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Power corrections to EEC meets conformal bootstrap

Hao Chen,[a] **Xinan Zhou**[b] **and Hua Xing Zhu**[c,d,*]

[a]*Zhejiang Institute of Modern Physics, School of Physics, Zhejiang University, Hangzhou, Zhejiang, 310027, China*

[b]*Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China*

[c]*School of Physics, Peking University, Beijing, 100871, China*

[d]*Center for High Energy Physics, Peking University, Beijing 100871, China*

E-mail: chenhao201224@zju.edu.cn, xinan.zhou@ucas.ac.cn, zhuhx@pku.edu.cn

Energy-Energy Correlation (EEC) measures the energy-weighted angular correlation of pair of particles in e^+e^- collisions. In the back-to-back limit, EEC exhibits typical Sudakov double logarithms that can be resummed using standard TMD factorization formula. In this talk we present a new approach to the resummation of Sudakov double logarithms for EEC, which involves the utilization of a local operator product expansion, conformal block, and large spin summation. We show that classical conformal symmetry of massless QCD lagrangian can be used to resummed certain power corrections in the back-to-back region. To illustrate the effectiveness of this approach, We demonstrate its application using the N=4 Supersymmetric Yang-Mills theory as an example. Generalization to the QCD case will be briefly discussed.

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1. Introduction

Energy distribution in the high energy scattering plays an important role in probing the dynamics of Quantum Chromodynamics (QCD) and understanding the general Lorentzian gauge theory. The energy flows are dominated by the formation of jets, which are bunches of collimated hadrons carry the dynamic information from the underlying quarks and gluons [1, 2]. To study QCD scattering perturbatively, we must construct infrared and collinear safe (IRC) observables. One famous example is the energy-energy correlator (EEC) [3–6], which characterizes the correlation of energy deposits along two different directions.

In the e^+e^- scattering, the definition of EEC is

$$\mathrm{EEC}(\zeta) = \frac{1}{\sigma_0}\sum_X \int d\sigma_{e^+e^-\to X} \sum_{b,c\in X} \frac{E_a E_b}{Q^2}\delta(\zeta - \frac{1-\cos\theta_{ab}}{2})\,. \tag{1}$$

Here, E_a, E_b are the energy of two final state particles measured in the center of mass frame and θ_{ab} is the angle between them. The variable $\zeta = (1-\cos\theta)/2$ is related to the angle θ between two calorimeters and Q is the total energy. On the other hand, EEC possesses an alternative field-theoretical definition as the correlation function of two energy flow operators:

$$\mathrm{EEC}(\zeta) = \frac{8\pi^2}{q^2\sigma_0}\int d^4x\, e^{iq\cdot x_{13}}\langle\Omega|J^\mu(x_1)\mathcal{E}(n_2)\mathcal{E}(n_4)J_\mu(x_3)|\Omega\rangle\,, \tag{2}$$

where $n_2 = (1,\vec{n}_2)$ and $n_4 = (1,\vec{n}_4)$ specify the directions of the calorimeters, $\zeta = \frac{(n_2\cdot n_4)q^2}{2(n_2\cdot q)(n_4\cdot q)}$, $q^\mu = (Q,0,0,0)$ is the total momentum, $J^\mu = \bar{\psi}\gamma^\mu\psi$ is the electromagnetic current, and the energy flow operator $\mathcal{E}(n_i)$ is a detector time integral of the stress tensor $T(x_i;\bar{n}_i) \equiv T_{\mu\nu}(x_i)\bar{n}_i^\mu\bar{n}_i^\nu$:

$$\mathcal{E}(n_i) = \int_{-\infty}^{\infty}\frac{d\,n_i\cdot x_i}{16}\lim_{\bar{n}_i\cdot x_i\to\infty}(\bar{n}_i\cdot x_i)^2 T(x_i;\bar{n}_i)\,. \tag{3}$$

The presence of two equivalent definitions makes EEC an interesting observable that benefits from both momentum space and position space techniques.

In the perturbation theory, EEC has end point divergences in the collinear limit $\zeta \to 0$ and back-to-back limit $\zeta \to 1$, which is illustrated by the LO result

$$\mathrm{EEC}(\zeta) = \frac{\alpha_s}{2\pi}C_F\frac{3-2\zeta}{4(1-\zeta)\zeta^5}\left[3\zeta(2-3\zeta)+2(2\zeta^2-6\zeta+3)\log(1-\zeta)\right] + O(\alpha_s^2)\,. \tag{4}$$

Away from these end points, the fixed order results are good approximations. Approaching the end points, large logarithms make the fixed order results unreliable that requires resummation of them to all orders in perturbation theory.

The focus of this talk is the back-to-back limit $\zeta \to 1$ which is the classical configuration of producing quark and anti-quark pair. Phenomenological application of the back-to-back includes precision measurement of strong coupling constant [25] and extraction of TMD functions [26–28]. Unlike the collinear limit, both collinear and soft emissions produce large logarithms. It features the famous Sudakov double logarithms

$$\mathrm{EEC}(\zeta) \sim \sum_{n=1}^{\infty}\sum_{m=0}^{2n-1}\alpha_s^n\left(c_{n,m}\frac{\log^m(1-\zeta)}{1-\zeta} + d_{n,m}\log^m(1-\zeta) + O(1-\zeta)\right)\,, \tag{5}$$

where we have truncated to the Leading Power (LP) and the Next-to-Leading Power (NLP). At LP, we can use Collins-Soper-Sterman (CSS) formalism [29] or the factorization formula from the Soft-Collinear Effective Theory (SCET) [30]

$$\mathrm{EEC}(\zeta) \sim \frac{1}{2}\int d^2\vec{k}_\perp \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp\cdot\vec{b}_\perp} H(Q) J^q(\vec{b}_\perp) J^{\bar{q}}(\vec{b}_\perp) S(\vec{b}_\perp)\delta\left(1-\zeta-\frac{\vec{k}_\perp^2}{Q^2}\right) \tag{6}$$

to resum the Sudakov logarithms in EEC, based on which the cutting-edge perturbative calculations have achieved $\mathrm{N^4LL}$ accuracy resummation [31].

However, it is not easy to extend the factorization analysis to the subleading powers. In CFT, Korchemsky observed that the back-to-back limit is related to the double lightcone limit of the local correlator via the Mellin representations [9]. Building upon this observation, we will introduce a new method of resumming Sudakov logarithms in EEC systematically beyond LP. This exploits the definition of EEC from the position space and uses techniques from modern conformal bootstrap program.

Compared with the factorization formalism, though less intuitive, we find several novel aspects of the position space techniques. First, we avoid IR divergences at all stages because the local correlator $\langle J^\mu(x_1)T(x_2;\bar{n}_2)T(x_4;\bar{n}_4)J_\mu(x_3)\rangle$ is IR finite. Second, the conformal symmetry becomes manifest (classically for QCD), which constrains (part of) the power corrections systematically. Last but not least, $\langle J^\mu(x_1)T(x_2;\bar{n}_2)T(x_4;\bar{n}_4)J_\mu(x_3)\rangle$ makes the crossing symmetry (*i.e.* invariance under the exchange of x_1 and x_3) manifest.

The procedure is summarized as follows. First, we elaborate on the connection between the back-to-back limit and the double lightcone limit. In terms of operator product expansion (OPE), the double lightcone limit is controlled by twist expansion and large spin expansion. In this way, we decompose the Sudakov resummation in EEC into the RG of local operators (governed by the anomalous dimensions) and the large spin resummation. Second, we make use of the twist conformal blocks [32] to resum the logarithms from the large spin tails. The Casimir equation of the conformal group plays an important role in obtaining TCB recursively. Further simplification comes from the crossing symmetry, which relates the twist corrections (in the large spin limit) to large spin corrections in the lower twists. We use the $\mathcal{N}=4$ super Yang-Mills (SYM) theory to illustrate this idea and achieve the first Leading and Next-to-Leading Logarithmic resummation at the NLP.

2. Position space techniques: Back-to-back v.s. double lightcone

To apply position space techniques to study Sudakov logarithms in the back-to-back limit, an important first step is to identify the relevant kinematic region that gives rise to such terms in the local IR finite correlators $\langle J^\mu(x_1)T(x_2;\bar{n}_2)T(x_4;\bar{n}_4)J_\mu(x_3)\rangle$ from (2). The trick is to choose a frame where detectors are exactly back-to-back $n_2=\bar{n}_4$ and the total momentum q acquires small transverse component $q_\perp^2 \ll q^2$ at the same time. From the Fourier factor $e^{iq\cdot x_{13}}$, this corresponds to the region $|x_{13}^\perp|^2 \gg x_{13}^+ x_{13}^-$ in the position space EEC $\langle\Omega|J^\mu(x_1)\mathcal{E}(n_2)\mathcal{E}(n_4)J_\mu(x_3)|\Omega\rangle$.

The position space EEC is the null integrations of the correlator $\langle J^\mu(x_1)T(x_2;\bar{n}_2)T(x_4;\bar{n}_4)J_\mu(x_3)\rangle$ with respect to x_2 and x_4 at null infinity. As illustrated in Figure 1, the configuration $|x_{13}^\perp|^2 \gg x_{13}^+x_{13}^-$ means the lightcone singularities of $x_{12}^2=0$ and $x_{23}^2=0$ are close (compared with $|x_{13}^\perp|$) and pinch

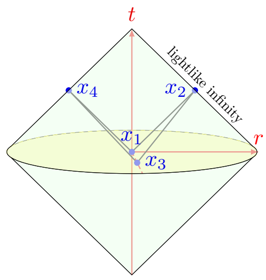

Figure 1: Penrose diagram for the double lightcone limit of the local correlator.

the integration contour of x_2 (due to Wightman ordering). Therefore, we conclude that the dominant contribution in the back-to back limit is from the double lightcone limit $x_{12}^2, x_{23}^2 \to 0$ of the local correlator.

To avoid the complexity from the tensor structures, we take the $\mathcal{N} = 4$ SYM as an example where the supersymmetry relates the EEC to local four point function of scalar operators [33]:

$$\text{EEC}(\zeta) = \frac{8\pi^2 Q^2}{\zeta^2} \prod_{i=2,4} \int_{-\infty}^{\infty} d(n_i \cdot x_i) \lim_{\bar{n}_i \cdot x_i \to \infty} \left(\frac{\bar{n}_i \cdot x_i}{2}\right)^2 \langle O(x_1) O(x_2) O(x_3) O(x_4) \rangle \,. \tag{7}$$

The functional form of the local correlator is constrained by the symmetry

$$\langle O(x_1) O(x_2) O(x_3) O(x_4) \rangle_{\text{dyn}} = \frac{1}{(2\pi)^4} \frac{x_{13}^4 x_{24}^4}{(x_{12}^2 x_{34}^2)^4} \mathcal{F}(u, v) \,, \tag{8}$$

where we have subtracted the contribution from the protected operators and $\mathcal{F}$ is the function of the conformal cross-ratios $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$. In the weak coupling limit $a = \frac{g^2 N_c}{4\pi^2} \ll 1$, $\mathcal{F}(u, v)$ reads

$$\mathcal{F}(u, v) = \sum_{n=0}^{\infty} a^n \mathcal{F}^{(n)}(u, v) = \mathcal{F}^{(0)}(u, v) + \frac{u^3}{v} \Phi(u, v) \,, \tag{9}$$

where the function $\Phi(u, v) = \sum_{n \geq 1} a^n \Phi^{(n)}(u, v)$ contains all the coupling-dependent information and is crossing symmetric, *i.e.*, $\Phi(u, v) = \Phi(v, u)$. The full three-loop of $\Phi(u, v)$ can be found in [34, 35]. The double lightcone limit, in terms of the cross-ratios, is the limit $u \to 0, v \to 0$ or $z \to 0, \bar{z} \to 1$. Upon expanding the perturbative results in the double lightcone limit, we have explicitly checked that the logarithmic terms in $\Phi(u, v)$ correctly produce the logarithmic terms in the EEC at both LP and NLP, which confirms the expectation that the Sudakov limit of EEC corresponds to the double lightcone limit of a local 4-point Minkowskian correlator (in the sense of logarithmic enhancement). In addition, we find that the double logarithmic series $\alpha_s^n \log^{2n-1}(1-\zeta)$ comes from the single logarithmic series $\alpha_s^n \log^n u \log^n v$.

3. Twist expansion and large spin expansion

It is known that the OPE in the lightcone limit is organized as the twist expansion. Suppose we consider the lightcone limit $x_{12}^2 \to 0$, the most singular contribution in the 1, 2-OPE channel are the lowest twist $\tau \equiv \Delta - \ell$ operators. However, single local operator contribution in the 1, 2-channel cannot produce the singularities in the other lightcone limit $x_{13}^2 \to 0$. In other words, the $x_{13}^2 \to 0$ singularities come from an infinite sum of operators with different spins at a given twist.

In CFT, operators are classified into two categories: primary operators and descendant operators, where descendants are the total derivatives of primary operators. The contributions of descendants in the OPE is completely determined by their associated primary operators. As a consequence, the correlator $\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle$ admits the superconformal block expansion

$$\mathcal{F}(u,v) = \sum_{\Delta}\sum_{\text{even }\ell} a_{\tau,\ell}G_{\Delta+4,\ell}(u,v)\,, \tag{10}$$

where the sums are over all superconformal primary operators with dimension Δ, spin ℓ and $a_{\tau,\ell}$ are the OPE coefficients. The functions $G_{\Delta,\ell}$ are called conformal blocks and have analytic expressions in $4d$ [36]

$$G_{\Delta,\ell}(u,v) = \frac{z\bar{z}}{\bar{z}-z}\left[k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})\right]\,, \tag{11}$$

where $k_\beta(x) = x^{\beta/2}{}_2F_1(\frac{\beta}{2},\frac{\beta}{2},\beta;x)$. They are eigenfunctions of the quadratic Casimir operator

$$\mathcal{D}_2 = z^2((1-z)\partial_z^2 - \partial_z) + \frac{(d-2)z\bar{z}}{z-\bar{z}}(1-z)\partial_z + (z \leftrightarrow \bar{z}) \tag{12}$$

with eigenvalues $\frac{1}{2}\left(\Delta(\Delta-4)+\ell(\ell+2)\right)$ [37].

The analytic expression of $4d$ conformal block is a good example to demonstrate the idea of twist expansion and large spin expansion in the double lightcone limit. Expanded in the $z \to 0$, $\bar{z} \to 1$ limit, the conformal block is $G_{\Delta,\ell}(z,\bar{z}) \sim z^{\tau/2}\log(1-\bar{z})$,where high twist causes suppression in the $z \to 0$ limit and the logarithmic divergence in $(1-\bar{z})$ is the effect of summing over infinitely many operators because each conformal block contains infinitely many descendants in the corresponding conformal family. However, physical correlators can exhibit more severe divergences than a single logarithm in $1-\bar{z}$, called enhanced divergences[1], necessitating contributions from infinitely many primary operators at large spin.

Noticed that there is a shift $\Delta \to \Delta + 4$ in the expansion (10) caused by superconformal symmetry, and we denote $\mathbf{G}_{\tau,\ell}(z,\bar{z}) = G_{\Delta+4,\ell}(u,v)$ for later convenience. The perturbative expansion enters the block decomposition via the twist $\tau = \tau_0 + \sum_{n=1}^{\infty} a^n\gamma_{\tau_0,\ell}^{(n)}$ and OPE coefficient $a_{\tau,\ell} = \sum_{n=0}^{\infty} a^n a_{\tau_0,\ell}^{(n)}$, where τ_0 is the classical twist and $\sum_{n\geq 1} a^n\gamma_{\tau_0,\ell}^{(n)} = \gamma_{\tau,\ell}$ is the anomalous dimension. In particular, after expanding with respect to the anomalous dimension $\gamma_{\tau,\ell}$ in the conformal blocks

$$\mathbf{G}_{\tau,\ell} = \sum_{n=0}^{\infty}\frac{1}{n!}\gamma_{\tau,\ell}^n\partial_{\tau_0}^n\mathbf{G}_{\tau_0,\ell}\,, \tag{13}$$

we find that $\partial_{\tau_0}^n\mathbf{G}_{\tau_0,\ell}$ contains at most $\log^n u$ in the small u limit. Therefore, we identify that $\log u$ is generated from the renormalization of operators in the twist expansion.

The origin of $\log v$ is hidden in the logarithmic growth of anomalous dimension in the large spin limit $\gamma_{\tau,\ell} \sim \Gamma_{\text{cusp}}\log\ell$, where the coefficient Γ_{cusp} is known as the cusp anomalous dimension. After summing over the spin, $\log\ell$ is converted into $\log v$. However, carrying out the sum is directly technically not easy. In the next section, we employ the tricks developed in the large spin perturbation theory to calculate the logarithms in v.

[1] Examples include power divergences $(1-\bar{z})^{m<0}$ and powers of logarithms $[\log(1-\bar{z})]^{k\geq 2}$.

4. Large spin perturbation theory

In this section, we will describe how to use the Large Spin Perturbation Theory [32, 38–44] to systematically handle the enhanced divergences at the leading twist. The starting point is the free correlator $\mathcal{F}^{(0)}$, which can be organized as a sum over classical twists

$$\mathcal{F}^{(0)}(z,\bar{z}) = \sum_{\tau_0=2,4,\ldots} H_{\tau_0}(z,\bar{z})\,, \qquad \text{where} \quad H_{\tau_0}(z,\bar{z}) = \sum_{\ell=0}^{\infty} \langle a^{(0)}_{\tau_0,\ell}\rangle \mathbf{G}_{\tau_0,\ell}(z,\bar{z})\,, \tag{14}$$

Each $H_{\tau_0}(z,\bar{z})$ contains huge degeneracy over spins and is known as a twist conformal block (TCB). In an interacting theory, the twist degeneracies are lifted by the anomalous dimensions and the free OPE coefficients receive quantum corrections. This leads to the modifications of $H_{\tau_0}(z,\bar{z})$ with the following form: $\sum_{\ell=0}^{\infty}\langle a^{(0)}_{\tau_0,\ell}\rangle \kappa_{\tau_0}(\ell)\mathbf{G}_{\tau_0,\ell}(z,\bar{z})$ .Performing the sum over spin explicitly with a general $\kappa_{\tau_0}(\ell)$ is extremely challenging. However, to capture the feature of large spin tails, it is sufficient to expand $\kappa_{\tau_0}(\ell)$ in the large ℓ limit. There is an equivalent expansion of $\kappa_{\tau_0}(\ell)$ around large conformal spins $J^2_{\tau,\ell} = (\ell+\frac{\tau}{2})(\ell+\frac{\tau}{2}-1)$: $\kappa_{\tau_0}(\ell) = \sum_{m=0}^{\infty}\sum_{i=0}^{N}\frac{K_{m,i}}{J^{2m}_{\tilde{\tau}_0,\ell}}\log^i J^2_{\tilde{\tau}_0,\ell}$.Here we introduce shifted twists $\tilde{\tau}_0 = \tau_0 + 4$ to take into account the dimension shift in the decomposition (10). An interesting feature for using the conformal spin is that only even negative powers appear in the expansion. Therefore, the correlator should be expanded in terms of a more general class of TCBs which accommodate large spin suppression and the logarithmic enhancement

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = \sum_{\ell} a^{(0)}_{\tau_0,\ell}\frac{\log^i J^2_{\tilde{\tau}_0,\ell}}{J^{2m}_{\tilde{\tau}_0,\ell}}\mathbf{G}_{\tau_0,\ell}(z,\bar{z})\,. \tag{15}$$

From the conformal block decomposition (10), the leading twist expansion of $\mathcal{F}^{(n)}$ up to NLL accuracy reads

$$\mathcal{F}^{(n)} = z^3\sum_{\text{even }\ell} a^{(0)}_{2,\ell}\left\{\frac{\left(\gamma^{(1)}_{2,\ell}\right)^n}{2^n n!}L_z^n + \frac{\left(\gamma^{(1)}_{2,\ell}\right)^{n-1}L_z^{n-1}}{2^{n-1}(n-1)!}\left[\frac{a^{(1)}_{2,\ell}}{a^{(0)}_{2,\ell}} + (n-1)\frac{\gamma^{(2)}_{2,\ell}}{\gamma^{(1)}_{2,\ell}} + \frac{\gamma^{(1)}_{2,\ell}\partial_\ell}{2}\right]\right\}k_{2\ell+6}(\bar{z}) + \cdots\,, \tag{16}$$

where we denote the logarithms as $\log(x) = L_x$ to lighten the notation. Expanded the needed perturbative data [8, 45, 46] in terms of $1/J_{6,\ell}$, we organize the correlator in terms of TCBs

$$\mathcal{F}^{(n)} = \frac{L_z^n}{2^n n!}\left(\sum_{i=0}^{n}\frac{n!(2\gamma_E)^i}{i!(n-i)!}H_2^{(0,n-i)} + \frac{n}{3}\sum_{i=0}^{n-1-i}\frac{(n-1)!(2\gamma_E)^i}{i!(n-1-i)!}H_2^{(1,n-1-i)} + \cdots\right)\,, \tag{17}$$

where, up to NLP in the large spin limit, only two kinds of TCBs $H_2^{(0,i)}$ and $H_2^{(1,i)}$ appear.

The trick to calculate the enhanced divergences in TCBs with logarithms is to make use of the recursion relations and analytic continuation [32, 46]. The conformal blocks $\mathbf{G}_{\tau_0,\ell}$ satisfy the shifted Casimir equation with the eigenvalues being the conformal spins

$$C_{\tilde{\tau}_0}\mathbf{G}_{\tau_0,\ell}(z,\bar{z}) = J^2_{\tilde{\tau}_0,\ell}\mathbf{G}_{\tau_0,\ell}(z,\bar{z})\,, \tag{18}$$

where $C_\tau = \mathcal{D}_2 + \frac{1}{4}\tau(2d-\tau-2)$ is the shifted conformal Casimir. This leads to the recursion relations for TCBs

$$H^{(m,i)}_{\tau_0}(z,\bar{z}) = C_{\tilde{\tau}_0}H^{(m+1,i)}_{\tau_0}(z,\bar{z})\,. \tag{19}$$

In $4d$, the TCBs take a factorized form when we ignoring the regular part in the $\bar{z} \to 1$ limit

$$H_{\tau_0}^{(m,i)}(z,\bar{z}) = \frac{z\, k_{\tau_0-2}(z)}{\bar{z}-z} \bar{H}_{\tau_0}^{(m,i)}(\bar{z}) \,, \tag{20}$$

and the recursion relation (19) becomes

$$\bar{H}_{\tau_0}^{(m,i)}(\bar{z}) = \bar{D}\bar{H}_{\tau_0}^{(m+1,i)}(\bar{z}) \,, \qquad \text{where} \quad \bar{D} = \bar{z}^2(1-\bar{z})\frac{d^2}{d\bar{z}^2} - \bar{z}(2-\bar{z})\frac{d}{d\bar{z}} + 2 - \bar{z} \,. \tag{21}$$

We start with $\bar{H}_2^{(0,0)}(\bar{z})$ in which sum can be explicitly done

$$\bar{H}_2^{(0,0)}(\bar{z}) = \frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z}\log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms} \,. \tag{22}$$

Here, we define $\epsilon = 1-\bar{z}$ and emphasize the enhanced divergences in the ϵ expansion. Repeatedly acting $\bar{D}$ on $\bar{H}_2^{(0,0)}(\bar{z})$ gives $\bar{H}_{\tau_0}^{(m,0)}(\bar{z})$ at negative integer m. The first few terms in small ϵ for negative integer m are given by (3.37) of [46]

$$\bar{H}_2^{(m,0)}(\bar{z}) = \frac{1}{2}\epsilon^{m-1}\Gamma(1-m)^2 + \frac{1}{6}m\left(2m^2 - 6m + 1\right)\epsilon^m\Gamma(-m)^2 + \cdots \,. \tag{23}$$

From the generic negative integer expression, we analytic continue in m and expanded around $m = 0$ and $m = 1$ to obtain $\bar{H}_2^{(0,i)}(\epsilon)$ and $\bar{H}_2^{(1,i)}(\epsilon)$. Substituting in the TCBs, we obtain $\mathcal{F}^{(n)}$ at LP in z and NLP in $1-\bar{z}$

$$\mathcal{F}^{(n)}(z,\bar{z}) = \frac{(-1)^n z^3}{2^{n+1}n!} L_z^n L_\epsilon^n \left[\frac{1-\epsilon}{\epsilon} + \frac{2n}{3L_\epsilon} + \frac{1}{L_z} + \cdots\right] + O(z^4) \,, \tag{24}$$

5. Power corrections to EEC in $\mathcal{N} = 4$ SYM

The NLP correction in EEC involves both the large spin power correction in the leading twist and the twist corrections in the large spin limit. The former is addressed using large spin perturbation theory in the previous section, while extracting the latter requires the dynamical information about next-leading twists. Remarkably, the crossing symmetry $\Phi(u,v) = \Phi(v,u)$ connects these two seemingly independent contributions. Using $\mathcal{F}^{(n)}(z,\bar{z}) = \frac{v}{u^3}\Phi^{(n)}(u,v)$, we get the NLL prediction for $\Phi^{(n)}$ at NLP in the double lightcone limit

$$\Phi^{(n)}(u,v) = \frac{(-1)^n}{2^n n!} L_u^n L_v^n \left\{\frac{1}{2} + (u+v) + \left[\left(\frac{n+1}{2}u + \frac{n}{3}v\right)\frac{1}{L_v} + (u \leftrightarrow v)\right] + \cdots\right\} \,, \quad n > 1 \,,$$

which agrees with the perturbative results up to three loops.

Now, we can transform the double lightcone limit of $\Phi(u,v)$ to the back-to-back limit of $\text{EEC}(\zeta)$ based on the position space definition (2). After detector integration, we obtain

$$\text{EEC}^{(n>1)} = \frac{(-1)^n}{2^n(n-1)!}\left[\frac{1}{2y}\left(L_y^{2n-1} + \cdots\right) + \left(\frac{n}{2n-1}L_y^{2n-1} + \frac{7n-5}{12}L_y^{2n-2} + \cdots\right) + \cdots\right] \,, \tag{25}$$

where $y = 1-\zeta$, which is in full agreement with the back-to-back limit expansion of full theory calculation up to $n = 3$ in [47].

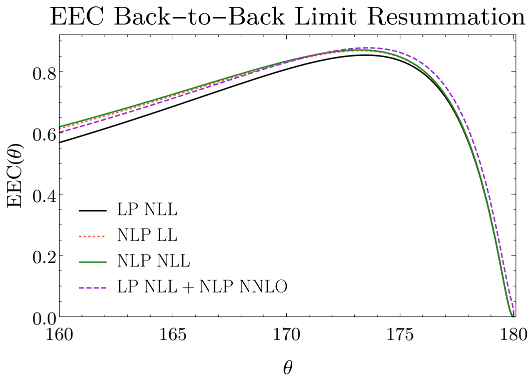

Figure 2: EEC as a function of θ in the back-to-back limit. We use $g^2/(4\pi) = 0.118$ to mimic the QCD strong coupling at Z pole. The dashed line refers to LP resummed to NLL, with the inclusion of NLP terms up to NNLO ($n \leq 3$).

After supplying the one-loop EEC to fix the constant at NLP, we can summing over n explicitly and obtain the following NLL formula at LP and NLP

$$\mathrm{EEC}(y) = -\frac{aL_y e^{-\frac{aL_y^2}{2}}}{4y} - \frac{1}{4}\left[\sqrt{\frac{\pi}{2}}\sqrt{a}\,\mathrm{erf}\left(\sqrt{\frac{a}{2}}L_y\right) + aL_y e^{-\frac{aL_y^2}{2}}\right] + \frac{a}{48}(7aL_y^2 - 4)e^{-\frac{aL_y^2}{2}} + \frac{a}{12} + \cdots\,, \tag{26}$$

where erf is the error function $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$. We plot the $\mathcal{N} = 4$ EEC in the back-to-back limit to illustrate the importance of NLP resummation in Figure 2. It can be seen that the LL and NLL series at NLP leads to substantial corrections for not too large θ.

6. Discussion

In this work, we have proposed a new method to resum Sudakov logarithms in EEC based on double lightcone OPE. In this method, the power corrections in the Sudakov region come from twist expansion and infinite spin expansion. Therefore, the Sudakov resummation is via the standard renormalization of local operators and the large spin summation with twist conformal blocks. In addition, we find crossing symmetry plays an important role in simplifying the calculation by relating higher twist information to the lower twist.

Our results motivates several future research directions. First of all, it is interesting to apply the same techniques to the QCD case, where one must deal with local correlators consisting of spinning local operators. The analytic results of EEC in QCD is available up to NLO recently [48–50], while the four point function of electromagnetic currents in QCD has also been computed at one loop [51]. We have checked that the lightcone OPE and the large spin perturbation theory can correctly predict the double lightcone limit of the one-loop electromagnetic currents correlator. Secondly, incorporating the running coupling effect into this method is important in the non-conformal theories, as it enters the NLL series and beyond in QCD. Thirdly, one can look forward to the generalization to more observables and a better comprehension the role of large spin physics and crossing symmetry in other conventional QCD event shapes.

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