

## Status of dark matter in $S_3$ -symmetric 3HDM

---

**Anton Kuncinas**<sup>a,\*</sup>

<sup>a</sup>*Centro de Física Teórica de Partículas, CFTP, Departamento de Física,  
Instituto Superior Técnico, Universidade de Lisboa,  
Avenida Rovisco Pais nr. 1, 1049-001 Lisboa, Portugal*

*E-mail:* [Anton.Kuncinas@tecnico.ulisboa.pt](mailto:Anton.Kuncinas@tecnico.ulisboa.pt)

The question of the dark matter nature is one of the important issues at the frontier of particle physics, astrophysics and cosmology. There are numerous models trying to explain the dark matter origin, assuming different underlying physical processes and proposing different candidates. One of the most simplistic dark matter candidates can arise due to the extension of the scalar electroweak sector. Such extensions are well motivated and could explain not only the dark matter origin but also other shortcomings of the Standard Model. The most natural way to accommodate a dark matter candidate in the extended scalar models, and to assure stability of dark matter, while controlling the number of free parameters is to assume an underlying symmetry. Governed by this, we review all possible cases of the  $S_3$ -symmetric three-Higgs-doublet models which can accommodate a scalar dark matter candidate. Within the model different cases arise due to different directions of vacua, which enforce different minimisation conditions. After classifying all possible dark matter candidates within the framework we explore two cases numerically.

*Corfu Summer Institute 2022 "School and Workshops on Elementary Particle Physics and Gravity",  
28 August - 1 October, 2022  
Corfu, Greece*

---

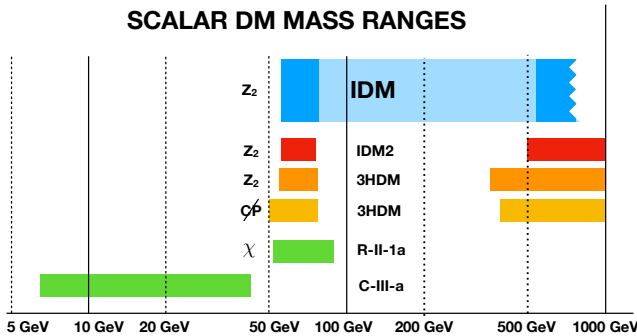
\*Speaker

## 1. Introduction

Cosmological observations, based on the standard cosmological model, indicate that we have a limited knowledge of the Universe [1]. The model implies that a significant part of the matter of the Universe is made up of a hypothetical non-luminous form of matter—Dark Matter (DM). There is no good candidate in the Standard Model (SM) of Particle Physics to explain what DM could possibly be. Still, different cosmological scales and phenomena point to our ignorance of the Universe. A variety of models have been proposed in order to explain DM. One of the simplest and appealing extensions of the SM, capable of explaining the origin of DM, invoke models with an extended scalar sector. Beyond the SM physics quite commonly relies on models with an extended scalar content. Such extensions may be associated with an extended scalar electroweak sector of the SM, governed by the fact that there may exist several replicas of the Higgs-like bosons and there is no firm experimental exclusion of the non-minimal Higgs sector [2].

One of the best studied extended scalar sector models with a DM candidate is the Inert Doublet Model (IDM) [3]. In the IDM the scalar sector of the SM is enlarged by an additional  $SU(2)$  Higgs doublet. In light of no evidence of realisability of the IDM in nature, one may consider models of a scalar potential with more  $SU(2)$  components added. Such models may allow for new interactions to explain the abundance of the hypothetical DM. However, such extensions come at a price—the number of degrees of freedom grows rapidly with more  $SU(2)$  components added to the scalar potential. The number of free parameters can be decreased by assuming an underlying symmetry. Symmetries can also enforce the DM candidate to become stable throughout the age of the Universe.

In this paper we analyse a three-Higgs-doublet model (3HDM) with an underlying  $S_3$  symmetry. There are several possibilities to implement a DM candidate within the  $S_3$ -3HDM model. These possibilities are due to different arrangements of the vacuum expectation values (vev), which yield different minimisation conditions. In light of that we catalogue different cases of the  $S_3$ -3HDM, also allowing for softly broken  $S_3$ -symmetric scalar potential, based on whether a specific implementation could accommodate a DM candidate. Such implementations can have vacua with one or two vanishing vevs. In total, we found that there are only three cases with “good” candidates not requiring soft symmetry breaking terms and yielding realistic fermionic content. The DM candidate is associated with a  $\mathbb{Z}_2$  symmetry which survives spontaneous symmetry breaking due to a vanishing vev and is a remnant of the  $S_3$  symmetry. Two models, without, R-II-1a [4], and with, C-III-a [5], spontaneous CP violation are explored numerically. The DM candidate mass ranges of the two cases along with other models considered in the literature are presented in Figure 1.



**Figure 1:** Sketch of allowed DM mass ranges up to 1 TeV in various models. Blue: IDM according to Refs. [6], the pale region indicates a non-saturated relic density. Red: IDM2 [7]. Ochre: 3HDM without [8] and with CP violation [9]. Green:  $S_3$ -symmetric 3HDM without CP violation (R-II-1a) [4] and with spontaneous CP violation (C-III-a) [5].

## 2. The $S_3$ -symmetric model

The  $S_3$ -symmetric 3HDM potential in the doublet-singlet representation can be written as [10]:

$$\begin{aligned}
V_2 &= \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \\
V_4 &= \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\
&\quad + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\
&\quad + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\
&\quad + \lambda_8 (h_S^\dagger h_S)^2,
\end{aligned}$$

soft symmetry breaking terms can be introduced, general form is given by

$$V'_2 = \mu_2^2 (h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{1}{2} \nu_{12}^2 (h_1^\dagger h_2 + \text{h.c.}) + \frac{1}{2} \nu_{01}^2 (h_S^\dagger h_1 + \text{h.c.}) + \frac{1}{2} \nu_{02}^2 (h_S^\dagger h_2 + \text{h.c.}).$$

Here all couplings are chosen to be real, complex couplings were covered in Ref. [11], and hence CP is not violated explicitly. There is still the possibility to have spontaneous CP violation when vevs are complex [12]. In the irreducible representation, the  $S_3$  fields can be decomposed as

$$h_i = \begin{pmatrix} h_i^+ \\ (w_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, \quad h_S = \begin{pmatrix} h_S^+ \\ (w_S + \eta_S + i\chi_S)/\sqrt{2} \end{pmatrix},$$

where the  $w_i$  and  $w_S$  are vevs that can be complex.

For the reducible defining representation consider Refs. [13].

## 3. Scalar dark matter candidates

The scalar potential exhibits a  $\mathbb{Z}_2$  symmetry under which  $h_1 \leftrightarrow -h_1$ . As a result, we do not need to impose any additional symmetries to stabilise the DM candidate when it is associated with the  $h_1$  doublet. Some of the vacuum configurations will enforce vanishing of  $\lambda_4 = 0$ . In this case the scalar potential becomes  $O(2)$  invariant. The  $O(2)$  symmetry can be spontaneously broken by vevs giving rise to additional unwanted Goldstone bosons [14]. Apart from this, additional continuous symmetries may arise due to specific vacua. The realisable continuous symmetries are summarised in Table 1. Such cases will require soft symmetry breaking terms if spontaneously broken by vevs.

Constraints	Continuous symmetries	# of massless states
$\lambda_4 = 0$	$O(2)$	1
$\dots \lambda_7 = 0$	$O(2) \otimes U(1)_{h_S}$	2
$\dots \lambda_2 + \lambda_3 = 0$	$SU(2)$ [ $O(2) \otimes U(1)_{h_1} \otimes U(1)_{h_2} \otimes U(1)_{h_S}$ ]	3

**Table 1:** Different continuous symmetries which may arise in  $S_3$ -3HDM as a result of the minimisation conditions. Each of the subsequent conditions in the constraints column is imposed over the previous ones. The number of additional unwanted Goldstone bosons is indicated in the last column.

Another important aspect of a model is to construct a realistic Yukawa Lagrangian. There are several possibilities to assign the  $S_3$  charges to fermions. When the singlet vev,  $w_S$ , is different from zero it is possible to couple fermions only to the  $h_S$  doublet in order to get realistic masses and mixing. In this case fermions are grouped into a trivial singlet representation. Such case would be identical to the SM-like structure with no specific restrictions on the Yukawa couplings. The other possibility is to assume that some of the fermions are assigned into the  $S_3$  doublet representations, e.g., a singlet-doublet representation with fermions transforming under  $S_3$  as:

$$\mathbf{2} : (Q_1 Q_2)^T, (u_{1R} u_{2R})^T, (d_{1R} d_{2R})^T \quad \text{and} \quad \mathbf{1} : Q_3, u_{3R}, d_{3R}.$$

It will not be always possible to have a vanishing vev and simultaneously get realistic quark masses and the Cabibbo–Kobayashi–Maskawa matrix when a non-trivial representation is considered.

We categorised all vacua in  $S_3$ -3HDM with at least a single vanishing vev. In Table 2 we list cases that could accommodate DM. As mentioned before, some of these cases will have additional unwanted Goldstone bosons, and as a result would require soft symmetry breaking terms, while other cases might not yield a realistic fermionic content. The unrealistic implementations from the point of view of the Yukawa Lagrangian are listed in the last column of Table 2.

Vacuum	vevs	$\lambda_4$	symmetry	# massless states	fermions under $S_3$
R-I-1	$(0, 0, w_S)$	$\sqrt{\phantom{x}}$	$S_3, h_1 \rightarrow -h_1$	none	trivial
R-I-2a	$(w, 0, 0)$	$\sqrt{\phantom{x}}$	$S_2$	none	unrealistic
R-I-2b,2c	$(w, \pm\sqrt{3}w, 0)$	$\sqrt{\phantom{x}}$	$S_2$	none	unrealistic
R-II-1a	$(0, w_2, w_S)$	$\sqrt{\phantom{x}}$	$S_2, h_1 \rightarrow -h_1$	none	trivial
R-II-2	$(0, w, 0)$	0	$S_2, h_1 \rightarrow -h_1, h_S \rightarrow -h_S$	1	unrealistic
R-II-3	$(w_1, w_2, 0)$	0	$h_S \rightarrow -h_S$	1	unrealistic
R-III-s	$(w_1, 0, w_S)$	0	$h_2 \rightarrow -h_2$	1	trivial
C-I-a	$(\hat{w}_1, \pm i\hat{w}_1, 0)$	$\sqrt{\phantom{x}}$	cyclic $\mathbb{Z}_3$	none	unrealistic
C-III-a	$(0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S)$	$\sqrt{\phantom{x}}$	$S_2, h_1 \rightarrow -h_1$	none	trivial
C-III-b	$(\pm i\hat{w}_1, 0, \hat{w}_S)$	0	$h_2 \rightarrow -h_2$	1	trivial
C-III-c	$(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$	0	$h_S \rightarrow -h_S$	2	non-trivial
C-IV-a	$(\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S)$	0	$h_2 \rightarrow -h_2$	2	trivial

**Table 2:** The  $S_3$  vacua that might accommodate DM due to a vanishing vev [12]. The “hat”,  $\hat{w}_i$ , denotes an absolute value and all  $w_i$  are real. In the second column vevs are listed in the irreducible representation. Symmetries in the fourth column refer to remnant symmetries explicit in the defining representation. For  $\lambda_4 \neq 0$  real vacua can at most break  $S_3 \rightarrow S_2$ . For  $\lambda_4 = 0$  the  $S_3$  symmetry can be fully broken by real vacua.

All in all, despite the variety of models, there are only three cases capable of accommodating a DM candidate and which do not require soft symmetry breaking and provide a realistic Yukawa sector. These cases are R-I-1  $(0, 0, w_S)$ , R-II-1a  $(0, w_2, w_S)$  and C-III-a  $(0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S)$ . The R-I-1 model was partially covered in Refs. [15]. The authors studied a more symmetric case— $S_3 \otimes \mathbb{Z}_2$ , which is actually  $O(2)$  symmetric since it was assumed that  $\lambda_4 = 0$ . Moreover, they lifted the mass degeneracy of the scalar states by introducing a soft symmetry breaking term. Another approach would be to notice that due to the loop effects there will be a mass splitting precisely due to the  $\lambda_4$  term. The other two implementations, R-II-1a and C-III-a, are discussed below.

#### 4. Numerical studies

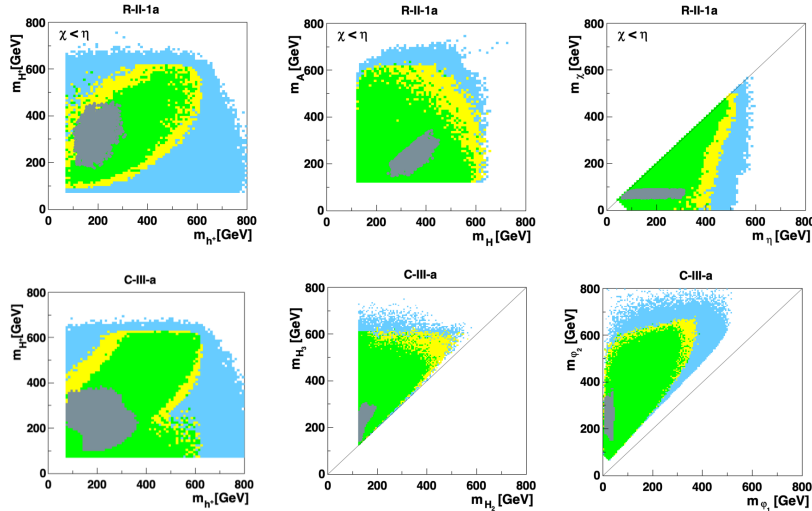
We analysed two cases numerically with vacua given by  $(0, w_2, w_S)$  in R-II-1a [4] and  $(0, \hat{w}_2 e^{i\sigma}, \hat{w}_S)$  in C-III-a [5]. The main difference between the cases is that in C-III-a there is a non-vanishing free phase  $\sigma$ , which is responsible for spontaneous CP violation [12]. Since vacua of the two cases differ by the  $\sigma$  phase one might expect that in the limit of  $\sigma \rightarrow 0$  C-III-a reduces to R-II-1a, which is not true [5]. The roman numerals of the models stand for the number of minimisation conditions. In C-III-a an additional constraint relates two couplings,  $\lambda_4$  and  $\lambda_7$ . As a result, the two cases cover different regions of parameter space. By observing the DM bands presented in Figure 1 it becomes obvious that the allowed DM mass regions differ for two cases.

Both R-II-1a and C-III-a were studied in terms of eight input parameters. We scanned over these parameters in order to identify the available parameter space. Several constraints were imposed:

- Cut 1: perturbativity, stability, unitarity checks, LEP constraints;
- Cut 2: SM-like gauge and Yukawa sector,  $S$  and  $T$  variables,  $\bar{B} \rightarrow X(s)\gamma$  decays;
- Cut 3: SM-like Higgs particle decays, DM relic density, direct searches;

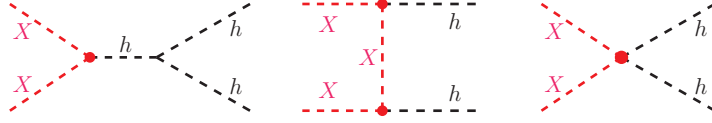
with each subsequent constraint being superimposed over the previous one. The applied numerical bounds are taken from the PDG [1]. We allow for a  $3\text{-}\sigma$  tolerance. In order to evaluate the Cut 3 constraints we used micrOMEGAs [16].

In the R-II-1a model due to no mixing between the inert neutral states either of the states could be a DM candidate. We found no data points satisfying all of the constraints when the state  $\eta$ , coming from  $h_1$ , with the mass-squared parameter  $m_\eta^2 \sim \lambda_4 v^2$  was associated with the DM candidate. The surviving parameter space of the two cases projected onto the allowed mass region for the DM candidate can be seen in Figure 1.



**Figure 2:** Scatter plots of masses that satisfy different sets of successive Cuts. Left column: the charged sector. Middle column: the active heavy neutral sector. Right column: the inert neutral sector. The blue region satisfies Cut 1. The yellow region accommodates a  $3\text{-}\sigma$  tolerance with respect to Cut 2, whereas the green region accommodates the  $2\text{-}\sigma$  bound. The grey region is compatible with Cut 1, Cut 2 and Cut 3.

One of the striking differences between the studied cases and the IDM is that there are no heavy DM candidates,  $m_{\text{DM}} \gtrsim 500$  GeV, in our models. We allowed for all of the scalars to be as heavy as 1 TeV. After applying Cut 1, theoretical constraints, the heaviest scalars are found to be around 800 GeV. The allowed parameter space further decreases after applying subsequent constraints. In the IDM, in the high-mass region, the observed relic density can be maintained by suppressing annihilation via intermediate neutral scalar bosons and into a pair of neutral scalar bosons, see Figure 3, while also requiring near mass degeneracy among the scalars coming from the inert sector. In both of the analysed cases it is not possible to maintain small portal couplings. In the R-II-1a case, in the SM-like limit, the portal coupling scales like  $\sim m_{\text{DM}}^2/v^2$ . In C-III-a it is not possible to have small portal couplings while also a mass gap between the neutral states of around 70 GeV develops for masses above 300 GeV.



**Figure 3:** Contribution to  $XX$  (particles of the inert sector) annihilation channels at high DM masses.

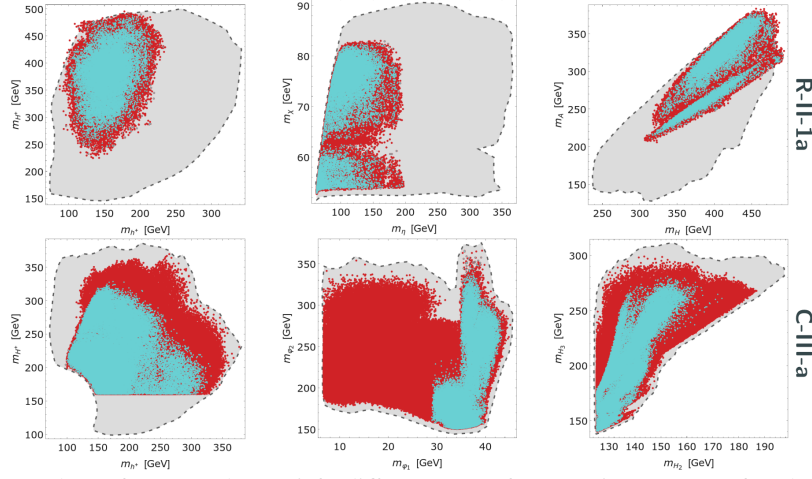
One of the most significant difference of the C-III-a case is that the allowed DM mass region is below 50 GeV. This region in other models is not consistent with fitting the relic density together with other relevant constraints, like direct DM detection. In C-III-a the primary decay channel for additional scalars is into states with the DM candidate. Such processes would be accompanied by large missing transverse momentum. The SM-like Higgs portal coupling can be very weak. This allows for the SM-like Higgs branching ratio to the DM scalar states to be really low, while also allowing for the direct DM searches to be satisfied.

After applying a selected set of constraints we determined possible DM mass ranges in R-II-1a and C-III-a. For R-II-1a we the allowed region is [52.5, 89] GeV and for C-III-a it is [6.5, 44.5] GeV.

During the preparation of the current review we performed an additional scan beyond Cut 1-3:

- LHC searches implemented in HiggsTools [17];
- Indirect DM detection constraints;
- Direct DM detection constraints from the currently running experiments;
- Significantly improved results on the SM-like Higgs boson decays into invisible states;

mainly motivated by the allowed parameter space of the C-III-a model. After applying the additional constraints the R-II-1a case remained almost unchanged; the allowed DM mass region was reduced from  $m_{\text{DM}} \in [52.5, 89]$  GeV to  $m_{\text{DM}} \in [53, 83]$  GeV [18]. On the other hand, the C-III-a implementation could be completely ruled out by the indirect DM detection searches, assuming some specific DM halo distribution profiles. If generous indirect DM detection bounds are applied the surviving region is reduced from  $m_{\text{DM}} \in [6.5, 44.5]$  GeV to  $m_{\text{DM}} \in [29, 44]$  GeV [18]. The allowed parameter space in terms of masses is shown in Figure 4.



**Figure 4:** Scatter plots of masses that satisfy different sets of successive Cuts. Left column: the charged sector. Middle column: the inert neutral sector. Right column: the active heavy neutral sector. The grey region satisfies Cut 3. The red region relies on the HiggsTools framework. The cyan region accommodates several constraints: indirect DM detection, currently running relevant direct DM detection experiments and assumes the branching ratio of the SM-like Higgs portal to DM to be within  $\text{Br}(h \rightarrow \text{inv.}) \leq 0.1$ .

## Acknowledgments

This work was partially supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the projects CFTP-FCT Unit UIDB/00777/2020, UIDP/00777/2020, CERN/FIS-PAR/0002/2021 and CERN/FIS-PAR/0008/2019, which are partially funded through POCTI (FEDER), COMPETE, QREN and EU. The work has also been supported by the FCT PhD fellowship with reference UI/BD/150735/2020.

## References

- [1] PARTICLE DATA GROUP collaboration, R. L. Workman et al., *Review of Particle Physics*, *PTEP* **2022** (2022) 083C01.
- [2] CMS collaboration, *A portrait of the Higgs boson by the CMS experiment ten years after the discovery*, *Nature* **607** (2022) 60–68, [2207.00043]; ATLAS collaboration, *A detailed map of Higgs boson interactions by the ATLAS experiment ten years after the discovery*, *Nature* **607** (2022) 52–59, [2207.00092].
- [3] R. Barbieri, L. J. Hall and V. S. Rychkov, *Improved naturalness with a heavy Higgs: An Alternative road to LHC physics*, *Phys. Rev.* **D74** (2006) 015007, [hep-ph/0603188]; L. Lopez Honorez, E. Nezri, J. F. Oliver and M. H. G. Tytgat, *The Inert Doublet Model: An Archetype for Dark Matter*, *JCAP* **0702** (2007) 028, [hep-ph/0612275]; Q.-H. Cao, E. Ma and G. Rajasekaran, *Observing the Dark Scalar Doublet and its Impact on the Standard-Model Higgs Boson at Colliders*, *Phys. Rev.* **D76** (2007) 095011, [0708.2939].
- [4] W. Khater, A. Kunčinas, O. M. Ogreid, P. Osland and M. N. Rebelo, *Dark matter in three-Higgs-doublet models with  $S_3$  symmetry*, *JHEP* **01** (2022) 120, [2108.07026].

- [5] A. Kunčinas, O. M. Ogreid, P. Osland and M. N. Rebelo, *Dark matter in a CP-violating three-Higgs-doublet model with  $S_3$  symmetry*, *Phys. Rev.* **D106** (2022) 075002, [2204.05684].
- [6] A. Belyaev, G. Cacciapaglia, I. P. Ivanov, F. Rojas-Abatte and M. Thomas, *Anatomy of the Inert Two Higgs Doublet Model in the light of the LHC and non-LHC Dark Matter Searches*, *Phys. Rev.* **D97** (2018) 035011, [1612.00511]; J. Kalinowski, W. Kotlarski, T. Robens, D. Sokolowska and A. F. Zarnecki, *Benchmarking the Inert Doublet Model for  $e^+e^-$  colliders*, *JHEP* **12** (2018) 081, [1809.07712].
- [7] B. Grzadkowski, O. M. Ogreid and P. Osland, *Natural Multi-Higgs Model with Dark Matter and CP Violation*, *Phys. Rev.* **D80** (2009) 055013, [0904.2173]; M. Merchand and M. Sher, *Constraints on the Parameter Space in an Inert Doublet Model with two Active Doublets*, *JHEP* **03** (2020) 108, [1911.06477].
- [8] V. Keus, S. F. King, S. Moretti and D. Sokolowska, *Dark Matter with Two Inert Doublets plus One Higgs Doublet*, *JHEP* **11** (2014) 016, [1407.7859]; V. Keus, S. F. King, S. Moretti and D. Sokolowska, *Observable Heavy Higgs Dark Matter*, *JHEP* **11** (2015) 003, [1507.08433]; A. Cordero, J. Hernandez-Sanchez, V. Keus, S. F. King, S. Moretti, D. Rojas et al., *Dark Matter Signals at the LHC from a 3HDM*, *JHEP* **05** (2018) 030, [1712.09598].
- [9] A. Cordero-Cid, J. Hernández-Sánchez, V. Keus, S. F. King, S. Moretti, D. Rojas et al., *CP violating scalar Dark Matter*, *JHEP* **12** (2016) 014, [1608.01673].
- [10] J. Kubo, H. Okada and F. Sakamaki, *Higgs potential in minimal  $S(3)$  invariant extension of the standard model*, *Phys. Rev.* **D70** (2004) 036007, [hep-ph/0402089]; T. Teshima, *Higgs potential in  $S_3$  invariant model for quark/lepton mass and mixing*, *Phys. Rev.* **D85** (2012) 105013, [1202.4528]; D. Das and U. K. Dey, *Analysis of an extended scalar sector with  $S_3$  symmetry*, *Phys. Rev.* **D89** (2014) 095025, [1404.2491].
- [11] A. Kuncinas, O. M. Ogreid, P. Osland and M. N. Rebelo, *Complex  $S_3$ -symmetric 3HDM*, 2302.07210.
- [12] D. Emmanuel-Costa, O. M. Ogreid, P. Osland and M. N. Rebelo, *Spontaneous symmetry breaking in the  $S_3$ -symmetric scalar sector*, *JHEP* **02** (2016) 154, [1601.04654].
- [13] E. Derman, *Flavor Unification,  $\tau$  Decay and  $b$  Decay Within the Six Quark Six Lepton Weinberg-Salam Model*, *Phys. Rev. D* **19** (1979) 317–329; E. Derman, H. S. Tsao,  *$SU(2) \times U(1) \times S(n)$  Flavor Dynamics and a Bound on the Number of Flavors*, *Phys. Rev. D* **20** (1979) 1207.
- [14] A. Kuncinas, O. M. Ogreid, P. Osland and M. N. Rebelo,  *$S_3$ -inspired three-Higgs-doublet models: A class with a complex vacuum*, *Phys. Rev.* **101** (2020) 075052, [2001.01994].
- [15] A. C. B. Machado and V. Pleitez, *A model with two inert scalar doublets*, *Annals Phys.* **364** (2016), 53–67 doi:10.1016/j.aop.2015.10.017, [1205.0995]; E. C. F. S. Fortes, A. C. B. Machado, J. Montaña and V. Pleitez, *Scalar dark matter candidates in a two inert Higgs doublet model*, *J. Phys.* **G42** (2015) 105003, [1407.4749].



- [16] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, *micrOMEGAs\_3: A program for calculating dark matter observables*, *Comput. Phys. Commun.* **185** (2014) 960–985, [1305.0237].
- [17] H. Bahl, T. Biekötter, S. Heinemeyer, C. Li, S. Paasch, G. Weiglein et al., *HiggsTools: BSM scalar phenomenology with new versions of HiggsBounds and HiggsSignals*, 2210.09332.
- [18] A. Kunčinas, O. M. Ogreid, P. Osland and M. N. Rebelo, *Revisiting two dark matter candidates in  $S_3$ -symmetric three-Higgs-doublet models*, in *8th Symposium on Prospects in the Physics of Discrete Symmetries*, 1, 2023. 2301.12194.