

Scale invariant SM and inhomogeneous universe

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The effects of the breaking of the scale symmetry by thermal corrections are discussed. It is argued, that the dynamical restoration of the scale symmetry at low energies and late times due to thermal corrections dragging the expectation value of the dilaton towards the origin can be avoided in realistic physical models. Cosmological evolution of the scale invariant theory may lead to a gravitational wave signal, a discovery of which would constrain the parameter space of specific models.

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1. Introduction - in memory of Graham Ross

Particle cosmology is an important playground for particle physics, particularly in the area of theories reaching beyond the Standard Model of fundamental interactions. The lack of direct evidence for new physics in accelerator experiments and the need for indirect arguments often based on cosmological considerations has been driving vast amount of theoretical work. A lot of effort concentrated on embedding the SM into more fundamental theories providing ultraviolet completion of both the SM and the Einstein gravity. This line of research became one of the leading themes in the scientific work of Graham Ross. The work reported in this note has been inspired by numerous discussions with Graham over the period of more than 35 years. The opportunity to participate in these discussion and to collaborate with Graham on common research projects was a great and inspiring privilege.

In the following discussion we shall concentrate on two related topics. Firstly, we shall advocate the scale invariant extension of the SM as its possible UV completion. Since this theory necessarily includes the scalar sector extended by at least one additional scalar and generalizes the SM Higgs sector, it exhibits an interesting cosmological evolution. This includes an inhomogeneous postinflationary initial condition for subsequent field evolution. The resulting field distribution will resemble a network of domain walls, which should vanish reasonably quickly to avoid leaving observable traces in CMB. However, a rapid evolution of the network could result in a gravitational wave signal. This work is based on published papers [1], [2].

2. Scale invariant universe

Let us consider a scale symmetric Lagrangian for the Higgs neutral component ϕ_1 and a new scalar singlet ϕ_0 , which we call dilaton. Coupling both fields with Einstein gravity via non-minimal couplings ξ_i we have:

$$\frac{\mathcal{L}}{\sqrt{g}} = -\frac{1}{12}(\xi_0\phi_0^2 + \xi_1\phi_1^2)R + \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 + \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - V(\phi_0, \phi_1), \quad (1)$$

where R is Ricci scalar in Riemannian geometry. In order to obtain scale symmetry at the tree level one needs to assume a vanishing Higgs mass parameter $m_H^2 = 0$. In terms of ϕ_0 and ϕ_1 fields, scale symmetric potential at the classical level is of the form:

$$V(\phi_0, \phi_1) = \lambda_0\phi_0^4 + \lambda_1\phi_0^2\phi_1^2 + \lambda_2\phi_1^4. \quad (2)$$

We choose certain hierarchy among couplings:

$$\lambda_2 \gg |\lambda_1| \gg \lambda_0 \quad (3)$$

and $\lambda_2 > 0$, $\lambda_1 < 0$, $\lambda_0 > 0$, so the new dilaton sector is weakly coupled to the Higgs sector. Origin of the dilaton field can be found in Weyl conformal geometry, which at low energies comes down to Einstein gravity. All essential properties of such formulation are described in the Appendix of [1] and in [3–31]. Stationary solutions give a flat direction:

$$\langle\phi_1^2\rangle = -\frac{\lambda_1}{2\lambda_2}\langle\phi_0^2\rangle, \quad \lambda_0 = \frac{\lambda_1^2}{4\lambda_2}, \quad \langle R\rangle = 0, \quad (4)$$

where the λ_0 dependence comes from the condition of zero cosmological constant at the ground state:

$$V(\langle\phi_0\rangle, \langle\phi_1\rangle) = 0.$$

Since theory is scale symmetric, only ratios of mass scales can be determined and $\langle\phi_0\rangle$ is arbitrary. With (3) we have the hierarchy $\langle\phi_0\rangle \gg \langle\phi_1\rangle$. When ϕ_0 acquires its vev, scale symmetry is broken and flat direction no longer exists. Because $\langle\phi_1\rangle$ is proportional to $\langle\phi_0\rangle$, dilaton generates Higgs vev and mass, so it can be considered as origin of mass scales.

2.1 Higgs potential parameters and Planck mass:

One can determine what values of λ_1 , λ_2 , ξ_0 and ξ_1 are available in our theory. First, we want the hierarchy (3) and two conditions to be fulfilled:

$$m_H^2 = (125 \text{ GeV})^2, \quad \langle\phi_1\rangle = 250 \text{ GeV} \quad (5)$$

and in our model we have:

$$m_H^2 = -4\lambda_1 \left(1 - \frac{\lambda_1}{2\lambda_2}\right) \langle\phi_0^2\rangle, \quad \langle\phi_1^2\rangle = -\frac{\lambda_1}{2\lambda_2} \langle\phi_0^2\rangle. \quad (6)$$

Satisfying all the conditions we get:

$$\lambda_2 = \frac{1}{32} (1 + 16\lambda_1), \quad -\frac{1}{48} \leq \lambda_1 \leq 0 \quad (7)$$

and example values are:

$$\lambda_2(\lambda_1 = -10^{-6}) \approx \lambda_2(\lambda_1 = -10^{-11}) \approx 0.03125. \quad (8)$$

Required value of $\langle\phi_0\rangle$ is then:

$$\langle\phi_0^2\rangle = -\frac{2\lambda_2}{\lambda_1} \langle\phi_1^2\rangle = -\frac{2\lambda_2}{\lambda_1} \cdot (250 \text{ GeV})^2. \quad (9)$$

Then, we want the Planck mass scale to be generated by ϕ_i fields, as in (1):

$$\frac{1}{6} \left(\xi_0 - \frac{\lambda_1}{2\lambda_2} \xi_1 \right) \langle\phi_0^2\rangle = M_{Planck}^2. \quad (10)$$

This will force constraints on values of ξ_i couplings. In realistic models, [19], we should have $\xi_1 \ll \xi_0$. Using (7), (9) and (10) we obtain relation of couplings:

$$\lambda_1 = \frac{-0.0625 \cdot \xi_0}{\xi_0 - \xi_1 + 1.43 \cdot 10^{34}}. \quad (11)$$

Example values:

$$\begin{aligned} \xi_0 = 10^5, \quad \xi_1 = 0.1 &\Rightarrow \lambda_1 = -4.37 \cdot 10^{-31}, \\ \xi_0 = 10^{10}, \quad \xi_1 = 0.1 &\Rightarrow \lambda_1 = -4.37 \cdot 10^{-26}, \\ \xi_0 = 10^{15}, \quad \xi_1 = 0.1 &\Rightarrow \lambda_1 = -4.37 \cdot 10^{-21}. \end{aligned} \quad (12)$$

Particle content and thermal masses

Obviously, the degrees of freedom ϕ_0 and ϕ_1 are present in our theory, and the mass eigenstates are their mixture. So we have two neutral scalars G (massless Goldstone) and H (massive "Higgs") with $n_G = n_H = 1$. The masses of this sector are the eigenvalues of the mass matrix in the scalar sector:

$$\begin{aligned} m_G^2 &= 2\lambda_1\phi_1^2 + O(\lambda_1^2) \\ m_H^2 &= 12\lambda_2\phi_1^2 + 2\lambda_1\phi_0^2 + O(\lambda_1^2). \end{aligned} \quad (13)$$

However, there are particles in SM, which give important contributions to thermally corrected effective potential, see [32–36]. These are:

- W^\pm boson: $m_W^2 = \frac{1}{4}g_2^2\phi_1^2$, $n_w = 6$,
- Z boson: $m_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)\phi_1^2$, $n_Z = 3$
- top quark: $m_t^2 = \frac{1}{2}h_t^2\phi_1^2$, $n_t = -12$,

where $g_1 \approx 0.35$, $g_2 \approx 0.65$ and $h_t \approx 1$ are correspondingly weak, strong and top yukawa coupling constants¹.

The thermal masses m_{eff}^2 for scalars are the eigenvalues of the thermally corrected scalar mass matrix:

$$\begin{aligned} (m_G^2)_{eff} &= 2\lambda_1\phi_1^2 + \frac{\lambda_1}{6}T^2 + O(\lambda_1^2), \\ (m_H^2)_{eff} &= 12\lambda_2\phi_1^2 + 2\lambda_1\phi_0^2 + \left(\lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_t^2}{4}\right)T^2 + O(\lambda_1^2). \end{aligned} \quad (14)$$

It should be noted, that the dependence of the temperature corrections on ϕ_0 enters via the tree-level mass terms in the scalar sector. Moreover, since the hierarchy of scales demands the hierarchy of couplings, one finds in the present case $\lambda_0 = \frac{\lambda_1^2}{4\lambda_2} \ll |\lambda_1| \ll \lambda_2$ and the dependence on ϕ_0 starts at the linear order in a small coupling λ_1 . At this point we refrain from discussing the issue of thermal equilibrium of the whole system and concentrate on the analysis of the thermal effective potential.

2.2 Symmetry breaking at high temperature

The scale symmetry breaking can be discussed reliably at the leading level of high temperature expansion, where

$$V_{eff} = V_{T=0} + \frac{1}{2}\phi_1^2 \cdot \left(\lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_t^2}{4}\right)T^2 + \frac{1}{2}\phi_0^2 \cdot \frac{\lambda_1}{6}T^2 = V_{T=0} + \frac{\gamma T^2}{2}\phi_1^2 + \frac{\lambda_1 T^2}{12}\phi_0^2. \quad (15)$$

One should note that the coefficients of the two terms quadratic in the temperature are completely independent, as in the case of ϕ_1 , which plays the role of the Higgs field, the thermal corrections are dominated by gauge couplings and by the coupling to the top quark, whereas in the case of the

¹It is well known that the Higgs effective potential in the Standard Model, calculated perturbatively, generically suffers from infrared (IR) divergences when the field-dependent tree-level mass of the Goldstone bosons in the Higgs doublet vanishes. Here we follow the analysis given in [37] and [38] and assume that such divergences can be cured by a resummation of IR-problematic terms to any order and neglect these troublesome contributions.

ϕ_0 they are proportional to the coupling of the mixing term in the scalar sector. As the result, the proportionality of the two scalar equations of motion which holds at $T = 0$ gets broken by the term

$$\left(\frac{\lambda_1}{6} - \gamma\right)T^2 \neq 0. \quad (16)$$

This is the amount of the scale symmetry breaking by the finite temperature effects. As the result, the only consistent solution to the corrected equations of motion becomes at this order

$$\phi_1 = 0, \quad \phi_0^2 = -\frac{\lambda_2}{6\lambda_1}T^2. \quad (17)$$

This shows, that at finite temperature the minimum of the potential picks up a finite expectation value of the dilaton underlying the fact that the scale symmetry remains broken, and the scale of the breaking given by the vev of the dilaton is proportional to the temperature - the new scale in the system. However, this indicates, that when the temperature goes to zero, the symmetry gets restored and the system goes into the unbroken phase, since the vevs of both scalars seem to be led to the origin. One should note that this would restore also the electroweak symmetry, which requires a nonvanishing vev of ϕ_0 . The point is that it is this vev which multiplied by the negative coupling λ_1 plays the role of the negative mass squared term in the Higgs sector. This would suggest a symmetric, unrealistic, vacuum emerging from the hot phase of the universe. However, the situation is more subtle. The point is that the Higgs field ϕ_1 easily comes to the equilibrium with rest of the universe through interaction with the Standard Model matter and gauge fields, which despite the fact that the its thermal average seems to vanish, produces a large rms value of the order of:

$$\langle \phi_1^2 \rangle_{T,p} = T \frac{P^3}{\omega_p^2} \quad (18)$$

per decade. For high temperatures and small masses this can be approximated as T^2 , and produces a large repulsive force in the equation of motion of ϕ_0 due to the mixing term in the potential:

$$\delta_m V = \lambda_1 \phi_0^2 \phi_1^2 \rightarrow \lambda_1 T^2 \phi_0^2, \quad (19)$$

giving in the eom the contribution:

$$-\frac{\partial \delta_m V}{\partial \phi_0} = -2\lambda_1 T^2 \phi_0, \quad (20)$$

which drives the dilaton away from the origin. In addition, as discussed later, the dynamical thermal equilibrium in the scalar sector, perhaps after the point of quasi-thermal initial production, is not to be maintained at the later stages of the evolution of the universe. Hence, in the realistic physical system the origin will not be achieved globally and there will be in the universe domains characterized by large expectation value of the dilaton, and hence the Higgs, which when the temperature drops down will have a chance to evolve dynamically into the zero temperature vacuum with spontaneously broken scale symmetry and electroweak symmetry.

A remark is in order here. We have assumed above the manifestly scale-invariant regularization as described in [26–31]. Then, due to scale invariance at the level of quantum corrections, the one-loop potential can be approximated by the tree-level formula with couplings understood as running

couplings. However, in the present case, where the hierarchy is based on the ratio of very small couplings, the non-thermal perturbative quantum corrections are small as proportional to higher powers of small couplings with respect to temperature corrections. Possible additional perturbative contributions violating explicitly scale symmetry, other than temperature effects, will shift the position of the scalar vevs, but note that the thermal shift of the ϕ_0^2 is proportional to a very large ratio of couplings $\frac{\lambda_2}{|\lambda_1|} \gg 1$, hence other perturbative shifts would be typically subdominant unless the temperature is very low. Here we concentrate on the role of temperature corrections, hence we assume the scale invariance at the loop level.

2.3 Electroweak Symmetry Breaking

To examine EWSB in our model, we consider time evolution of the effective potential for the Higgs neutral component ϕ_1 , i.e.:

$$V_{\phi_1}(t) \equiv V_{eff}(\phi_0(t), \phi_1, T(t)), \quad (21)$$

where $\phi_0(t)$ comes from solution of evolution for realistic model parameters with $\lambda_2 = 0.03125$, $\lambda_1 = -4.37 \cdot 10^{-26}$ and time dependent temperature $T(t)$. In Figure 1 we show plot of the shape of $V_{\phi_1}(t)$ for different time of evolution. It is easy to see that the phase transition associated with EWSB in tested model is of the second order.

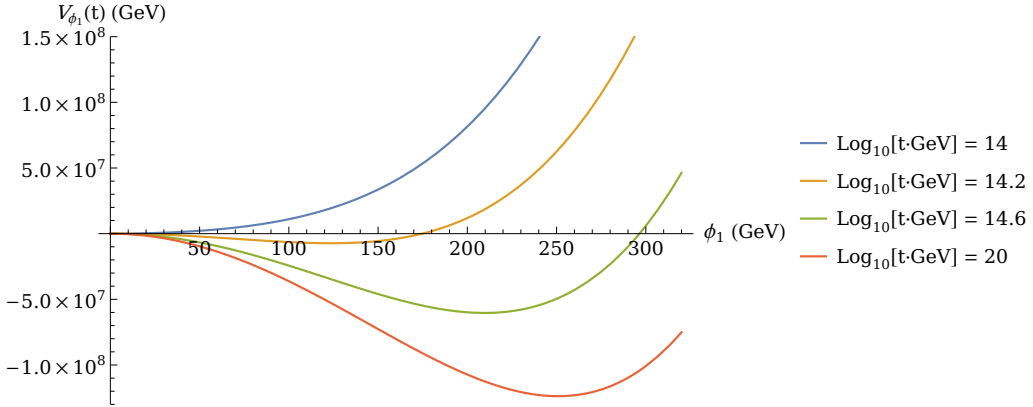


Figure 1: Plot of the potential for Higgs neutral component $V_{\phi_1}(t)$ for different time values during evolution in hot Universe.

3. Additional scalars

Parameter space obtained from fulfilling numerical conditions on mass scales m_H^2 , v^2 and M_P^2 is rather constrained. One can relax these constraints by adding more scalar singlets to the model. Here we consider briefly addition of one more scalar singlet for simplicity. Then low energy Lagrangian density will take the form

$$\frac{\mathcal{L}_{mod}}{\sqrt{g}} = -\frac{1}{12}(\xi_0\phi_0^2 + \xi_1\phi_1^2 + \xi_2\phi_2^2)R + \sum_{i=0}^2 \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - V_{mod}(\phi_0, \phi_1, \phi_2), \quad (22)$$

where potential V_{mod} is assumed to be

$$V_{mod}(\phi_0, \phi_1, \phi_2) = \lambda_0 \phi_0^4 + \lambda_1 \phi_0^2 \phi_1^2 + \lambda_2 \phi_1^4 + \lambda_3 \phi_1^2 \phi_2^2 + \lambda_4 \phi_2^4, \quad (23)$$

so the new scalar ϕ_2 doesn't couple directly to the dilaton. Such a structure could be justified for instance by the locality of couplings in extra dimensions (space-time or internal). Taking λ_2 and λ_4 order one while $\lambda_1, \lambda_3 \ll 1$, one can easily arrange for a ground state displaying a hierarchy of vevs:

$$\langle \phi_0^2 \rangle \gg \langle \phi_1^2 \rangle \gg \langle \phi_2^2 \rangle \quad (24)$$

and zero cosmological constant condition:

$$V_{mod}(\langle \phi_0 \rangle, \langle \phi_1 \rangle, \langle \phi_2 \rangle) = 0, \quad (25)$$

where the explicit solution reads:

$$\langle \phi_1^2 \rangle = \frac{2\lambda_1 \lambda_4}{\lambda_3^2 - 4\lambda_2 \lambda_4} \langle \phi_0^2 \rangle, \quad \langle \phi_2^2 \rangle = -\frac{\lambda_3}{2\lambda_4} \langle \phi_1^2 \rangle = -\frac{\lambda_1 \lambda_3}{\lambda_3^2 - 4\lambda_2 \lambda_4} \langle \phi_0^2 \rangle, \quad \lambda_0 = \frac{\lambda_1^2 \lambda_4}{4\lambda_2 \lambda_4 - \lambda_3^2}. \quad (26)$$

Then the effective Planck mass may become almost independent from the field playing the role of the Higgs and assuming the smallest vev:

$$M_P^2 \sim \xi_0 \phi_0^2 + \xi_1 \phi_1^2. \quad (27)$$

Such an extension of the scalar sector would allow building additional hierarchy of scales, while the thermal evolution for each pair of coupled scalars may follow approximately the scenario outlined above, this leading to the physically acceptable final state.

4. Inhomogeneities

We have argued that at late times the scale invariant scalar potential may lead to the evolution reproducing to large extent the one known from the Standard Model, see Figure 1. Therefore inhomogeneities in the initial state of the fields at the beginning of the post-inflationary epoch may lead to formation of a network of domain walls, whose collapse may lead to a signal in the spectrum of the gravitational wave background as discussed in [2]. To study generic effects a family of potentials has been constructed whose shape around the potential barrier and the level of degeneracy of minima can be set independently. This set of potential includes representatives of the scale invariant potentials in the form projected on the direction of the physical Higgs field in the space of scalar degrees of freedom. The evolution of networks of domain walls in models given by potentials from the constructed family has been investigated using lattice simulations based on the constant width PRS algorithm. After performing thousands of simulations Authors of [2] have found that the final state of the decay of the network is determined by the bias of the initial probability distribution. Even though other factors can shorten or enlarge the life-time of the network, the excess of lattice points belonging to one of basins of attraction of minima of the potential drive the evolution of domain walls into corresponding vacuum. The findings are represented in the Figure 2, details can be found in [2].

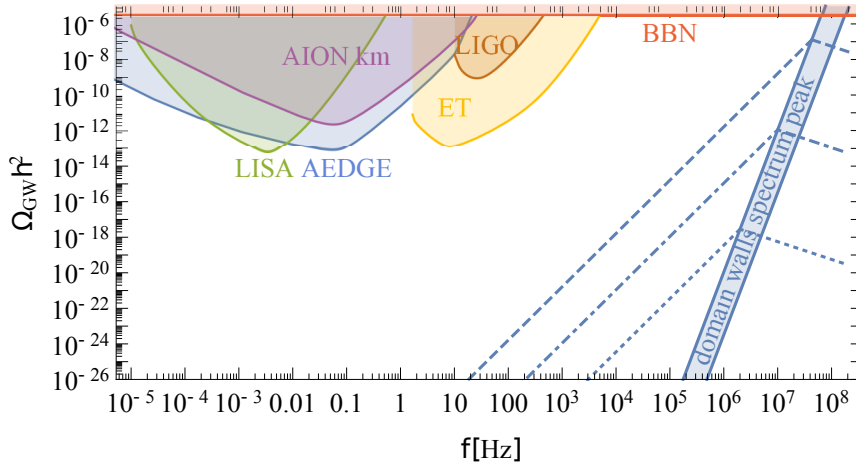


Figure 2: The blue band shows hypothetical peak amplitudes of GWs emitted from cosmological domain walls as a function of the peak frequency f . The width of the band comes from the possible range on the prefactor controlling the amplitude of the signal. The shape of the spectra peaking in the allowed region is indicated by the dashed blue lines. These should be compared to predicted sensitivities of currently operating and planned detectors (LIGO, LISA, AEDGE, AION-1km, ET) as well as to an upper bound induced by the CMB/BBN.

5. Summary and conclusions

In this note we have discussed possible thermal corrections to the cosmological evolution of the scale symmetric scalar sector extending the standard Higgs sector. All sources of the explicit breaking of scale invariance other than the temperature corrections have been neglected. Hierarchy of generated mass scales relies on a hierarchy of small couplings, which stays perturbatively stable. We have visualized the effects of the breaking of the scale symmetry by the thermal corrections and argued, that the dynamical restoration of the scale symmetry at low energies and late times due to thermal corrections dragging the expectation value of the dilaton towards the origin can be avoided in realistic physical models. We have also demonstrated with specific examples that starting with the acceptable initial conditions one can reach the physically relevant vacuum configuration as the result of the evolution of the scale symmetric scalar sector in the hot universe.

We have pointed out that the cosmological evolution of the scale invariant theory may lead to a gravitational wave signal, a discovery of which would constrain the parameter space of specific models.

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References

- [1] Z. Lalak and P. Michalak, “Spontaneous scale symmetry breaking at high temperature,” [arXiv:2211.09045 [hep-ph]].

- [2] T. Krajewski, J. H. Kwapisz, Z. Lalak and M. Lewicki, “Stability of domain walls in models with asymmetric potentials,” *Phys. Rev. D* **104** (2021) no.12, 123522 doi:10.1103/PhysRevD.104.123522 [arXiv:2103.03225 [astro-ph.CO]].
- [3] M. Heikinheimo, A. Racioppi, M. Raidal, C. Spethmann and K. Tuominen, “Physical Naturalness and Dynamical Breaking of Classical Scale Invariance,” *Mod. Phys. Lett. A* **29** (2014), 1450077 doi:10.1142/S0217732314500771 [arXiv:1304.7006 [hep-ph]].
- [4] K. Kannike, A. Racioppi and M. Raidal, “Embedding inflation into the Standard Model - more evidence for classical scale invariance,” *JHEP* **06** (2014), 154 doi:10.1007/JHEP06(2014)154 [arXiv:1405.3987 [hep-ph]].
- [5] K. Kannike, G. Hütsi, L. Pizza, A. Racioppi, M. Raidal, A. Salvio and A. Strumia, “Dynamically Induced Planck Scale and Inflation,” *PoS EPS-HEP2015* (2015), 379 doi:10.22323/1.234.0379
- [6] K. Kannike, M. Raidal, C. Spethmann and H. Veermäe, “The evolving Planck mass in classically scale-invariant theories,” *JHEP* **04** (2017), 026 doi:10.1007/JHEP04(2017)026 [arXiv:1610.06571 [hep-ph]].
- [7] K. Kannike, N. Koivunen, A. Kubarski, L. Marzola, M. Raidal, A. Strumia and V. Vipp, “Dark matter-induced multi-phase dynamical symmetry breaking,” *Phys. Lett. B* **832** (2022), 137214 doi:10.1016/j.physletb.2022.137214 [arXiv:2204.01744 [hep-ph]].
- [8] M. Shaposhnikov and D. Zenhausern, “Scale invariance, unimodular gravity and dark energy,” *Phys. Lett. B* **671** (2009), 187-192 doi:10.1016/j.physletb.2008.11.054 [arXiv:0809.3395 [hep-th]].
- [9] M. Shaposhnikov and D. Zenhausern, “Quantum scale invariance, cosmological constant and hierarchy problem,” *Phys. Lett. B* **671** (2009), 162-166 doi:10.1016/j.physletb.2008.11.041 [arXiv:0809.3406 [hep-th]].
- [10] J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, “Higgs-Dilaton Cosmology: From the Early to the Late Universe,” *Phys. Rev. D* **84** (2011), 123504 doi:10.1103/PhysRevD.84.123504 [arXiv:1107.2163 [hep-ph]].
- [11] M. Shaposhnikov, A. Shkerin, I. Timiryasov and S. Zell, “Einstein-Cartan gravity, matter, and scale-invariant generalization ,” *JHEP* **10** (2020), 177 doi:10.1007/JHEP08(2021)162 [arXiv:2007.16158 [hep-th]].
- [12] G. K. Karananas, M. Shaposhnikov, A. Shkerin and S. Zell, “Scale and Weyl invariance in Einstein-Cartan gravity,” *Phys. Rev. D* **104** (2021) no.12, 124014 doi:10.1103/PhysRevD.104.124014 [arXiv:2108.05897 [hep-th]].
- [13] M. Shaposhnikov and A. Tokareva, “Anomaly-free scale symmetry and gravity,” [arXiv:2201.09232 [hep-th]].

- [14] P. G. Ferreira, C. T. Hill and G. G. Ross, “Scale-Independent Inflation and Hierarchy Generation,” *Phys. Lett. B* **763** (2016), 174-178 doi:10.1016/j.physletb.2016.10.036 [arXiv:1603.05983 [hep-th]].
- [15] P. G. Ferreira, C. T. Hill and G. G. Ross, “Weyl Current, Scale-Invariant Inflation and Planck Scale Generation,” *Phys. Rev. D* **95** (2017) no.4, 043507 doi:10.1103/PhysRevD.95.043507 [arXiv:1610.09243 [hep-th]].
- [16] P. G. Ferreira, C. T. Hill, J. Noller and G. G. Ross, “Inflation in a scale invariant universe,” *Phys. Rev. D* **97** (2018) no.12, 123516 doi:10.1103/PhysRevD.97.123516 [arXiv:1802.06069 [astro-ph.CO]].
- [17] P. G. Ferreira, C. T. Hill, J. Noller and G. G. Ross, “Scale-independent R^2 inflation,” *Phys. Rev. D* **100** (2019) no.12, 123516 doi:10.1103/PhysRevD.100.123516 [arXiv:1906.03415 [gr-qc]].
- [18] D. M. Ghilencea and H. M. Lee, “Weyl gauge symmetry and its spontaneous breaking in the standard model and inflation,” *Phys. Rev. D* **99** (2019) no.11, 115007 doi:10.1103/PhysRevD.99.115007 [arXiv:1809.09174 [hep-th]].
- [19] D. M. Ghilencea, “Spontaneous breaking of Weyl quadratic gravity to Einstein action and Higgs potential,” *JHEP* **03** (2019), 049 doi:10.1007/JHEP03(2019)049 [arXiv:1812.08613 [hep-th]].
- [20] D. M. Ghilencea, “Stueckelberg breaking of Weyl conformal geometry and applications to gravity,” *Phys. Rev. D* **101** (2020) no.4, 045010 doi:10.1103/PhysRevD.101.045010 [arXiv:1904.06596 [hep-th]].
- [21] D. M. Ghilencea, “Weyl R^2 inflation with an emergent Planck scale,” *JHEP* **10** (2019), 209 doi:10.1007/JHEP10(2019)209 [arXiv:1906.11572 [gr-qc]].
- [22] D. M. Ghilencea, “Gauging scale symmetry and inflation: Weyl versus Palatini gravity,” *Eur. Phys. J. C* **81** (2021) no.6, 510 doi:10.1140/epjc/s10052-021-09226-1 [arXiv:2007.14733 [hep-th]].
- [23] D. M. Ghilencea, “Standard Model in Weyl conformal geometry,” *Eur. Phys. J. C* **82** (2022) no.1, 23 doi:10.1140/epjc/s10052-021-09887-y [arXiv:2104.15118 [hep-ph]].
- [24] D. M. Ghilencea and T. Harko, “Cosmological evolution in Weyl conformal geometry,” [arXiv:2110.07056 [gr-qc]].
- [25] D. M. Ghilencea, “Non-metricity as the origin of mass generation in gauge theories of scale invariance,” [arXiv:2203.05381 [hep-th]].
- [26] D. M. Ghilencea, “Manifestly scale-invariant regularisation and quantum effective operators,” *Phys. Rev. D* **93** (2016) no.10, 105006 doi:10.1103/PhysRevD.93.105006 [arXiv:1508.00595 [hep-ph]].
- [27] D. M. Ghilencea, “One-loop potential with scale invariance and effective operators,” *PoS CORFU2015* (2016), 040 doi:10.22323/1.263.0040 [arXiv:1605.05632 [hep-ph]].

- [28] D. M. Ghilencea, Z. Lalak and P. Olszewski, “Two-loop scale-invariant scalar potential and quantum effective operators,” *Eur. Phys. J. C* **76** (2016) no.12, 656 doi:10.1140/epjc/s10052-016-4475-0 [arXiv:1608.05336 [hep-th]].
- [29] D. M. Ghilencea, Z. Lalak and P. Olszewski, “Standard Model with spontaneously broken quantum scale invariance,” *Phys. Rev. D* **96** (2017) no.5, 055034 doi:10.1103/PhysRevD.96.055034 [arXiv:1612.09120 [hep-ph]].
- [30] D. M. Ghilencea, “Quantum implications of a scale invariant regularization,” *Phys. Rev. D* **97** (2018) no.7, 075015 doi:10.1103/PhysRevD.97.075015 [arXiv:1712.06024 [hep-th]].
- [31] Z. Lalak and P. Olszewski, “Vanishing trace anomaly in flat spacetime,” *Phys. Rev. D* **98** (2018) no.8, 085001 doi:10.1103/PhysRevD.98.085001 [arXiv:1807.09296 [hep-th]].
- [32] S. R. Coleman and E. J. Weinberg, “Radiative Corrections as the Origin of Spontaneous Symmetry Breaking,” *Phys. Rev. D* **7** (1973), 1888-1910 doi:10.1103/PhysRevD.7.1888
- [33] M. Sher, “Electroweak Higgs Potentials and Vacuum Stability,” *Phys. Rept.* **179** (1989), 273-418 doi:10.1016/0370-1573(89)90061-6
- [34] M. E. Carrington, “The Effective potential at finite temperature in the Standard Model,” *Phys. Rev. D* **45** (1992), 2933-2944 doi:10.1103/PhysRevD.45.2933
- [35] M. Quiros, “Finite temperature field theory and phase transitions,” [arXiv:hep-ph/9901312 [hep-ph]].
- [36] A. Megevand and A. D. Sanchez, “Supercooling and phase coexistence in cosmological phase transitions,” *Phys. Rev. D* **77** (2008), 063519 doi:10.1103/PhysRevD.77.063519 [arXiv:0712.1031 [hep-ph]].
- [37] S. P. Martin, “Taming the Goldstone contributions to the effective potential,” *Phys. Rev. D* **90** (2014) no.1, 016013 doi:10.1103/PhysRevD.90.016013 [arXiv:1406.2355 [hep-ph]].
- [38] J. Elias-Miro, J. R. Espinosa and T. Konstandin, “Taming Infrared Divergences in the Effective Potential,” *JHEP* **08** (2014), 034 doi:10.1007/JHEP08(2014)034 [arXiv:1406.2652 [hep-ph]].