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High-Energy and Soft-Collinear Resummation for Jet Production at the LHC

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We discuss the progress in the development of HEJ+Pythia framework, which accounts for both the high-energy, and soft-collinear logarithms as implemented in the *High Energy Jets* partonic Monte Carlo, and Pythia8, respectively. The method to combine the two all-order descriptions point-by-point in phase-space is presented, and we verify the validity of the framework through comparisons of predictions produced with this merging scheme and ATLAS data.

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1. Introduction

Most of the cross-sections and distributions that are measured with high precision at the Large Hadron Collider (LHC) are predicted well by state-of-the-art fixed-order calculations, which typically include next-to-next-to-leading order (NNLO) corrections for the majority of processes of interest and next-to-next-to-next-to-leading order (N^{3}LO) for some low-multiplicity processes [1]. However, it is well-known that fixed-order computations are not entirely sufficient to deliver precise theoretical predictions and that in certain regions of phase space, the convergence of the perturbative series relies on all-order input.

For instance, the analysis of collider data collected from both the Tevatron [2] ($\sqrt{s}$ = 1.96 TeV) and the LHC [3–8] ($\sqrt{s}$ = 7, 8 TeV) shows that including terms beyond fixed order is necessary when describing observables in processes that involve at least two jets in the phase-space region where the partonic centre-of-mass energy $\sqrt{\hat{s}}$ is significantly larger than the typical transverse momentum scale $k_\perp$, $\sqrt{\hat{s}}/k_\perp > 5$. The large corrections in this region of phase-space, known as the *high-energy* or *multi-Regge kinematic* (MRK) limit, are described by the theory of Balitsky-Fadin-Kuraev-Lipatov (BFKL) [9–12]. The high-energy limit for a scattering of $2 \to n$ partons with momenta $p_a, p_b \to p_1, p_2 \ldots, p_{n-1}, p_n$ is defined by a strong ordering between the rapidities of each parton, denoted by y_i:

$$y_1 \gg y_2 \gg \cdots \gg y_{n-1} \gg y_n, \quad p_{\perp,i} \approx k_\perp \quad \forall i \in \{1, 2, \cdots, n-1, n\}. \tag{1}$$

In this case, large logarithms $\ln(\hat{s}_{ij}/p_{\perp,i}p_{\perp,j})$ corresponding to a large rapidity span between partons $\Delta y_{i,j} \equiv |y_i - y_j|$ appear to all orders in perturbation theory [13] and need to be re-summed. The resummation of high-energy logarithms is implemented in the framework of the *High Energy Jets* (HEJ) [14–19] Monte Carlo. HEJ applies the *leading logarithmic* (LL) accurate high-energy corrections to processes which at tree-level contribute to leading and subleading FKL configurations [20] and performs leading order matching. The HEJ framework currently contains description of logarithmic corrections to the perturbative series in α_s to the processes $pp \to hjj$, $pp \to W^{+/-}(\to l, \nu)jj$ and $pp \to Z(\to l^+l^-)jj$, as well as recently $pp \to hj$ [21]. Progress towards next-to-leading logarithmic (NLL) accuracy in HEJ was discussed during this conference series [22].

An important application of the HEJ formalism is the study of the production of a Higgs boson in association with dijets that have a large rapidity span. In this instance, the stringent experimental cuts select as the region of interest the part of the cross-section where large logarithmic corrections to all orders in perturbation theory appear. As is depicted in figure 1, reproduced from [23], these corrections are relevant in describing the QCD background for the study of weak boson fusion (WBF) at large dijet invariant masses $m_{j_1j_2}$ [24–26]. In this case, $m_{j_1j_2} > 400$ GeV and corrections in logarithms of $m_{j_1j_2}/k_\perp$ appear to all orders in the strong coupling, α_s, expansion, and need to be controlled at every order to enable precise determinations of WBF, as well as to investigate the CP-properties of the Higgs boson coupling to gluons [25–27]. The corrections described by HEJ have significant impact precisely in the signal region [23].

However, the high-energy limit is not the only source of large logarithms which can spoil the convergence of the perturbative series. For instance, many experimental analyses use selection cuts where the *parton shower* effects play a significant role. This is for example the case when the

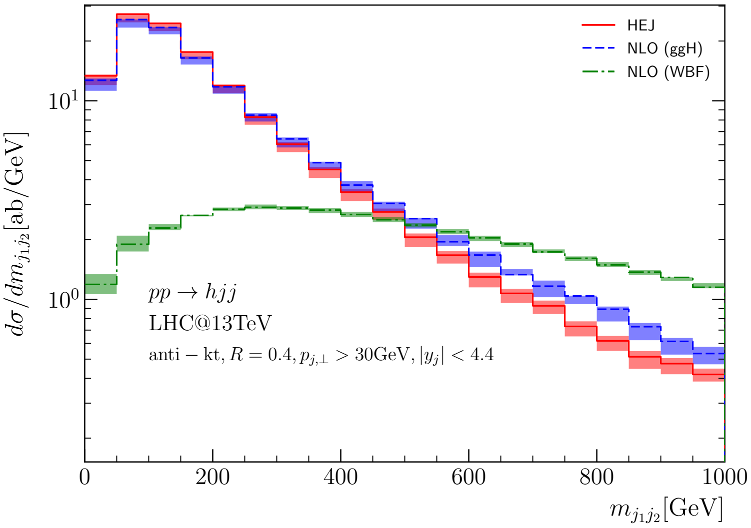

Figure 1: Plot showing the spectrum of the invariant mass between the two hardest jets in $pp \to hjj$ process. The WBF and gluon fusion channels are calculated at next-to-leading order and within HEJ framework. Plot reproduced from [23].

jets are required to be narrow, with $R = 0.4$, but not particularly hard, with $k_\perp > 30$ GeV. In this limit, the soft and collinear parton splittings become important and a different class of large double logarithms emerge. These typically contain ratios of transverse energy scales and may be recast into the product of a soft logarithm and a collinear logarithm, which diverge when partons in the event have low transverse momenta or have small angular separation, respectively.

In order to improve the theoretical predictions, the *inclusive* fixed-order events are typically *merged* with parton shower resummation. Several procedures for merging fixed-order events with parton showers exist including CKKW-L [28, 29] merging for input events accurate to leading order. Matching of parton shower events to higher fixed-orders in perturbation theory can be achieved using methods such as MC@NLO [30] and POWHEG [31]. MENLOPS [32, 33] and UNLOPS [34, 35] are further generalisations of next-to-leading order (NLO) merging. The result of these procedures are *exclusive* showered events containing description of the soft and collinear splittings to all-orders in the strong coupling expansion.

Thus far, we have discussed strategies to incorporate into the theoretical prediction only one source of possible all order corrections, such as high-energy or soft-collinear radiation, at a time. In general, there will exist regions of phase-space where different all-order effects play a role and their individual contribution is not straightforward to isolate in observables of interest. Therefore, it becomes desirable to combine the all-order high-energy resummation with that of the all-order parton shower.

The idea of merging the high-energy corrections to the hard scattering of dijet processes with parton shower resummation has been introduced previously in [36] and later also implemented in Pythia [37]. However, the results in these papers include only the merging up to the first parton shower emission beyond the high-energy input. They also do not take into account parts of the cross-section included in HEJ through the fixed-order component (i.e. the *non*-HEJ-*resummable* part of the cross-section [20]). This proceeding is based on the publication [38], where we present the implementation of a new procedure for merging the *exclusive* high-energy resummation performed by the HEJ framework with the Pythia8 [39] *exclusive* parton shower resummation. This *exclusive-exclusive* merging procedure builds significantly on previous works by allowing for unlimited number of Pythia8 emissions in the event history of HEJ and systematically removing any double counting between the two. This procedure is implemented in the HEJ+Pythia software [38, 40, 41].

In the following, we discuss the merging procedure and present results for n-jet processes. For predictions including electroweak bosons, see [38].

2. Merging Procedure

In this section we outline the main steps and considerations of the merging procedure, where we cover the phase-space twice: first with HEJ and then with Pythia, but subtracting what has already been performed in the first covering. For a complete discussion of the subtleties arising in this subtraction procedure we direct the reader to [38, 41].

To produce resummed predictions which contain both the high-energy component as implemented in the HEJ framework and the soft-collinear contributions from Pythia, we express the resummation performed in HEJ in the language of the parton shower. To achieve this, we define a splitting kernel which corresponds to the matrix elements of HEJ, which are valid in the high-energy limit, as shown below.

We begin with QCD, where the splitting functions are obtained by [20, 42]:

$$dk_\perp^2 dz \int d\phi \frac{1}{16\pi^2} \frac{\left|\mathcal{M}^{n+1}\right|^2}{\left|\mathcal{M}^n\right|^2} \sim \frac{dk_\perp^2}{k_\perp^2} dz \frac{\alpha_s}{2\pi} P(z), \tag{2}$$

where $\mathcal{M}^n$ is a usual QCD matrix element with n partons, $k_\perp$ is the transverse momentum of the emitted parton, z is its longitudinal momentum fraction, and ϕ is the angle with respect to the emitter.

The HEJ splitting function is defined analogously, where the only change is that the full QCD matrix elements are substituted by the matrix elements of HEJ valid in the high-energy limit, these are denoted with a superscript HEJ:

$$P^{\text{HEJ}} = \frac{1}{2} \frac{1}{16\pi^2} \frac{\overline{\left|\mathcal{M}^{n+1}_{\text{HEJ}}\right|^2}}{\overline{\left|\mathcal{M}^{n}_{\text{HEJ}}\right|^2}}. \tag{3}$$

The additional factor of $1/2$ in the splitting probability of HEJ originates in the treatment of colour configurations. Namely, we take into account two possible colour configurations when inserting an additional particle in the high-energy limit [43, 44]. Each is weighted equally in the HEJ method. P^{HEJ} implicitly contains the quantities in Eq. (2) which are marked in blue.

Similarly to the approach adopted in [37], the method presented here borrows from CKKW-L merging and introduces a procedure where the parton shower emissions are vetoed according to the probability that HEJ has already produced these emissions:

$$\mathcal{P}^{\text{veto}} = \frac{P^{\text{HEJ}}}{P^{\text{Pythia}}} \cdot \Theta(P^{\text{Pythia}} - P^{\text{HEJ}}) + 1 \cdot \Theta(-P^{\text{Pythia}} + P^{\text{HEJ}}). \tag{4}$$

The P^{Pythia} in the above equation are the Altarelli-Parisi splitting kernels which have been weighted by $\alpha_s/2\pi k_\perp^2$. To correctly reproduce the backwards DGLAP evolution, in the instance of an emission from the initial state $i \longrightarrow jk$, the splitting kernel must be reweighted by the ratio of parton distribution functions (PDFs):

$$P \longrightarrow P \cdot \frac{x_i f_i(x_i, \mu_F^2)}{x_j f_j(x_j, \mu_F^2)}, \tag{5}$$

where $f_i(f_j)$ is the PDF at evaluated at energy fraction $x_i(x_j)$ and the relevant factorisation scale, denoted by μ_F. The overall effect of the described procedure is that trial emissions with $P^{\text{Pythia}} < P^{\text{HEJ}}$ are vetoed with 100% probability. Therefore, the shower emissions are generated with a modified Sudakov form factor:

$$\Delta^S(k_{\perp,i}^2, k_{\perp,i+1}^2) = \exp\left\{-\int_{k_{\perp,i+1}^2}^{k_{\perp,i}^2} dk_\perp^2 dz\, \Theta\left(P^{\text{Pythia}} - P^{\text{HEJ}}\right)\left[P^{\text{Pythia}}(k_\perp^2, z) - P^{\text{HEJ}}(k_\perp^2, z)\right]\right\}. \tag{6}$$

Effectively, the modified Sudakov form factor removes from the Pythia splitting kernel the HEJ equivalent, avoiding double counting.

Practically, for the leading order input events which were dressed by HEJ (i.e. the HEJ-resummable input events), the so-called *histories* are constructed. Taking the example of inclusive dijet production, histories are sequences of splittings that connect the $2 \longrightarrow 2$ leading order scattering process to the $2 \longrightarrow n$ input event. The splittings are ordered in the Pythia shower evolution variable, $k_\perp^2$. Each history is assigned a weight related to the product of HEJ splitting probabilities, and one is selected according to their weights. These configurations are merged according to the following procedure:

1. Beginning with $i = 0$, we start the trial shower with an emission from state i in the history at the scale $k_{\perp,i}^2$.
   (a) if $k_{\perp,i+1}^2 < k_{\perp,i}^2$, we continue to the next state in the history, and we set $i \longrightarrow i+1$. Then, we return to step 1. In case that this is the original input event, we proceed to step 2.
   (b) if $k_{\perp,i+1}^2 > k_{\perp,i}^2$, the emission is vetoed with probability $\mathcal{P}^{\text{veto}}$ as given in Eq. (4).
      i. If the emission is vetoed, we generate a new trial emission at the current scale, and return to step 1a.
      ii. If the emission is not vetoed, we keep the trial emission, and append it to each subsequent node in the history $i+1, i+2, \ldots$, taking care of the recoils. We then generate a new trial emission from the current scale and go back to step 1a.
2. A final trial emission is performed. This is to account for the case where no emissions have been appended due to the vetoing procedure. We then exit the merging after an emission is accepted according to the veto probability in 1b. We then use the updated event record (with any appended emissions) to initiate the later shower.
3. We now have the input HEJ event dressed with the additional shower emissions at all stages of the history, i.e. the merged event. We continue the parton shower evolution using the merged event and we veto each trial emission with probability $\mathcal{P}^{\text{veto}}$, until the scale of hadronisation is reached.

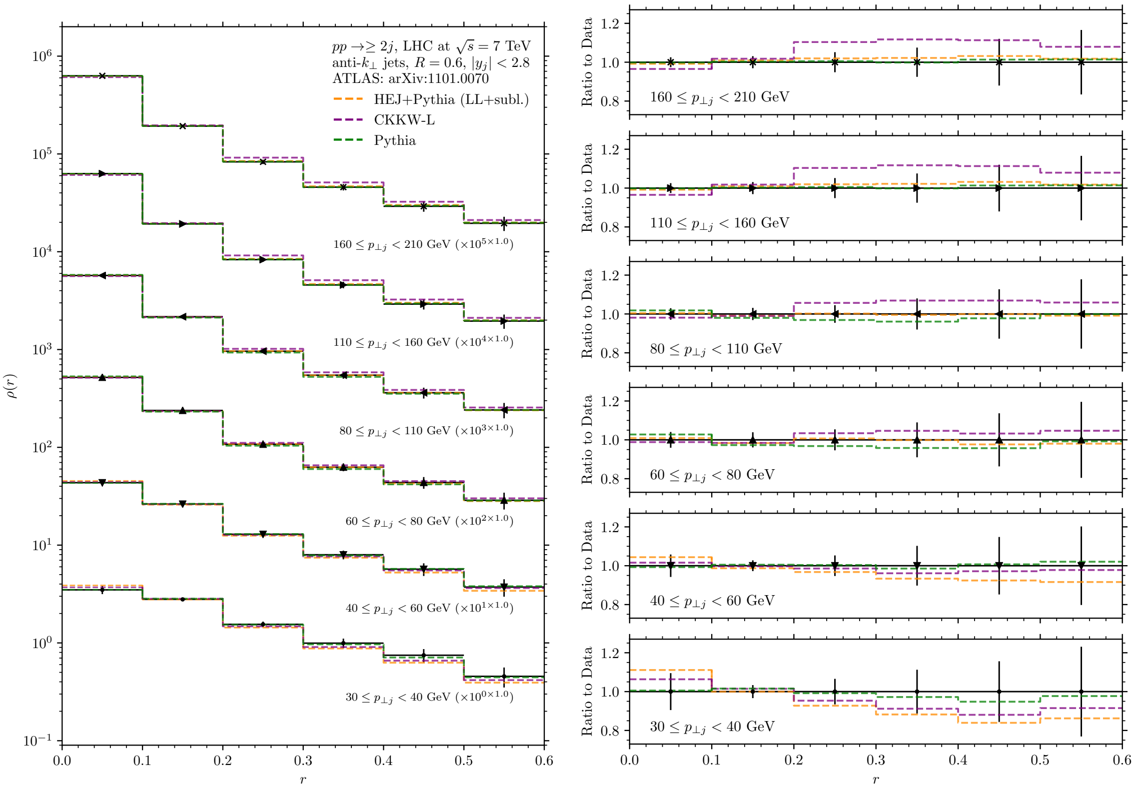

Figure 2: The differential jet profile $\rho(r)$ predictions obtained using HEJ+Pythia, CKKW-L, and Pythia. The predictions are split into bins depending on the jet transverse momentum, $p_{\perp j}$, and we show the ratio of the predictions to data from ATLAS [45]. The showered predictions use the Monash 2013 tune [46]. We display the analysis cuts in the figure. Since the physical effects resummed in HEJ do not affect this region of phase space, the HEJ+Pythia prediction should follow CKKW-L closely, which can indeed be seen to be the case in the plots in this figure.

4. The event is hadronised.

As we have already mentioned, one of the major developments with respect to the previous merging of high-energy and soft-collinear resummation [37] is that all of the original HEJ partons from the input event are retained (i.e. not only up to first parton shower emission). Moreover, in the framework developed in [38, 41] we also include the non-HEJ-resummable states (i.e. states which correspond to input event configurations that do not receive all-order corrections in HEJ). These configurations are merged using the CKKW-L method. Therefore, we keep, for the first time, both the full leading order matched accuracy of HEJ and the logarithmic accuracy of Pythia.

3. Results

We have performed a number of validation studies of the HEJ+Pythia framework and present here a selection. For a complete investigation including also electroweak bosons see [38].

We first discuss the differential jet profile, $\rho(r)$, which is defined by the normalised sum of transverse momenta in an annulus of width Δr, in $y - \phi$ space, inside a jet of total width R as

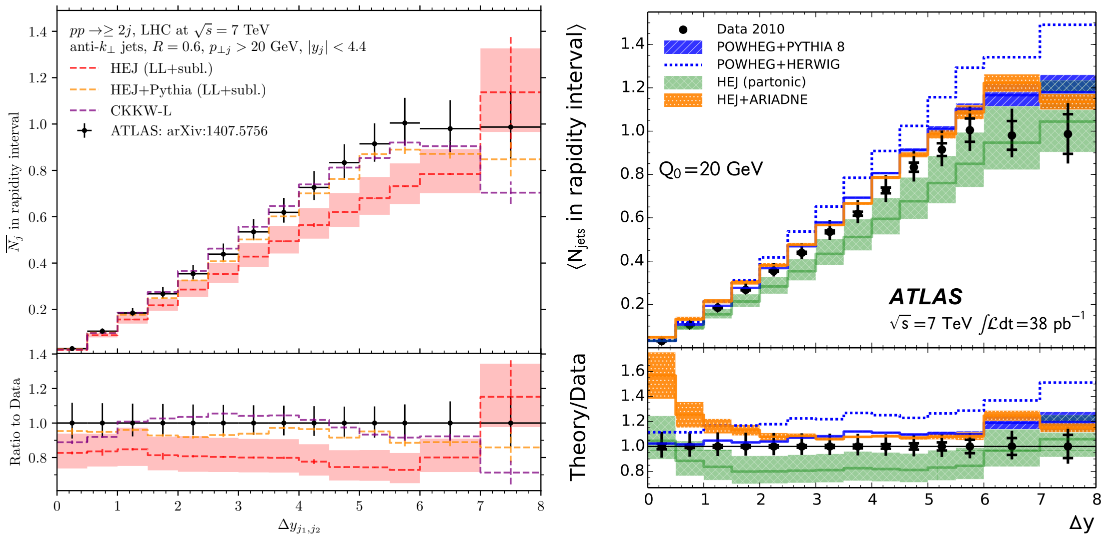

Figure 3: The mean number of jets in the rapidity interval bounded by the dijet system. Data and analysis cuts obtained from ATLAS [4] are displayed on the figures. (left) predictions obtained with HEJ, HEJ+Pythia, and CKKW-L. (right) predictions obtained with HEJ,HEJ+ARIADNE, POWHEG+PYTHIA8 and POWHEG+HERWIG. Right plot reproduced from [4].

follows:

$$\rho(r) = \frac{1}{\Delta r} \frac{1}{N_{\text{jets}}} \sum_{\text{jets}} \frac{p_T(r - \Delta r/2, r + \Delta r/2)}{p_T(0, R)}, \tag{7}$$

where the $p_T(r_1, r_2)$ is scalar sum of transverse momenta of partons between radii r_1 and r_2. We compared the theoretical predictions for this observable to ATLAS experimental data collected at $\sqrt{s} = 7$ TeV [47], where the jets are clustered with $R = 0.6$ and have a minimum transverse momentum of $p_{\perp,j} > 30$ GeV. The jets were also required to be central in rapidity i.e. $|y_j| < 2.8$. The main physical effects for this observable arise from soft and collinear splittings. This region of phase space populated by these emissions is described well by parton showers, whereas in HEJ, jet profiles are dominated by a single hard parton. Therefore, the expectation is that the HEJ+Pythia prediction follows closely the one of CKKW-L, which is indeed the case as can be seen in figure 2.

In the left plot of figure 3, we present the predictions for mean number of jets in the rapidity interval bounded by the two hardest jets obtained using HEJ, HEJ+Pythia, and CKKW-L. The jets are clustered with $R = 0.6$ and have a transverse momentum of at least 20 GeV. The predictions are compared against data from ATLAS [4]. We can see that in this case both HEJ and CKKW-L yield considerable contributions to this distribution due to the very soft jets. This allowed us to test the merging procedure which ensures that a proper subtraction of double-counted contributions within the HEJ+Pythia framework is performed. Otherwise, large overshooting would be observed as both HEJ and Pythia populate this region of phase-space. The improvement in the merging of high-energy and soft-collinear resummation is noticeable in the comparison of the HEJ+Pythia against the HEJ+ARIADNE prediction shown in the right plot of figure 3. We can see that properly accounting for all HEJ and parton shower emissions describes better the low $\Delta y_{j_1 j_2}$ region where HEJ+ARIADNE exhibits significant growth, whereas HEJ+Pythia remains well-behaved.

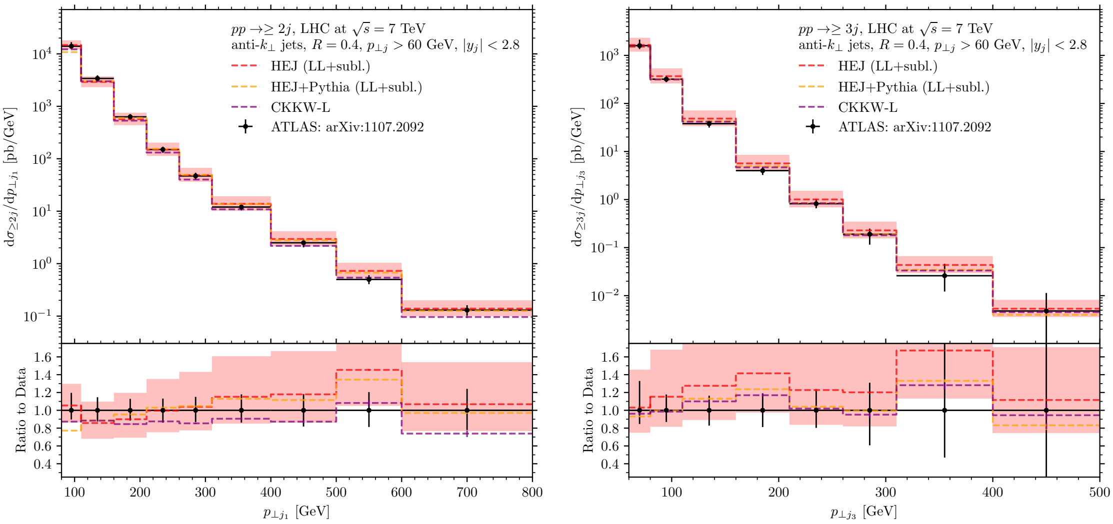

Figure 4: Distributions of the transverse momentum of the hardest (left) and third-hardest (right) jet in inclusive dijet and three-jet events, respectively. We show predictions obtained with HEJ, HEJ+Pythia, and CKKW-L, data is obtained from [48] and the analysis cuts are given in the figures.

Lastly, in the left panel of figure 4 we show the transverse momentum distribution of the hardest jet in inclusive dijet events, and in the right panel we show the transverse momentum distribution of the third-hardest jet in three-jet events. It is clear that in most of the phase-space, the HEJ+Pythia framework successfully describes the data by incorporating both the HEJ resummation and the soft-collinear effects. It is most encouraging to see that the HEJ+Pythia line is not the average between the HEJ and CKKW-L lines, but rather stays closer to the CKKW-L line when this has dominant effects and closer to HEJ when the converse is true.

4. Conclusions

In this article, we have summarised the progress in the development of the HEJ+Pythia framework [38, 40, 41], which accounts both for the high-energy behaviour as described by HEJ partonic Monte Carlo, and the soft-collinear effects in Pythia. We have outlined the merging procedure which allows to capture both towers of logarithms to all orders in the strong coupling expansion and which avoids double counting. Finally, we have explored the validity of the framework by showing results for observables where either one, or both, effects are important, and by demonstrating that the HEJ+Pythia framework delivers reliable predictions in each case. More details and studies including electroweak bosons are presented in [38].

Acknowledgements

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