PROCEEDINGS OF SCIENCE

Planck formula for the gluon parton distribution in the proton

Loredana Bellantuono,[a] **Roberto Bellotti**[b] **and Franco Buccella**[c,*]

[a] *Dipartimento di Scienze Mediche di Base, Neuroscienze ed Organi di Senso, Universita'degli Studi di Bari Aldo Moro, Bari I-70124, Italy*
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Bari I-70125, Italy

[b] *Dipartomento Interateneo di Fisica, Universita'degli Studi di Bari Aldo Moro,Bari I-70126, Italy*
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Bari I-70125, Italy

[c] *INFN, Sezione di Napoli, Italy*

E-mail: loredana.bellantuono@bari.infn.it, roberto.bellotti@bari.infn.it, buccella@roma1.infn.it

Twentyone years after the proposal of a role of quantum mechanics statistics for the parton distributios and some years after the assumption that the boundary conditions for the DGLAP equations imply Fermi-Dirac functios for the quarks and a Planck function for the gluons, it is useful to recall the facts, which confirm the validity of the parametrization implied by those statements. In fact both at small and high x the uncertainties in the regions, where the experimental information is scarce, are smaller than the ones obtained by the standard polynomial parametrizations.

Corfu Summer Institute 2022 "School and Workshops on Elementary Particle Physics and Gravity",
28 August - 1 October, 2022
Corfu, Greece

*Speaker

PoS(CORFU2022)008

1. Introduction

For about a decade quarks [1] were considered mathematical objects to describe the consequences of $SU(3)$ flavor symmetry [2] [3] and its extension to $SU(6)$ flavor spin [4] [5], despite the prediction of the ratio of the magnetic moments, $-\frac{2}{3}$, [6] and the successful assumption that the commutators of the weak and electromagnetic currents, current algebra, were well described in terms of them [7] . The prediction of the ratio $\frac{G_A}{G_V}$ [8] [46] and the saturation of the commutators of the chiral $SU(3) \times SU(3)$ algebra within a set of hadron states [10] lead to the discovery of the generator of the transformation between costituent and current quarks [61] with the prediction of linear Regge trajectories [12] and of the signs and of the orders of magnitude of the contributions of the resonances for the processes $\pi + N \longrightarrow \pi + \Delta$ [13] [14] and of the $\rho - \pi$ and $\Delta - N$ mass differences [15] . The brilliant Veneziano formula [16] for the reaction $\pi + \pi \longrightarrow \pi + \omega$ encouraged to describe strong interactions in the framework of S matrix theory and the symmetric function for three quarks in a baryon did not encourage to consider quarks as real particles . The scale invariance of the strucure function in deep inelastic scattering [17] found at SLAC [18] lead Feynman to propose the parton model consisting in the hypothesis that at large values of Q^2, the transverse momentum transmitted by the incident lepton to the hadron, this particle behaves as an incoherent set of pointlike charged objects interacting elastically with the incident lepton with a given probability of carrying the percentage x of its momentum in the frame of reference of the final hadrons [19] .
The study of deep inelastic scattering induced by (anti)neutrinos lead to identify the charged objects with the quarks [20] .
A first attempt to describe scale invariance in the framework of quantum field theory was performed by relating the structure functions to the singularities on the lightcpne of the products of two currents [21] .
The study of the non abelian gauge theories, which imply a negative sign of the β function for the renormalization group equations [22] [23], lead to the proposal of quantum chromodynamics, QCD, as the quantum field theory of strong interactions with fundamental fields the quarks and the gauge bosons, the gluons, which were identified with the neutral particles carrying about half of the hadron momentum : the baryons singlets with respect to the gauge group $SU(3)$ color restored the antisymmetry for the quark wave function and the factor 3 was welcome to reproduce the production of hadrons in electron-positron collisions and the lifetime of $(\pi)^0$ [24] . QCD accounts for the confinement of the quarks ("infrared slavery") and for the scale invariance of the structure functions, which describe deep inelstic scattering ("asymptotic freedhom") .
The proton and the other baryons, which at small Q^2 behave as states with three quarks combined into a color singlet, at high Q^2 behave as an incoherent set of quarks, gluons and antiquarks with distributions, which obey the sum rules of the parton model as the condition that at high p_z :

$$\int_0^1 \Sigma_i x p_i(x) dx = 1 \tag{1}$$

where x is the fraction of the proton momentum carried by the parton i .

QCD implies logarithmic violations of scale invariance described by DGLAP [25] [26] [27] equations, which allow to deduce the parton distributions at a Q^2 larger than a sufficiently high Q_0^2

from the ones at Q_0^2 . The standard parametrization at Q_0^2 is :

$$Ax^B(1-x)^C P(x) \tag{2}$$

with the parameter A, B e C and the polynome P(x) depending on the parton and such a form is taken for the non polarized $q(x) = q^\uparrow(x) + q^\downarrow(x)$ and for the polarized $\Delta q(x) = q^\uparrow(x) - q^\downarrow(x)$ distributions .

Parton model and the consequent scale invariance hold for large values of Q^2 and $(p+q)^2 = M^2 + Q^2(\frac{1}{x} - 1)$ larger than M^2 and therefore the values $x = 0$ e $x = 1$ are exscluded as well as their neighboroods with amplitudes decreasing with Q^2 .

Therefore to fix the power behaviour around these points has not a strong motivation .

To fix the distributions at Q_0^2 one may be ispired by experiment, which suggests a role of quantum statistical mechanics .

2. The Parton Distributions Suggested by Quantum Statistical Mechanics

The experimental search of the parton distributions soon showed non trivial flavor properties for the antiquark sea and for the valence partons . In fact in the proton there is the isospin asymmetry related to the fact that $\bar{d}(x)$ is larger than $\bar{u}(x)$. The study of the ratio $\frac{F_2^n(x)}{F_2^p(x)}$ shows a fast decrease in the intermediate region of x, in the range (0.2, 0.5) and a slower decrease above 0.5 [28].

Both these facts may be related to the fact that in the proton there are two valence u quark and one valence d quark : in fact Pauli principle has been advocated [29] [30] to account for the isospin asymmetry in the proton sea and its role requires that quantum mechanical statistics plays a role, which implies in analogy with Fermi sphere that the $u(x)$ distribution is broader than the $d(x)$ distribution . The role of Pauli principle implies that the occupation numbers, which depend both on the flavor and the helicity of the quark, are not small .

Therefore one has to consider at the same time the unpolarized and polarized parton distributios given respectively by $p^\uparrow(x) + p^\downarrow(x)$ and $p^\uparrow(x) - p^\downarrow(x)$. A first attempt to relate the unpolarized and polarized distributions has been made in [**?**] with the assumption :

$$2u^\downarrow(x) = d(x) \tag{3}$$

which implies

$$u^\uparrow(x) - u^\downarrow(x) = u(x) - d(x) \tag{4}$$

and relates the contribution of the l. h. s. to $g_1^p(x)$ to the one of the r. h. s. to the difference $F_2^p(x) - F_2^n(x)$. After previous attempts to use Fermi-Dirac functions for the quarks and Bose-Einstein for the gluons [32], [33], [34], [35], a good description of several deep inelastic and Drell-Yan pair production data has been obtained by assuming [36] at $Q_0^2 = 4\frac{(GeV)^2}{c^4}$ chosen for the boundary conditions :

$$xq^h(x) = \frac{AX_q^h x^b}{(\exp\frac{x - X_q^h}{\bar{x}} + 1)} + \frac{\tilde{A}x^{\tilde{b}}}{(\exp\frac{x}{\bar{x}} + 1)} \tag{5}$$

$$x(\bar{q})^h = \frac{\bar{A}x^{2b}}{X_q^{-h}(\exp\frac{x+X_q^{-h}}{\bar{x}}+1)} + \frac{\tilde{A}x^{\tilde{b}}}{(\exp\frac{x}{\bar{x}}+1)} \tag{6}$$

$$xG(x) = \frac{A_G x^{b_G}}{(\exp\frac{x}{\bar{x}}-1)} \tag{7}$$

For the strange partons it has been assumed :

$$s(x) = \bar{s}(x) = \frac{\bar{u}(x)+\bar{d}(x)}{4} \tag{8}$$

The fermion distributions are given by the sum of a non diffractive term depending on the flavor and helicity of the quark and a diffractive term isoscalar, unpolarized and invariant with respect to C, in such a way to do not contribute to the sum rules for the first moments of the partons .
The diffractive term has a low x behaviour related to the one of the gluons corresponding to an infinite number of partons and its symmetry properties imply that only the non diffractive terms contribute to the quark number and to the Bjorken [37] sum rules . The potentials X_p of the valence quarks and of their particles are constrained by the equilibrium conditions proposed by Bhalerao and his collaborators [42] [43] [44] with respect to the processes, which are responsible for the DGLAP equations, the emission of a gluon by a fermion and the conversion of a gluon into a $q\bar{q}$ pair .
The important consequences are that a fermion and its antiparticle with opposite helicity have opposite potentials and the potentials of the gluons vanish and they are unpolarized and described by a Planck formula (Bose-Einstein with a vanishing potential) . As long for the fermions the isospin and spin asymmetries of the proton sea are related to the potentials of the valence quarks, obeying the conditions :

$$\Delta\bar{d}(x) < 0 < \Delta\bar{u}(x) < \bar{d}(x) - \bar{u}(x) < \Delta\bar{u}(x) - \Delta\bar{d}(x) \tag{9}$$

which follow from the inequalities :

$$X_u^{\uparrow} > X_d^{\downarrow} > X_u^{\downarrow} > X_d^{\uparrow} \tag{10}$$

implied by he quark number and the Bjorken [37] sum rules, which require :

$$u^{\uparrow} > d_d^{\downarrow} > u^{\downarrow} > d^{\uparrow} \tag{11}$$

The first two inequalities of Eq.(9) are confirmed by the asymmetries for the production of the charged weak bosons at RHIC [45] [46] with polarized particles .
The third one agrees with the isospin asymmetry implied by the defect [47] in the Gottfried sum rule [48] , while the fourth one requires a more precise measurement of the isovector spin asymmetry, for which there is uncertainty on the value [**?**] .
It is interesting to consider the entropy, which for a fermion is given by :

$$S(q) = \Sigma_i -n_i \ln n_i - (1-n_i)\ln(1-n_i) \tag{12}$$

vanishing for $n_1 = 0$ or 1, which is the case for $\bar{x} = 0$, showing the validity of the third principle of thermodynamics . The condition of equilibrium with respect to the elementary processes of the

DGLAP equations corresponds to the maximum of the entropy, which is reached at the separation between the non-perturbative and the perturbative regimes, which we assumed to be $Q_0^2 = 4\frac{(GeV)^2}{c^4}$
.
The "ad hoc" factors X_q^h and $\frac{1}{X_q^h}$ have been introduced to comply with data, which have been successfully described in terms of the temperature $\bar{x}$, the four "potentials" for the valence quarks u and d with both helicities, the two exponents b and b_G and the four factors, A, $\bar{A}$, A_G and $\tilde{A}$, constrained by the moment sum rule and by the quark number sum rules .
The values of the parameters have been $\bar{x} = 0.099$, $X_u^{\uparrow} = 0.461$, $X_d^{\downarrow} = 0.301$, $X_u^{\downarrow} = 0.299$, $X_{=}^{\uparrow}0.225$, $b = 0.41$, $b_G = 0.747$, $A = 1.75$, $\bar{A} = 1.91$, $A_G = 14.3$ and $\tilde{A} = 0.183$.
The statistical approach implies a common Boltzmann behaviour exp $\frac{-x}{\bar{x}}$ for x larger of the highest "potential" , $Xu^{\uparrow} = 0.46$. in good agreement with experiment.
The predictions for the polarized structure functions of the nucleons measured after [36] have been shown in agreement with experiment [49] [50] .
While the behaviour of the ratio $\frac{\bar{d}(x)}{\bar{u}(x)}$ in a first experiment [38] agreed at $x = 0.18$ with the value predicted in [34], in a second one only up to a certain value of x [39] with [**?**] ; a third experiment [40], more precise, extended the agreement to all the x measured .

3. The Extension to the Transverse Momenta

To account for the "ad hoc" factors previously mentioned one considered the transverse degrees of freedhom and their form coming from the sum rule proposed for the transverse energy , defined as the difference between the energy and the longitudinal component of the momentum [51] .
For the hadron of the target the transverse energy is given by $P_0 - P_z$, approximately equal at large P_z to $\frac{M^2}{2P_z}$.
For a massless parton with the longitudinal component of the momentum xP_z and the transverse p_T the transverse energy is given by :

$$\frac{p_T^2}{p_z + \sqrt{p_z^2 + p_T^2}} = \frac{p_T^2}{P_z(x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}})} \tag{13}$$

where P_z is the momentum of the initial hadron in the reference system of the final hadrons and is given, neglecting terms in $(xM)^2$, by :

$$P_z^2 = \frac{Q^2}{4x(1-x)} \tag{14}$$

Multiplying $\times 2P_z$ we obtain a sum rule with M^2 in the right hand side .
The sum rule for the transverse energy fixes the dependence on P_T of the transverse distribution, which is given by :

$$\frac{2}{[\exp(\frac{(p_T)^2}{x+\sqrt{x^2+(\frac{p_T}{P_z})^2}(\mu)^2} - Y_q^h) + 1]}. \tag{15}$$

where Y_q is the "transverse potential and μ has the dimension of a mass and it is fixed by the sum rule for the transverse energy .
The transformation :

$$p_T^2 = \frac{\mu^2 \eta (x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}})}{2} \tag{16}$$

gives rise to the integral in the variable η, which has the value, neglecting terms proportional to the ratio $(\frac{(\mu)^2}{Q^2})$:

$$\ln [(1 + \exp (Y_q^h - \frac{k}{x})] \tag{17}$$

where $k = (\frac{m_q}{\mu})^2$ and may be neglected for the lightest partons u and d, but not for the strange partons . For the non diffractive contribution of the non valence fermion partons $Y_q^h = 0$, which implies for the lightest partons in the mesons the value $\ln 2$ for the integral defined in Eq.(17), while the non diffractive term for the strange partons is reduced at low x and for the strange valence partons the factor in Eq.(17) takes the value $\ln 2$, when $x = \frac{k}{Y_s^h}$.
Therefore for the light valence partons one should have instead of the factors AX_q^h the factors $A' \ln (1 + \exp Y_q^h)$ and one could recover the form proposed in [36] for the valence quarks simply assuming the proportionality between X_q^h and $\ln (1 + \exp Y_q^h)$. Indeed in [52], where both X_q^h and Y_q^h are fixed by comparing with the fermion distributions proposed in [53] the proportionality holds with a good approximation .
For the non diffractive part of their antiparticles one has a slight change, since $\ln (1 + \exp Y_q^h) \ln [1 + \exp (-Y_q^h)]$ is not costant, but the product gets its maximun, $(\ln 2)^2$, at $Y_q^h = 0$ and the more relevant change concerns $\bar{u}^{\downarrow}$, which has a small non diffractive contribution .
There is an important difference with respect to the standard form $Ax^B(1-x)^C P(x)$ at high x, where the different parton distributions are fixed by the exponent C, which comes out different for the different valence quarks with the conseguence that the limit $\frac{d(x)}{u(x)}$ for $x \to 1$ is 0 or infinity. In the fit by Hera [53] the parameter C is larger for u than for d, while for the sea is still smaller with the consequence to be dominant in that limit.
To agree with the experimental behaviour of the ratio $\frac{d(x)}{u(x)}$ the "ad hoc" factor $(1 + 9.7x^2)$ is introduced for the parton u .
Instead for the statistical approach above the highest "potential", $X_u^{\uparrow} = 0.461$ all the distributions approach the universal Boltzmann behaviour proportional to $\exp (\frac{-x}{\bar{x}})$, which describes for a larger range the gluon and te diffractive distributions as a consequence of their vanishing potential .
For the valence partons the costant, which multiples $\exp (\frac{-x}{\bar{x}})$ at high$x$, depends on their "potential" .
An important feature of the statistical approach is to describe at the same time the unpolarized and the polarized parton distributions .

4. Comparison with HERA and NNQCD

When HERA presented the parton distributions derived by the combined fit to H1 and ZEUS data, Jacques Soffer immediately realized their similarity with the ones found in [36] .

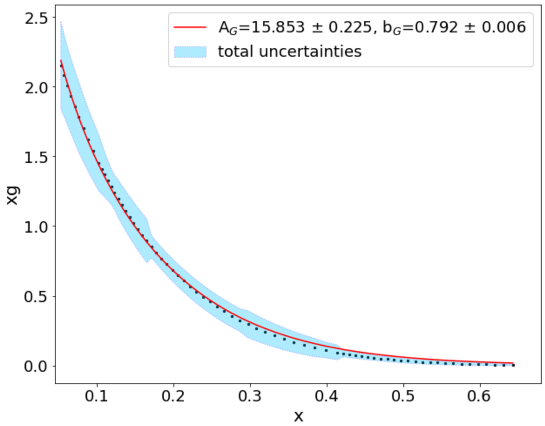

Figure 1: The red curve represents the best fit of the gluon momentum distribution $xg(x)$ obtained in the ATLAS experiment, performed using the functional form in Eq.(9), with A_G and b_G as free parameters, and $\bar{x} = 0.099$. The dots correspond to the experimental points, and the (cyan) shaded area to their uncertainty.

Therefore in [52] the free parameters introduced in [36] were fixed by minimizing the difference from the unpolarized distributions proposed by HERA and from the polarized in [36], which were found in very good agreement with experiment .
Instead of the "ad hoc" factors X_q^h and $\frac{1}{X_q^h}$ the factors $\ln(1 + \exp Y_q^h)$ and $\ln(1 + \exp -Y_q^h)$ coming from the extension to the transverse momenta were considered .
In Table 1 we compare the "temperature" and the "potentials" found in [36] with the ones obtained in [33] and in [52] from the comparison with HERA.

Parameter	[36]	[33]	[52]
$\bar{x}$	0.099	0.090	0.102
X_u^+	0.461	0.475	0.446
X_u^-	0.298	0.307	0.297
X_d^+	0.228	0.245	0.222
X_d^-	0.302	0.309	0.320

Table 1: Values of the statistical model parameters found in previous works. The temperature $\bar{x}$ is involved in both the fermion and gluon distributions. The "potentials" X_u^+, X_u^-, X_d^+ and X_d^- determine the non-diffractive parts of the fermion distributions.

Also the proportionality between X_q^h and $\ln(1 + \exp Y_q^h)$ is very well respected .
Instead for the gluons the agreement holds up to about $x = 0.2$, while above the different parametrization lead to a faster decrease for the distribution proposed by HERA . The comparison was repeated [41] with the distributions found by [55] better in agreement with [52] than with [53] . More recently the Planck formula for the gluons was compared [56] with the distribution found by ATLAS [57] and the very good agreement shown in Fig. 1 is obtained with the same $\bar{x}$ and very similar values for A_G and b_G found in [36].

5. The Parton Distributions of the Charged Mesons

By studyng the production of Drell-Yan pairs and J/ψ particles in the scattering of charged pions and kaons on nuclear targets, one may reach information on the parton distributions of the incident particles in the framework of the statistical approach . The increasing ratio at high x of the Drell-Yan pairs produced by negative pions or antiprotons on nuclear targets may be easily explained by a large value of the "potentials" of the valence partons in the charged pions .
In fact a description of the Drell-Yan pairs produced in $(\pi)^-$ nuclei reactions are described with a high value of $X_{\bar{u}} = 0.75$, and with the value of the "temperature", $\bar{x} = 0.102$, [59] near to the value found for the nucleons, 0.099 .
The ratios for the J/ψ and Drell-Yan production by negative kaon or pion scattering on nuclei have a similar behaviour, near 1 up to a certain x and decreasing above . This may be reproduced with a distribution of $\bar{u}(x)$ softer in the negative kaon than in the pion [**?**] .
This property may be understood, since the valence strange parton are expected to be rare at small x as a consequence of their mass and to obey the quark number sum rule should be harder than the $\bar{u}$ and take a larger percentage of the kaon momentum, while in the pion isospin symmetry implies that the two valence partons have the same distribution .
In the statistical approach one can describe the ratios with a smaller potential for the non diffractive term of the $\bar{u}$ in the negative kaon than in the negative pion .
Also the equilibrium condition implies that the non diffractive terms of the antiparticles of the two valence partons are negligible, because their potentials, opposite to the ones of the valence partons, are very negative . The quark number sum rule implies that the first moments of the valence partons in the mesons are almost equal, very near to 1 . This leads to the intriguing property that the non diffractive terms of $\bar{u}(x)]$ in K^- and $(\pi)^-$ limit the same area : the first, smaller at high x, should cross the second at a certain x and become higher below . To keep the ratio for the physical processes studied near to one this behaviour should be compensated by a gluon distribution for the kaon smaller than the one for the pion at small x and larger at high x . So we expect in the kaon softer $\bar{u}$ and harder gluons than in the pion [**?**] .

6. Conclusion

The proposal that the boundary conditions at $Q_0^2 = 4\frac{(GeV)^2}{c^4}$ for DGLAP equations are Fermi-Dirac functions for the quarks and a PlancK formula for the gluons allows to make many predictions in agreement with experiment and to write both the unpolarized and polarized distributions in terms of few parameters, which are rather stable with respect to the comparison with new data .
The degeneracy of the gas of the valence partons realizes the idea proposed in [29] and [30] that Pauli principle accounts for the isospin asymmetry in the proton sea.
In the phase transition from $Q^2 = 0$ to the deep inelastic regime the DGLAP equations may be applied, when the narrowing of the distributions implied by them is consistent with the increase of the available phase space and this happens at a certain Q_0^2, which a posteriori is around the value chosen in [36] . As it happened for the transformation between constituent and current quarks [60] [**?**] the transverse degrees of freedom for the constituents of a hadron with large P_z play an important role [62] [51] .

References

[1] Zweig, *Cern Preprint*, bf 8419/TH.412 (1964)

[2] M. Gell-Mann, *A Theory of Strong Interaction Symmetry*, bf CTSL-20 (1961), Unpublished

[3] Y . Neeman, *Nucl. Phys.* **26** (1961) 222

[4] B. Sakita, *Phys. Rev.* **B136** (1964) 1756

[5] F. Gursey and L. A. Radicati, *Phys. Rev. Lett.* **13** (1964) 173

[6] M. B. Beg, Lee and A. Pais, *Phys. Rev. Lett.*, **13** (1964) 656

[7] M. Gell-Mann, *Physics* bf 1 (1964) 63

[8] W. I. Weissberger, *Phys. Rev. lett.* **14** (1965) 1947

[9] S. L. Adler, *Phys. Rev. lett.* **14** (1965) 1951

[10] R. Gatto, L. Maiani and G. Preparata, *Phys. Rev.* **B140** (1965) 579

[11] F.Buccella, E. Celeghini, H. Kleinert, C. A. Savoy and E. Sorace, *Nuovo Cimento* **69** (1970) 133

[12] F. Buccella; E. Celeghini and C. A. Savoy, *Nuovo Cimento* bf 7A (1972) 281

[13] F. J. Gilman, M. Kugler and S. Meshkov, *Phys. Lett.* **B45** (1973) 481 and *Phys. Rev.* **D9** (1974) 715

[14] D. Faiman and J. Rosner, *Phys. Lett.*, **B45** (1973) 357

[15] F. Buccella, F. Nicolo' and A. Pugliese, *Lett. al Nuoco Cimento* bf 8 (1973) 244

[16] G. Veneziano *Nuovo Cimento* **57** (1968) 190

[17] J. D. Bjorken and E. A. Paschos, *Phys.Rev.* **D1** (1970) 3151

[18] D. H. Coward et al., *Phys. Rev. Lett.* **20** (1968)
E. D. Bloom et al., *Phys. Rev. Lett.* **25** (1969)
M. Briedenbach et al., *Phys. Rev. Lett.* **25** (1969) 935

[19] R. P. Feynman, *Photon-Hadron Interactions* (Taylor and Francis, 1972)

[20] C. H. Llewellynn Smith, *Phys. Rev.* **C3** (1964) 264

[21] R. Brandt and G. Preparata, *Phys. Rev.* **25** (1970) 1530 and *Nucl. Phys.* **B57** (1972) 541
G. Preparata *Rivista del Nuovo Cim.* **3** (1973) 180

[22] H. D. Politzer, *Phys. Rev. Lett.* **30** (1973) 1346

[23] D. J. Gross and F. Wilczek,*Phys. Rev. Lett.* **30** (1973) 1343

[24] H. Fritzsch, M. Gell-Man and H. Leutwyler, *Phys. Lett.* **B47** (1973) 365

[25] V. N. Gribov and L. N. Lipatov, *Sov. Nucl.Phys.* **15** (1972) 438,
L. N. Lipatov, *Sov. J. Nucl.Phys.*, **20** (1975) 94

[26] G. Altarelli and G. Parisi, *Nucl.Phys.* **B126** (1977) 298

[27] Y. Dockshitzer, *J. Exp. Theor. Phys.*, **46** (1977) 1946

[28] T. Sloan, G. Smadja and R. Voss, *Phys. Rev.* **162** (1988) R45

[29] A. Niegawa and K. Sasaki, *Progr. Theor. Phys.* **54** (1975) 192

[30] R. P. Feynman and J. D. Field, *Phys. Rev.* **D15** (1977) 2590

[31] F.Buccella and J. Soffer, *Mod.Phys. Lett.* **A8** (1993) 225

[32] C. Bourrely, F. Buccella, G. Miele, G. Migliore, J. Soffer and V. Tibullo, *Z. Phys.* bf C62 (1994) 431

[33] C. Bourrely and J. Soffer,*Phys. Rev.* **D51** (1995) 2108 and *Nucl. Phys.* **B445** (1995) 341

[34] F.Buccella, G. Miele, G. Migliore and V. Tibullo, *Z. Phys.* **C68** (1995) 631

[35] F.Buccella, G. Miele and N. Tancredi, *Progr. Theor. Phys.* **96** (1996) 749

[36] , C. Bourrely, F. Buccella and J. Soffer, *Eur. Phys. J.* **C23** (2002) 187

[37] J. Bjorken, *Phys. Rev.* **D1** (1970) 376

[38] A. Baldi et al. (NA51 Collaboration), *Phys. lett.* **B332** (1994) 244

[39] (FNAL, Nusea Collaboration), E. A Hawker et al. *Phys. Rev. Lett.* **80** (1998) J. C. Peng et al., *Phys. Rev.* **D58** (1998) 092004 R. S. Towell et al., *Phys. Rev.* **D64** (2001) 0552

[40] J. Dove (Sea Quest Experiment), *PoS.* **DIS2019** (2019) 011

[41] F.Buccella, S. Sohaily and F.Tramontano *J. Stat. Mech : Theory Exp.* **2019** (2019) 073302

[42] R. S. Bhalerao, *Phys. Lett.* **B380** (1996) 1 *Eur. Phys. J.* **C41** (2005) 327

[43] R. S. Bhalerao, N. G. Keipar and G. Ram, *Phys. Lett.* **B476** (2000) 285

[44] R. S. Bhalerao, *Phys. Rev.* **C63** (2001) 025208

[45] L. Adamczyk et al., *Phys. Rev. Lett.* **C113** (2014) 072301

[46] J. Adam et al., *Phys. Rev.* **CD09** (2019) 051102)

[47] M. Arneodo et al. (NMC collaboration), *Phys. Rev.* **D50** (1994) R1
P. Amaudruz et al., *Phys. Rev. Lett.* **66** (1991) 2712 and *Nucl. Phys.* **B371** (1995) 3

[48] K. Gottfried *Phys. Rev. Lett.* **18** (1967) 1174

[49] C. Bourrely, F. Buccella and J. Soffer, *Mod. Phys. Lett.* **A18** (2003) 771

[50] C. Bourrely, F. Buccella and J. Soffer, *Eur. Phys. J.* **C41** (2005) 327

[51] C. Bourrely, F. Buccella and J. Soffer, *Mod. Phys. Lett.* **A18** (2006) 143 and *Int. Jour. of Mod. Phys.* **28** (2013) 13500

[52] F.Buccella and S. Sohaily *Mod. Phys. Lett.* **A30** (2015) 1550203 .

[53] F. D. Aaron (HERA Collaboration), *JHEP.* **109** (2010) 1001

[54] C. Bourrely and J. Soffer, *Nucl. Phys.* **A941** (2015) 307

[55] R. D. Ball et al. (NNQCD Collaboration) *Eur. Phys. J.* **C77** (2017) 663

[56] L. Bellantuono, R. Bellotti and F. Buccella, arXiv : 2201.07640 [hep-ph] (to appear in MPLA)

[57] G. Aad et al. (ATLAS Collaboration), *Eur. Phys. Jour.* **C82** (2022) 438

[58] C.Bourrely, F. Buccella and J. C. Peng, *Phys. Lett.* bf B213 (2021) 136021

[59] C.Bourrely, F. Buccella, W. C. Cheng and J. C. Peng, (in preparation)

[60] C. Boldrighini, F. Buccella, E. Celeghini, E. Sorace and L. Triolo, *Nucl. Phys.* bf B22 (1970) 651

[61] F. Buccella, E. Celeghini, H. Kleinert, C: A. Savoy and E. Sorace, *Nuovo Cimento*, bf 69A (1970) 133

[62] H. J. Melosh, *Phys.. Rev.* bf D9 (1975) 1995