

Abundant radiation of soft photons: a puzzle lasting four decades

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The observed enhancement of low- k_T photons in comparison with incorrect calculations, should not be treated as a puzzle. The paper by Low considered a large rapidity gap process of diffractive excitation of a hadron, $h \rightarrow h + \gamma$, rather than multiple hadron production spanning all over the rapidity interval between colliding hadrons. The optical theorem connects these two processes, and what is inner bremsstrahlung, suppressed according to Low, corresponds to radiation from final state hadrons. Thus, the main result of the Low theorem, based on gauge invariance of the diffractive bremsstrahlung amplitude, supplemented with the optical theorem, contradicts the so-called bremsstrahlung model. The latter has been used for comparison with data, leading to the longstanding soft photon puzzle.

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1. The Low theorem revisited

The process under consideration in the Low paper [1] is quasi-elastic, hadronic scattering with radiative excitation of one of the hadrons,

$$h_1 + h_2 \rightarrow h'_1 + \gamma + h'_2 \quad (1)$$

The mechanisms of radiation can be grouped to external and internal radiation as is depicted in Fig. 1.

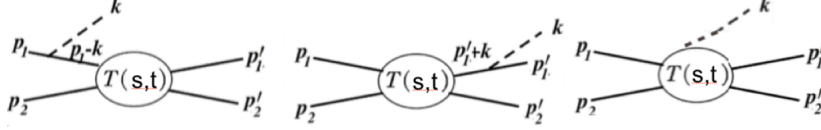


Figure 1: External (1st and 2d figs.) and internal (3rd fig.) terms in the radiation amplitude.

Correspondingly, the amplitude of (1) reads, $M = e_\mu M_\mu$, where e_μ is the photon polarization vector, and

$$M_\mu = M_\mu^{ext} + M_\mu^{int} \quad (2)$$

The amplitudes of external radiation contains Feynman propagators,

$$M_\mu^{ext} = \left(\frac{p'_{1\mu}}{p'_1 k} - \frac{p_{1\mu}}{p_1 k} \right) T(s, t) + \dots \quad (3)$$

The factor in parentheses is divergent at $k \rightarrow 0$. The elastic hadronic amplitude $h_1 h_2 \rightarrow h'_1 h'_2$ is depicted in Fig. 1 by blobs.

The two terms in Eq. (2) are related since the amplitude is Gauge invariant [1], $k_\mu M_\mu = 0$. Therefore, the term presenting internal radiation, must be finite at $k \rightarrow 0$, i.e. is suppressed in comparison with external radiation. This is the main observation of the Low theorem [1].

1.1 Landau-Pomeranchuk principle

The Low theorem can be also treated as a formal proof of the Landau-Pomeranchuk principle [2], which states that any variations of the electric current within a short distance do not affect the spectrum of radiation with a much longer radiation, or coherence length,

$$l_c^\gamma = \frac{2E_h x(1-x)}{k_T^2 + x m_h^2}, \quad (4)$$

where $x = k_+^\gamma / p_+^h$ is the fractional light-cone hadronic momentum carried away by the photon. The important condition for the Low theorem is a long radiation coherence length, which Low calls “the distance a particle can move with energy imbalance $\Delta E = k$ ”. Notice that l_c^γ and E_{h1} are not Lorentz invariant, so must be taken within the same reference frame.

Important is to keep the incoming ($l < -l_c^\gamma$) and outgoing ($l > l_c^\gamma$) currents unaffected by the current variations on a short length scale of strong interaction. This means that only extrinsic radiation from initial and final hadrons h_1, h'_1 matters.

1.2 Fock state representation

According to the optical theorem forward elastic amplitude is a "shadow" of inelastic interactions. This is why the elastic process is usually called diagonal diffraction. Good and Walker [3] proposed an extended version of the optical theorem, relating the off-diagonal diffractive amplitude with a linear combination of diagonal amplitudes. This can be explained basing on the Fock-state expansion of the hadronic wave function. For radiative diffractive excitation for the sake of simplicity we single out two Fock components of the hadron, just a hadron $|h\rangle_0$, and a hadron accompanied by a photon $|h\gamma\rangle$ [4],

$$|h\rangle = C_0|h\rangle_0 + C_1|h\gamma\rangle + \dots \quad (5)$$

The Fock components are eigenstates of the elastic amplitude operator, $\hat{f}|i\rangle = f_i|i\rangle$.

Diffractive excitation occurs only due to diversity of the elastic eigenamplitudes, otherwise the incoming wave packet would remain unchanged, so only elastic scattering was possible. Indeed, according to (5) the off-diagonal diffractive amplitudes read,

$$\langle h\gamma|\hat{f}|h\rangle = C_1^*C_0(f_{h\gamma} - f_h) \quad (6)$$

Differently from the Feynman diagrams, having no space-time structure, one cannot say whether the photon is radiated before or after the interaction. The radiation amplitude is a linear combination of elastic eigenamplitudes.

Notice that in both Fock components $|h\rangle$ and $|h\gamma\rangle$ only the hadron constituent h interacts, so at first glance $f_h = f_{h\gamma}$ and the diffractive amplitude must vanish. Nevertheless, the impact parameters b are shifted by the radiation, so the eigen-amplitudes $f_h(b) \neq f_{h\gamma}(b')$ cancel only in the b -integrated amplitude, i.e. at $t = 0$. Indeed, the Low amplitude Eq. (3) vanishes in forward direction, $t = 0$, what corresponds to integration of the amplitude over all impact parameters.

2. Photon radiation in inelastic collisions

2.1 Use and misuse of the Low theorem

The so-called bremsstrahlung model (BM) pretends to extend the Low theorem to inelastic collisions with multi-particle production. Photons are assumed to be radiated by participating charge particles, either incoming, or outgoing.

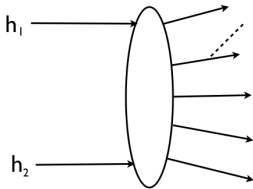


Figure 2: Multi-hadron production and photon radiation in inelastic collision.

It might be tempting to generalize the Low expression (3) to a larger number of produced charged hadrons, as was proposed in [6] (see more references in [5]).

$$M = M_0(p_1, p_2; p_3 \dots p_N) \left(\sum_i \frac{\eta_i e_i p_i \cdot \epsilon}{2p_i \cdot k} \right), \quad (7)$$

where $\eta_i = \pm 1$ for outgoing and incoming particles respectively, and $M_0(p_1, p_2; p_3 \dots p_N)$ is the amplitude of $2 \rightarrow N$ inelastic collision without radiation. Such an unjustified extension of the Low result derived for a diffractive process, is illegitimate [7]. Moreover, it strictly contradicts the Low theorem, as is demonstrated below. Not

a surprise that BM contradicts data.

2.2 Bremsstrahlung Model vs data

A puzzling enhancement of low- k_T photons in high-energy hadronic and hadron-nucleus collisions in comparison with BM expectations has been repeatedly observed since 1984. A collection of relevant experimental results [5], is displayed in Fig. 3. Data are compared with

Experiment	Collision Energy	Photon k_T	Photon/Brem Ratio
K^+p , CERN,WA27, BEBC (1984)	70 GeV/c	$k_T < 60$ MeV/c	4.0 ± 0.8
K^+p , CERN,NA22, EHS (1993)	250 GeV/c	$k_T < 40$ MeV/c	6.4 ± 1.6
π^+p , CERN,NA22, EHS (1997)	250 GeV/c	$k_T < 40$ MeV/c	6.9 ± 1.3
π^-p , CERN,WA83,OMEGA (1997)	280 GeV/c	$k_T < 10$ MeV/c	7.9 ± 1.4
π^+p , CERN,WA91,OMEGA (2002)	280 GeV/c	$k_T < 20$ MeV/c	5.3 ± 0.9
pp , CERN,WA102,OMEGA (2002)	450 GeV/c	$k_T < 20$ MeV/c	4.1 ± 0.8

Figure 3: The list of experiments detected an anomalous enhancement of low- k_T photon in high-energy inelastic hadronic collisions. The last column shows the ratio of the observed yield of soft photons to the prediction of the BM. The table is taken from [5]

predictions of the BM Eq. (7). We see that the yield of low- k_T photons substantially exceeds data. Apparently, the suspect is the model.

2.3 The Low process at high energies

At high energies the Low process essentially simplifies. Diffraction is a large rapidity gap process. That means that the hadronic amplitude $T(s, t)$ is dominated by Pomeron exchange in terms of Regge phenomenology [8], or by colorless gluonic exchange [9] in terms of QCD.

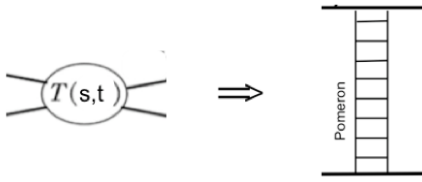


Figure 4: The hadronic amplitude $T(s, t)$ is dominated by the Pomeron exchange.

terms [1] can be neglected.

2.4 Unitarity relation

Optical theorem relates the inelastic cross section for multi-particle production, with the imaginary part of the forward elastic amplitude, as is depicted in Fig. 5 for the Pomeron amplitude.

The ladder graph Pomeron depicted in Fig. 4 is simplified. In topological expansion the Pomeron has non-planar, or cylindrical structure [8].

According to Regge phenomenology the Regge intercept of the Pomeron trajectory is slightly above one, $\alpha_{Pom} = 1 + \Delta$, so the elastic amplitude $T(s, t) \propto s^\Delta$. Since $\Delta \ll 1$ the energy derivatives $\partial T(s, t)/\partial s \sim 1/s$ vanish. This makes the relation (3) more accurate, because the skipped derivative

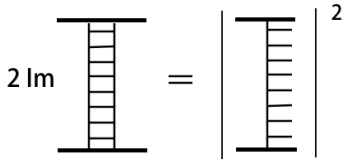


Figure 5: Unitarity cut of the Pomeron.

number of inelastic collision.

2.5 Charge screening

The specific feature of the Pomeron is electric neutrality. This means that electric charges of the colliding hadrons $h_{1,2}$ do not flow toward the central rapidities. A charge can be transported through large rapidity intervals only by Reggeons, having intercepts $\alpha_R \approx 1/2$ and lower. Therefore production of a charged hadron with rapidity y is suppressed by $\exp(-(Y - y)/2)$, where $Y = \ln(\sqrt{s}/2)$. Thus, the electric charges of the colliding hadrons remain at maximal and minimal rapidities and photons cannot be radiated at medium rapidities as is incorrectly assumed in the BM Eq. (7).

Of course one could introduce quark loops in the gluonic latter of the Pomeron, however photon radiation from quarks and antiquarks cancels due to charge screening.

2.6 Why Feynman rules cannot be applied to Fig. 2

The infra-red divergent Feynman propagator $\eta_i e_i p_i \cdot \epsilon / 2 p_i \cdot k$ in (7) in coordinate representation reads,

$$\frac{1}{p^2 - m_h^2} = \frac{l_c^Y}{2E} \quad (8)$$

When the propagator diverges as $1/k$ the coherence length rises to infinity. This means that a hadron radiating soft photons, cannot emerge instantaneously, but its production takes a long time.

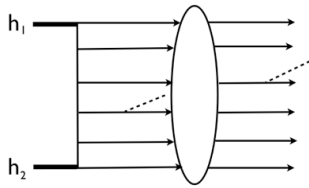


Figure 6: Space-time pattern of radiative inelastic interaction.

This is illustrated in Fig. 6 which shows that all final hadrons radiating a photon must preexist long time before interaction. This is a standart space-time picture of interaction in the conventional parton model.

Thus, the process cannot be treated like in (7) as radiation either from the two incoming, or from N outgoing charges. All the produced partons pre-exist prior the interaction. Due to factorization of long distances (Partonic wave function of a hadron) and short-scale interaction the cross section is subject to the factorization theorem.

This inconsistency is another reason for incorrectness of the Bremsstrahlung Model.

3. Color-dipole description of photon radiation

As far as the BM turns out to be incorrect, an alternative description of soft photon radiation is required [11].

The cross section of bremsstrahlung by a projectile quark can be calculated within the color dipole phenomenology [4, 12–16], adjusted to precise data on DIS from HERA,

$$\frac{d\sigma(qN \rightarrow \gamma X)}{d \ln \alpha d^2 k_T} = \frac{1}{(2\pi)^2} \int d^2 r_1 d^2 r_2 e^{i\vec{k}_T(\vec{r}_1 - \vec{r}_2)} \Psi_{\gamma q}^*(\alpha, \vec{r}_1) \Psi_{\gamma q}(\alpha, \vec{r}_2) \sigma_\gamma(\vec{r}_1, \vec{r}_2, \alpha), \quad (9)$$

where α is the fractional light-cone momentum, carried by the photon;

$$\sigma_\gamma(\vec{r}_1, \vec{r}_2, \alpha) = \frac{1}{2} \left\{ \sigma_{\bar{q}q}(\alpha r_1) + \sigma_{\bar{q}q}(\alpha r_2) - \sigma_{\bar{q}q}[\alpha(\vec{r}_1 - \vec{r}_2)] \right\}. \quad (10)$$

The quark-photon distribution function in (9) reads,

$$\Psi_{\gamma q}(\alpha, \vec{r}) = \frac{\sqrt{\alpha_{em}}}{2\pi} e^{\vec{s}} \chi_f \left\{ im_q \alpha^2 [\vec{n} \times \vec{\sigma}] + \alpha \left[\vec{\sigma} \times \vec{\nabla} \right] - i(2 - \alpha) \vec{\nabla} \right\} \chi_i K_0(\alpha m_q r), \quad (11)$$

where $\chi_{i,f}$ are the quark spinors.

The $\bar{q}q$ dipole-nucleon cross section $\sigma_{\bar{q}q}(r)$ in (10) at a soft scale has been parametrized in a saturation form and fitted to DIS and photoproduction data from NMC and HERA. The details can be found in [4, 16].

The quark distribution function in the proton at a hard scale is related to the structure function $F_2(x, Q^2)$, which is well known from DIS data, but only at a hard scale. At a soft scale one should rely on models, and we employ the well developed quark-gluon string model (QGSM) [21], or a similar dual parton model [22]. Both models assume Regge behavior at the end-points $x \rightarrow 1$ or $x \rightarrow 0$ of the quark distribution functions, and a simple, but ad hoc, interpolation at medium x . We skip the simple, but lengthy expressions. The details can be found e.g. in [23].

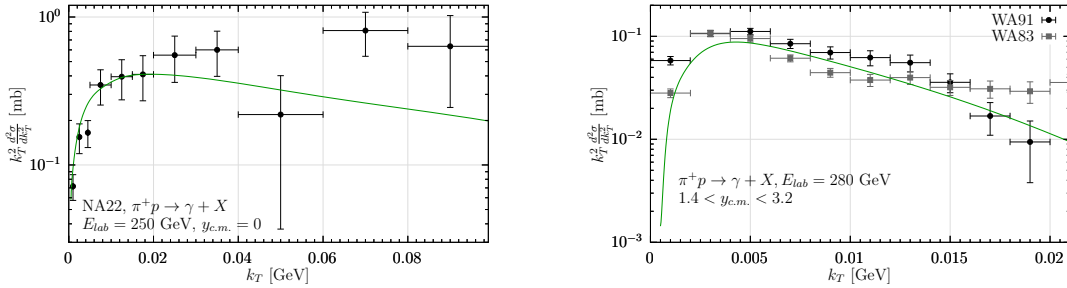


Figure 7: *Left:* Comparison with data of the NA22 experiment [17] for $\pi^+ p \rightarrow \gamma X$ at $E_{lab} = 250 GeV$. *Right:* Comparison with data of the WA91 [18] and WA83 [19] experiments for $\pi^+ p \rightarrow \gamma X$ at $E_{lab} = 280 GeV$.

At somewhat higher energy $E_{lab} = 450 GeV$ our calculations depicted by solid curve in Fig. 8, apparently overestimate the data of the WA102 experiment. However, the experiment had specific cuts, namely, events with number of charge tracks $N_{ch} > 8$, were excluded. To calculate the multiplicity distribution we assume the Poisson distribution of the number of unitary cut Pomerons, and employed the result of QGSM. So we obtained a suppression factor $\delta = \sum_{N_{ch}=0}^8 / \sum_{N_{ch}=0}^{\infty} = 0.39$.

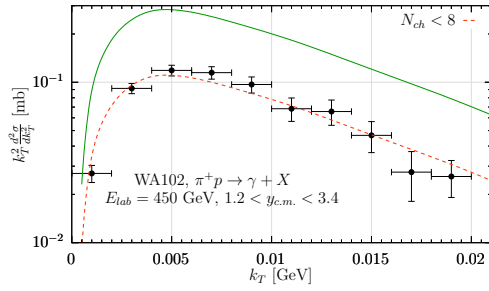


Figure 8: Comparison with data of the WA102 [20] experiment for $\pi^+Be \rightarrow \gamma X$ at $E_{lab} = 450 GeV$.

Pomeron photon radiation at the mid-rapidities cancels. This interference effect is missed in BM.

- The observed enhancement of low- k_T photons in comparison with incorrect BM calculations, should not be treated as a puzzle.

- Calculations based on the color dipole description of photon radiation well agree with data.

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