi_Z + Y\psi_X\psi_Z + Z\psi_X\psi_Y\right) + \text{h.c.}, \tag{6.41c}$$

describing $\widehat{\delta\phi}$ decay into a pair of N^c_j with masses $M_{jN^c} = \lambda_{jN^c} M$, H_u and H_d and three MSSM (s)-particles X, Y, Z, respectively.

6.2 Lepton-Number and Gravitino Abundances

For $T_{\mathrm{rh}} < M_{iN^c}$, the out-of-equilibrium decay of N^c_i generates a lepton-number asymmetry (per N^c_i decay), ε_i. The resulting lepton-number asymmetry is partially converted through sphaleron effects into a yield of the observed BAU

$$Y_B = -0.35\cdot\frac{5}{2}\frac{T_{\mathrm{rh}}}{\widehat{m}_{\delta\phi}}\sum_i \frac{\widehat{\Gamma}_{\delta\phi\to N^c_i N^c_i}}{\widehat{\Gamma}_{\delta\phi}}\varepsilon_i \quad \text{with} \quad \varepsilon_i = \sum_{j\neq i}\frac{\mathrm{Im}\left[(m^\dagger_{\mathrm{D}} m_{\mathrm{D}})^2_{ij}\right]}{8\pi\langle H_u\rangle^2 (m^\dagger_{\mathrm{D}} m_{\mathrm{D}})_{ii}}\Big(F_{\mathrm{S}}\left(x_{ij}, y_i, y_j\right) + F_{\mathrm{V}}(x_{ij})\Big). \tag{42}$$

Here $\langle H_u\rangle \simeq 174$ GeV, for large $\tan\beta$, F_{S} $[F_{\mathrm{V}}]$ are the functions entered in the vertex and self-energy contributions computed as indicated in Ref. [22] and m_{D} is the Dirac mass matrix of neutrinos, ν_i, arising from the forth term in Eq. (2.2a). Employing the seesaw formula we can then obtain the light-neutrino masses $m_{i\nu}$ in terms of $m_{i\mathrm{D}}$ and M_{iN^c} given by Eq. (2.2d). As a consequence, nTL

PARAMETER	BEST FIT $\pm 1\sigma$	
	NORMAL	INVERTED
	HIERARCHY	
$\Delta m^2_{21}/10^{-5}\mathrm{eV}^2$	$7.5^{+0.22}_{-0.20}$	
$\Delta m^2_{31}/10^{-3}\mathrm{eV}^2$	$2.55^{+0.02}_{-0.03}$	$2.45^{+0.02}_{-0.03}$
$\sin^2\theta_{12}/0.1$	3.18 ± 0.16	
$\sin^2\theta_{13}/0.01$	$2.2^{+0.069}_{-0.062}$	$2.225^{+0.064}_{-0.070}$
$\sin^2\theta_{23}/0.1$	5.74 ± 0.14	$5.78^{+0.10}_{-0.17}$
δ/π	$1.08^{+0.13}_{-0.12}$	$1.58^{+0.15}_{-0.16}$

Table 4: *Low energy experimental neutrino data for normal or inverted hierarchical neutrino masses.*

can be nicely linked to low energy neutrino data. We take into account the recently updated best-fit values [23] of that data listed in Table 4. Furthermore, the sum of $m_{i\nu}$'s is bounded from above at 95% c.l. by the data [17, 23]

$$\textstyle\sum_i m_{i\nu} \leq 0.23\ \mathrm{eV}\quad \text{for NO } m_{i\nu}\text{'s}\quad \text{or}\quad \sum_i m_{i\nu} \leq 0.15\ \mathrm{eV}\quad \text{for IO } m_{i\nu}\text{'s}, \tag{43}$$

where NO [IO] stands for *normal [inverted] ordered* neutrino masses $m_{i\nu}$'s.

The validity of Eq. (42) requires that the $\widehat{\delta\phi}$ decay into a pair of N^c_i's is kinematically allowed for at least one species of the N^c_i's and also that there is no erasure of the produced Y_L due to N^c_1 mediated inverse decays and $\Delta L = 1$ scatterings. These prerequisites are ensured if we impose

$$\text{(a)}\ \widehat{m}_{\delta\phi} \geq 2M_{1N^c}\quad \text{and}\quad \text{(b)}\ M_{1N^c} \gtrsim 10T_{\mathrm{rh}}. \tag{44}$$

Finally, Eq. (42) has to reproduce the observational result [17]

$$Y_B = (8.697 \pm 0.054)\cdot 10^{-11}. \tag{45}$$

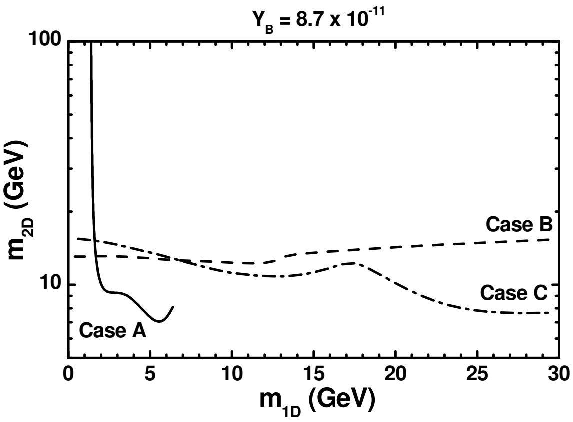
The required T_{rh} in Eq. (42) must be compatible with constraints on the $\widetilde{G}$ abundance, $Y_{3/2}$, at the onset of *nucleosynthesis* (BBN), which is estimated to be

$$Y_{3/2} \simeq 1.9\cdot 10^{-22}\ T_{\mathrm{rh}}/\mathrm{GeV}, \tag{46}$$

where we take into account only thermal production of $\widetilde{G}$, and assume that $\widetilde{G}$ is much heavier than the MSSM gauginos. On the other hand, $Y_{3/2}$ is bounded from above in order to avoid spoiling the success of the BBN. For the typical case where $\widetilde{G}$ decays with a tiny hadronic branching ratio, we have

$$Y_{3/2} \lesssim \begin{cases} 10^{-13} \\ 10^{-12} \end{cases} \text{for}\quad m_{3/2} \simeq \begin{cases} 10.6\ \mathrm{TeV} \\ 13.5\ \mathrm{TeV} \end{cases} \text{implying}\quad T_{\mathrm{rh}} \lesssim 5.3\cdot \begin{cases} 10^8\ \mathrm{GeV}, \\ 10^9\ \mathrm{GeV}. \end{cases} \tag{47}$$

The bounds above can be somehow relaxed in the case of a stable $\widetilde{G}$.



CASES : Hierarchy :	A NO	B NO	C IO
$m_{r\nu}$ / eV	0.001	0.05	0.005
$\Sigma_i m_{i\nu}$ / eV	0.06	0.074	0.1
m_{3D} / GeV	100	100	33
φ_1	$-\pi/2$	$\pi/6$	$\pi/4$
φ_2	0	0	$-\pi/3$
M_{1N^c} / 10^{12} GeV	0.9 - 2.2	0.01 - 6.4	0.01 - 3.8
M_{2N^c} / 10^{12} GeV	2 - 447	5 - 78	4.3 - 11
M_{3N^c} / 10^{15} GeV	2.3 - 9.5	0.58	0.1

Figure 2: *Contours, yielding the central Y_B in Eq.* (6.9) *consistently with the inflationary requirements, in the $m_{1\rm D} - m_{2\rm D}$ plane. We take $K = K_2$ with $N_X = 2$, $\lambda_\mu = 10^{-6}$ and the values of $m_{i\nu}$, $m_{1\rm D}$, $m_{3\rm D}$, φ_1 and φ_2 which correspond to the cases A* (*solid line*)*, B* (*dashed line*) *and C* (*dot-dashed line*).

6.3 Results

Confronting Y_B and $Y_{3/2}$ – see Eqs. (42) and (46) – with observations we can constrain the parameters of neutrino sector. This is because Y_B and $Y_{3/2}$ depend on $\widehat{m}_{\delta\phi}$, $T_{\rm rh}$, M_{iN^c} and $m_{i\rm D}$ and can interconnect IHI with neutrino physics. We follow the bottom-up approach detailed in Ref. [12], according to which we find the M_{iN^c}'s by using as inputs the $m_{i\rm D}$'s, a reference mass of the ν_i's – $m_{1\nu}$ for NO $m_{i\nu}$'s, or $m_{3\nu}$ for IO $m_{i\nu}$'s –, the two Majorana phases φ_1 and φ_2 of the PMNS matrix, and the best-fit values for the low energy parameters of neutrino physics shown in Table 4.

The outcome of our computation is presented in Fig. 2, where we depict the allowed values of $m_{2\rm D}$ versus $m_{1\rm D}$ for $K = K_2$ with $N_X = 2$, $\lambda_\mu = 10^{-6}$ and the remaining parameters shown in the Table of Fig. 2. The conventions adopted for the various lines is depicted in the plot label. In particular, we use solid, dashed and dot-dashed line when the remaining inputs – i.e. $m_{i\nu}$, $m_{3\rm D}$, φ_1, and φ_2 – correspond to the cases A, B and C of the Table of Fig. 2 respectively. We consider NO (cases A and B) and IO (case C) $m_{i\nu}$'s. In all cases, the current limit in Eq. (43) is safely met. The gauge symmetry considered here does not predict any particular Yukawa unification pattern and so, the $m_{i\rm D}$'s are free parameters. This fact offers us a convenient flexibility for the fulfilment of all the imposed requirements. Care is also taken so that the perturbativity of λ_{iN^c} holds, i.e., $\lambda^2_{iN^c}/4\pi \leq 1$. The inflaton $\widehat{\delta\phi}$ decays mostly into N_1^c's. In all cases $\widehat{\Gamma}_{\delta\phi\to N_i^c N_i^c} < \widehat{\Gamma}_{\delta\phi\to H_u H_d} < \widehat{\Gamma}_{\delta\phi\to XYZ}$ and so the ratios $\widehat{\Gamma}_{\delta\phi\to N_i^c N_i^c}/\widehat{\Gamma}_{\delta\phi}$ in Eq. (42) introduce a considerable reduction in the derivation of Y_B. For the considered cases in Fig. 2 we obtain:

$$0.01 \lesssim M_{1N^c}/10^3\ \text{EeV} \lesssim 6.4,\quad 2 \lesssim M_{2N^c}/10^3\ \text{EeV} \lesssim 447\quad \text{and}\quad 0.1 \lesssim M_{2N^c}/10^6\ \text{EeV} \lesssim 9.5, \tag{6.48a}$$

where 1 EeV = 10^9 GeV. As regards the other quantities, in all we obtain

$$1.4 \lesssim Y_{\widetilde{G}}/10^{-13} \lesssim 1.7 \quad \text{with} \quad 7.5 \lesssim T_{\rm rh}/10^8\text{GeV} \lesssim 9\,. \tag{6.48b}$$

As a bottom line, nTL is a realistic possibility within our setting provided that $m_{3/2} \sim 10$ TeV as deduced from Eqs. (47) and (6.48b). These values are in nice agreement with the ones needed for the solution of the μ problem within CMSSM in regions (I) and (IV) of Table 3.

7. Conclusions

We investigated the realization of IHI in the framework of a $B-L$ extension of MSSM endowed with the condition that the GUT scale is determined by the renormalization-group running of the three gauge coupling constants. Our setup is tied to the super- and Kähler potentials given in Eqs. (1) and (2.3a) – (2.3b). Our models exhibit the following features:

(i) they predict the correct $n_{\rm s}$ and low r thanks to the induced-gravity and the GUT requirements;

(ii) they ensure the validity of the effective theory up-to $m_{\rm P}$;

(iii) they inflate away cosmological defects;

(iv) they offer a nice solution to the μ problem of MSSM, provided that λ_μ is somehow small;

(v) they allow for baryogenesis via nTL compatible with $\widetilde{G}$ constraints and neutrino data. In particular, we may have $m_{3/2}\sim 10$ TeV, with the inflaton decaying mainly to N_1^c and N_2^c – we obtain M_{iN^c} in the range $(10^{10}-10^{15})$ GeV.

It remains to introduce a consistent soft SUSY breaking sector – see, e.g., Ref. [24] – to obtain a self-contained theory – cf. Ref. [25, 26]. Moreover, since our main aim here is the demonstration of the mechanism of IHI in SUGRA, we opted to utilize the simplest GUT embedding. Extensions to more structured GUTs are also possible – see e.g. Ref. [9, 13] – with similar inflationary observables.

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