

The quantum nature of the “minimal” SO(10) GUT

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The minimal non-supersymmetric SO(10) Grand Unified Theory consists of the scalar sector $\mathbf{45} \oplus \mathbf{126} \oplus \mathbf{10}_C$. Since this model is expected to provide robust proton decay predictions, its analysis is of great interest, but has been hampered by tree-level tachyonic instabilities and thus requires an assessment at one-loop level. We describe in these proceedings the latest progress in such efforts. We find that viable regions of parameter space exist where the scalar spectrum is non-tachyonic, perturbative and compatible with unification. The latest developments show, however, an obstruction in the Yukawa sector: realistic fermion masses require the Standard Model Higgs doublet to be fine-tuned to the EW scale in such a way that it is an admixture of states from both the $\mathbf{10}$ and $\mathbf{126}$, which turns out to be in tension with either tachyonicity or perturbativity of the scalar spectrum. This strongly indicates the model is not perturbatively viable, albeit due to a subtle and perhaps unexpected reason.

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1. Introduction

The most compact non-supersymmetric $SO(10)$ Grand Unified Theory (GUT) model is based on the scalar sector $\mathbf{45} \oplus \mathbf{126} \oplus \mathbf{10}_C$. While the representations $\mathbf{45}$ and $\mathbf{126}$ are used to break the GUT symmetry down to the Standard Model (SM) group, the representations $\mathbf{126}$ and $\mathbf{10}$ should allow for a realistic Yukawa-sector fit to the SM. Due to the economy with which representations are made use of, this is considered the “minimal” $SO(10)$ GUT model.

It has long been known that this model suffers from tachyonic instabilities at tree-level, but that one-loop corrections to the effective potential might provide a cure [1–4]. Although this greatly complicates its analysis, the model remains interesting due to the expectation of a robust proton decay prediction [5].

We present in these proceedings the latest developments in the analysis of the model based on our work in [6] and some further work in progress (to appear soon). We organize the paper as follows: we describe the model and define all associated quantities in Section 2, and then proceed with its analysis as a sequence of arguments in Section 3. We conclude in Section 4 pointing to strong indications the model is in fact not phenomenologically viable.

2. Model definition

The GUT model we consider is a Yang-Mills gauge theory with the gauge group $SO(10)$, and the following field content:

$$\text{fermions : } 3 \times \mathbf{16}_F, \quad \text{scalars : } \mathbf{45} \oplus \mathbf{126} \oplus \mathbf{10}_C. \quad (1)$$

Each fermionic representation $\mathbf{16}_F$ contains an entire generation of SM particles together with a right-handed neutrino ν^c . The scalar representation $\mathbf{45}$ is real, while $\mathbf{126}$ and $\mathbf{10}_C$ consist of complex scalars. Note: while the $\mathbf{10}$ is a self-conjugate (real) representation of the group $SO(10)$, we complexify it due to the requirements of the Yukawa sector.

We briefly present the structure and salient points of the scalar sector in Section 2.1 and of the Yukawa sector in Section 2.2.

2.1 Scalar sector

The representation $\mathbf{45}$ has 2 real SM-singlet fields, the $\mathbf{126}$ one complex SM-singlet field and the $\mathbf{10}$ no SM singlets, hence it is not involved in the spontaneous symmetry breaking process. We label the vacuum expectation values (VEVs) of SM-singlet fields as follows:

$$\langle (1, 1, 1, 0)_{45} \rangle \equiv \sqrt{3} \omega_{BL}, \quad \langle (1, 1, 3, 0)_{45} \rangle \equiv \sqrt{2} \omega_R, \quad \langle (1, 1, 3, +2)_{126} \rangle \equiv \sqrt{2} \sigma, \quad (2)$$

where for unambiguous identification we use labels $\langle R_S \rangle$, where R and S denote the left-right $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SO(10)$ origin of the SM-singlets, respectively.

Symmetry breaking to the SM group is envisioned to happen in two stages:

$$SO(10) \xrightarrow{\langle 45 \rangle} G \xrightarrow{\langle 126 \rangle} SU(3)_c \times SU(2)_L \times U(1)_Y, \quad (3)$$

case	direction of $\langle \mathbf{45} \rangle$	approximate relation	intermediate symmetry G
(a)	$\omega_{BL} = 0$	$ \omega_{BL} \ll \omega_R $	$SU(4)_C \times SU(2)_L \times U(1)_R$
(b)	$\omega_R = 0$	$ \omega_R \ll \omega_{BL} $	$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
(c)	$\omega_{BL} = \omega_R$	$\omega_{BL} \approx \omega_R$	$SU(5) \times U(1)$
(d)	$\omega_{BL} = -\omega_R$	$\omega_{BL} \approx -\omega_R$	$SU(5)' \times U(1)'$
(e)	generic	generic $ \omega_{BL}/\omega_R \sim \mathcal{O}(1)$	$SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$

Table 1: Various intermediate symmetries G that result from VEV direction of $\langle \mathbf{45} \rangle$. The subscript letter indicate c or C for color, L for left, R for right, $B - L$ for “baryon minus lepton number”, and primes denote the flipped-SU(5) case. Derived in e.g. [2].

where G is the intermediate symmetry group. It depends on the direction the VEV of the $\mathbf{45}$ takes, see Table 1. The 2nd breaking stage is triggered by the VEV σ in the $\mathbf{126}$, and establishes an intermediate scale that phenomenologically corresponds to the seesaw scale.

The scalar potential V can be split into two parts according to the presence of $\mathbf{10}_C$:

$$V = V_{SB} + V_{10}, \quad (4)$$

where V_{10} consists of all terms that involve the representation $\mathbf{10}$ and V_{SB} of those that do not. The part V_{10} will not be further considered here, while V_{SB} is relevant for symmetry breaking in Eq. (3) and takes the explicit form

$$\begin{aligned} V_{SB} = & -\frac{1}{4}\mu^2(\phi\phi)_0 + \frac{1}{4}a_0(\phi\phi)_0(\phi\phi)_0 + \frac{1}{4}a_2(\phi\phi)_2(\phi\phi)_2 - \frac{1}{5!}v^2(\Sigma\Sigma^*)_0 + \frac{1}{(5!)^2}\lambda_0(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \\ & + \frac{1}{(4!)^2}\lambda_2(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 + \frac{1}{(3!)^2(2!)^2}\lambda_4(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{1}{(3!)^2}\lambda'_4(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} + \\ & + \frac{1}{4!}i\tau(\phi)_2(\Sigma\Sigma^*)_2 + \frac{1}{2\cdot 5!}\alpha(\phi\phi)_0(\Sigma\Sigma^*)_0 + \frac{1}{4\cdot 3!}\beta_4(\phi\phi)_4(\Sigma\Sigma^*)_4 + \frac{1}{3!}\beta'_4(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} + \\ & + \frac{1}{(4!)^2}\eta_2(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{1}{4!}\gamma_2(\phi\phi)_2(\Sigma\Sigma)_2 + h.c.. \end{aligned} \quad (5)$$

We denote $\mathbf{45} \sim \phi_{ij}$ and $\mathbf{126} \sim \Sigma_{ijklm}$, where $SO(10)$ indices run from 1 to 10 and both tensors are completely antisymmetric. The parentheses indicate a contraction that leaves the number of indices specified in the subscript uncontracted, see e.g. [6] for details. The prefactors next to the parameters are conventional, and all parameters have real values except for the complex η_2 and γ_2 .

2.2 Yukawa sector

The Yukawa sector of the model consists of operators in which fermions couple to scalar representations $\mathbf{10}$ and $\mathbf{126}$, schematically written as

$$\mathcal{L}_{\text{Yuk}} = \mathbf{16}_F^a \left(Y_{10}^{ab} \mathbf{10} + \tilde{Y}_{10}^{ab} \mathbf{10}^* + Y_{126}^{ab} \mathbf{126}^* \right) \mathbf{16}_F^b + h.c.. \quad (6)$$

We denoted family indices by a and b , and there are three 3×3 symmetric Yukawa matrices in the model: Y_{10} , \tilde{Y}_{10} and Y_{126} . All gauge and Lorentz indices have been suppressed in the notation.

There are 2 SM-doublets $(1, 2, +\frac{1}{2})$ and 2 anti-doublets $(1, 2, -\frac{1}{2})$ in the scalar sector, and we label their electroweak (EW) VEVs by

$$v_{10}^u = \langle (1, 2, +\frac{1}{2})_{10} \rangle, \quad v_{126}^u = \langle (1, 2, +\frac{1}{2})_{126^*} \rangle, \quad (7)$$

$$v_{10}^d = \langle (1, 2, -\frac{1}{2})_{10} \rangle, \quad v_{126}^d = \langle (1, 2, -\frac{1}{2})_{126^*} \rangle. \quad (8)$$

The SM Higgs doublet is a linear combination of all four doublet fields, from which we get the normalization condition $v_{SM}^2 = |v_{10}^u|^2 + |v_{10}^d|^2 + |v_{126}^u|^2 + |v_{126}^d|^2$ with $v_{SM} \approx 174$ GeV. These definitions lead to fermion mass matrices

$$M_U = Y_{10} v_{10}^u + \tilde{Y}_{10} v_{10}^{d*} + Y_{126} v_{126}^u, \quad M_D = Y_{10} v_{10}^d + \tilde{Y}_{10} v_{10}^{u*} + Y_{126} v_{126}^d, \quad (9)$$

$$M_\nu^{\text{Dirac}} = Y_{10} v_{10}^u + \tilde{Y}_{10} v_{10}^{d*} - 3Y_{126} v_{126}^u, \quad M_E = Y_{10} v_{10}^d + \tilde{Y}_{10} v_{10}^{u*} - 3Y_{126} v_{126}^d, \quad (10)$$

where U , D , E and ν denote the up, down, charged lepton and neutrino sectors, respectively.

Note that having a complex representation **10** leads to 2 different Yukawa terms involving it, hence the two different Yukawa matrices Y_{10} and \tilde{Y}_{10} . Complexification of the **10** is necessary to avoid the relation $v_{10}^u = v_{10}^{d*}$, which would prevent a proper fit of the up relative to the down sector.

The simplest case of a renormalizable $SO(10)$ -symmetric Yukawa sector, in which the SM masses and mixing angles can be successfully fit, consists of the terms Y_{10} and Y_{126} , e.g. see [7]. Eq. (6) extends it by also having the \tilde{Y}_{10} -term, so it can clearly be fit as well.

Finally, we comment on the necessity of \tilde{Y}_{10} . Although such a Yukawa term can be forbidden by introducing a global $U(1)$ Peccei-Quinn (PQ) symmetry under which the **10** and **126** are charged, this also forbids the γ_2 term in the scalar potential of Eq. (5) (**45** is real, hence it has zero PQ charge). The γ_2 parameter, however, was found to be crucial for the scalar spectrum to be non-tachyonic, cf. [6], hence PQ symmetry cannot be imposed here.

3. Progress in model analysis

Having specified the model under consideration in Section 2, we now turn to the progress in analyzing its viability. As we shall see below, the study is complicated by the realization that the lowest consistent perturbative order for its description is at one-loop level rather than tree level, implying that the model is indeed quantum in nature.

We present the developments as a series of key insights. This provides a good summary of the existing literature, the newest of which is our work in [6]. We finish by going beyond and presenting a key insight in our current research (to appear soon), which seems to offer a final negative judgment on the model’s viability.

3.1 Tachyonic instabilities

The first crucial insight is gained by determining the vacuum and the scalar spectrum of the model. For later convenience, let us first define the *dimensionless ratio* χ of VEVs as

$$\chi := \frac{\omega_{BL}\omega_R}{|\sigma|^2}. \quad (11)$$

The stationarity conditions for the scalar potential V come only from V_{SB} of Eq. (5), and solving them for massive parameters μ^2 , v^2 and τ gives (see e.g. [4])

$$\mu^2 = (12a_0 + 2a_2)\omega_{BL}^2 + (8a_0 + 2a_2)\omega_R^2 + (4\alpha + 4\beta'_4 + 2a_2\chi)|\sigma|^2, \quad (12)$$

$$v^2 = 3(\alpha + 4\beta'_4)\omega_{BL}^2 + 2(\alpha + 3\beta'_4)\omega_R^2 + (12\beta'_4\chi + 4\lambda_0)|\sigma|^2 + a_2\chi(\omega_{BL} + \omega_R)(3\omega_{BL} + 2\omega_R), \quad (13)$$

$$\tau = 2\beta'_4(3\omega_{BL} + 2\omega_R) + a_2\chi(\omega_{BL} + \omega_R). \quad (14)$$

Inserting this solution into the second derivative of the potential, we obtain the following expressions for the tree-level mass-squares of SM color-octets $(8, 1, 0)$ and weak-triplets $(1, 3, 0)$:

$$M^2(8, 1, 0) = +2a_2(\omega_{BL} - \omega_R)(\omega_R + 2\omega_{BL}), \quad (15)$$

$$M^2(1, 3, 0) = -2a_2(\omega_{BL} - \omega_R)(2\omega_R + \omega_{BL}). \quad (16)$$

These expressions are both non-tachyonic only if $a_2 > 0$ and the regime $\omega_{BL}/\omega_R \in (-2, -\frac{1}{2})$ applies. This suggests a flipped-SU(5) or left-right intermediate symmetry, i.e. cases (d) and (e) in Table 1, which is at odds with unification constraints [1, 8, 9].

It has been first argued in [2], and more specifically for the $\mathbf{45} \oplus \mathbf{126}$ Higgs model in [3], that this problem can be cured and phenomenologically relevant cases (a) and (b) might be reached if quantum corrections to the scalar potential are taken into account. One might hope to cure the tachyonic instabilities in the octet and triplet, provided that the a_2 parameter is small, so that their tree-level masses are spuriously small. The states of Eqs. (15)–(16) with a_2 -proportional mass-squares have been referred to in the literature as *pseudo-Goldstone bosons* (PGBs).

An analytic computation of the one-loop spectrum in the $\gamma_2, \eta_2, \sigma \rightarrow 0$ limit has been performed in [4], with all indication that the tachyonic instabilities can indeed be cured. It has also been pointed out that one of the SM-singlet $(1, 1, 0)$ mass-eigenstates should also function as a PGB, along with any other SM representations required to complete intermediate-symmetry multiplets.

3.2 Symmetry breaking and perturbativity

A further observation regarding symmetry breaking patterns can be made by considering the role of the dimensionless ratio χ , cf. [6].

To have the phenomenologically motivated seesaw-scale sufficiently below the GUT scale, a hierarchy $|\sigma|^2 \ll \omega_{BL}^2 + \omega_R^2$ is required, i.e. the VEV $\langle \mathbf{126} \rangle$ is much smaller than $\langle \mathbf{45} \rangle$. Consider the role of χ in the vacuum solution of Eqs. (12)–(14). For the massive parameters to be perturbative, i.e. sufficiently below the Planck scale, either $|\chi| \lesssim 1$ or $|\omega_{BL} + \omega_R| \approx |\sigma|^2$. Since the latter option leads to case (d) and is incompatible with unification, see Section 3.1, the former condition must apply, hence either $|\omega_{BL}| \ll |\sigma| \ll |\omega_R|$ or $|\omega_R| \ll |\sigma| \ll |\omega_{BL}|$, i.e. only cases (a) and (b) from Table 1 can be phenomenologically relevant for the model.

3.3 Restrictions on parameter space

Still further insight has been gained in our recent work [6] with a full numerical analysis at one-loop for the Higgs model $\mathbf{45} \oplus \mathbf{126}$, i.e. restricted to the part V_{SB} of V in Eq. (4).

The ability to compute the entire one-loop spectrum opens up the possibility to assess perturbativity. There is no canonical or unambiguous way to judge whether a particular parameter point of the model is perturbative; nevertheless we constructed two quantitative measures \bar{t} and $\bar{\Delta}$ that allow for comparisons of points and reveal something about the degree of perturbativity. The measure \bar{t} is defined as the geometric mean

$$\bar{t} := \sqrt{t_- t_+}, \quad (17)$$

where $t_{\pm} := \log_{10}[\mu_{R\pm}/\mu_R]$ with μ_R being the renormalization scale of the chosen parameter values and $\mu_{R\pm}$ the higher (lower) scale above (below) which the beta-functions for dimensionless scalar parameters blow up.

The measure $\bar{\Delta}$ is defined as the ratio of the largest one-loop correction to mass-squares $\delta m_{(1)}^2$ searched for among all fields, and the average tree-level mass-square $\bar{m}_{(0)}^2$ of the heavy fields (those at GUT scale, excluding any PGBs):

$$\bar{\Delta} := \frac{\max \delta m_{(1)}^2}{\bar{m}_{(0)}^2}. \quad (18)$$

Taking advantage of all available information from the one-loop calculation, a parameter point is considered viable if it fulfills the following three requirements:

1. **Non-tachyonicity:** all one-loop scalar mass-squares of physical fields must be non-negative.
2. **Perturbativity:** the one-loop corrections and RGE running are sufficiently under control by taking the very mild restrictions $\bar{\Delta} < 1$ and $t_+ > 0.5$.
3. **Unification:** a top-down one-loop RGE analysis of gauge couplings gives successful unification, i.e. the low energy SM gauge couplings can be successfully fit.

Conditions 1 and 2 stem purely from considerations of mathematical consistency, while condition 3 is phenomenological.

A comprehensive numerical scan of the parameter space was performed in [6] for the two relevant cases (a) and (b). Viable regions were found in both cases, but the viability constraints severely restrict the allowed parameter values, as demonstrated by Figure 1. The results also show that case (a) admits points with a higher degree of perturbativity and also has energy scales consistent with unification and phenomenology, unlike case (b). Case (a) has the seesaw scale in a reasonable pheno range between 10^{11} – $10^{11.8}$ GeV and the GUT scale at around 14.9 GeV, while case (b) has a too-low intermediate scale of 10^8 GeV and a GUT scale 10^{18} GeV, which is too close to the Planck scale for renormalizability or perturbativity to apply.

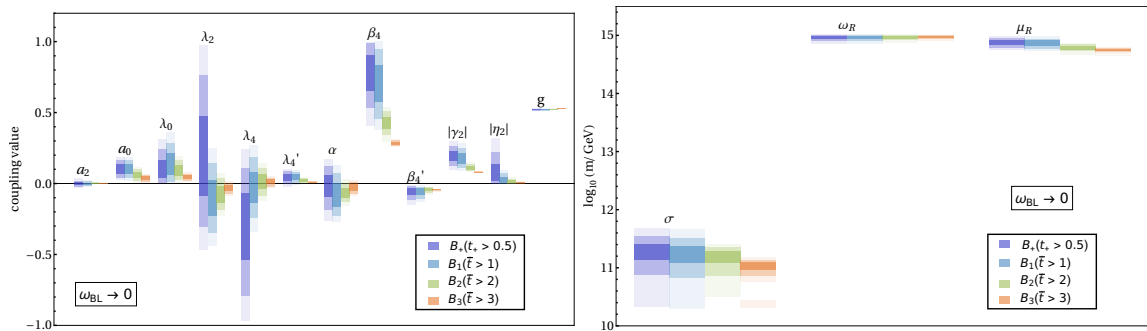


Figure 1: The allowed ranges of dimensionless (left) and massive (right) parameters of the $45 \oplus 126$ Higgs model for case (a). The various colors indicate searches with increasingly stricter \bar{t} perturbativity measure, allowing for ever smaller parameter ranges. The increasing transparency levels of bars indicate 1-, 2- and 3-sigma ranges in the dataset.

We therefore conclude that only case (a) remains phenomenologically viable, albeit with a GUT scale that is somewhat low for the experimental bounds on proton decay. We do caution that a more thorough proton decay analysis is required before dismissal, since two-loop RGE effects or tuning some scalar states to be lower in mass might sufficiently raise the unification scale.

3.4 Doublet fine-tuning and beyond

We’ve now reached the latest development in the story of the minimal $SO(10)$ model that is part of our current work in progress.

As discussed in Section 2.2, there are four SM doublets $(1, 2, \pm 1/2)$, i.e. two in each of the representations **126** and **10**. Their tree-level mass-square matrix can therefore be written schematically based on their $SO(10)$ -origin in terms of 2×2 blocks as

$$M^2(1, 2, +\frac{1}{2}) = \begin{pmatrix} M_{126}^2 & M_{\text{mix}}^2 \\ M_{\text{mix}}^{2\dagger} & M_{10}^2 \end{pmatrix}. \quad (19)$$

The block M_{126}^2 depends on the parameters in V_{SB} , while M_{10}^2 and the mixing block M_{mix}^2 involve the representation **10** and thus consist of parameters from V_{10} , cf. Eq. (4). In the limit of case (a) in Table 1, the sizes of the diagonal-block entries are $M_{10}^2, M_{126}^2 \sim \omega_R^2$, while $M_{\text{mix}}^2 \sim |\sigma|^2$.

A SM Higgs doublet at 125 GeV requires one mass in Eq. (19) to be fine-tuned from the GUT scale down to the EW scale. Furthermore, it was argued in Section 2.2 that a Yukawa-sector fit is possible, but this assumes the VEVs of Eqs. (7) and (8) can all take fit-specified non-zero values. The SM Higgs must therefore be an admixture of both doublets from the **10** as well as **126**. The only way this is possible in Eq. (19) is to first mini-tune one doublet combination in each of the diagonal blocks M_{126}^2 and M_{10}^2 down to $|\sigma|^2$, so that M_{mix} can mix the two states, and then perform one full fine-tuning down to the EW scale.

Consider now the tree-level masses of the doublet and two more states in case (a), cf. [6]:

$$M^2(1, 2, +\frac{1}{2})_{\text{lightest}} = \left(\frac{\beta_4}{2} - 5\beta'_4 - 2a_2\chi - \sqrt{(4\beta'_4 + a_2\chi)^2 + 4|\gamma_2|^2} \right) \omega_R^2, \quad (20)$$

$$M^2(\bar{3}, 1, +\frac{1}{3})_{\text{lightest}} = (\beta_4 - 4\beta'_4 - 2a_2\chi - 4|\gamma_2|) \omega_R^2, \quad (21)$$

$$M^2(1, 3, -1) = -2(2\beta'_4 + a_2\chi) \omega_R^2. \quad (22)$$

Figure 1 shows that the viable region is found inside the multi-dimensional box defined by

$$\beta_4 \in [0.2, 1], \quad \beta'_4 \in [-0.2, -0.01], \quad a_2 \in [-0.05, 0.05], \quad \chi \in [-1, 1], \quad |\gamma_2| \in [0.1, 0.3]. \quad (23)$$

Fine-tuning the doublet mass-square in Eq. (20) to effectively zero (or $|\sigma|^2$) in this region results in tachyonicity in either Eq. (21) or (22). Thus an obstruction exists for the required mini-tuning in the M_{126}^2 block.

There is only one possible remedy to this obstruction. Since all mass-square entries receive quantum corrections, one might hope for *perturbative fine-tuning*, where the mini-tuning of M_{126}^2 is performed only down to next-order corrections, and hope these corrections then align properly for a further cancellation. This is only a necessary and not a sufficient condition, since there is no guarantee that the next-order corrections of the doublet indeed align for cancellation, while at the same time those of states in Eqs. (21) and (22) align for non-tachyonicity.

We are currently working on a new parameter-space scan which includes perturbative fine-tuning of doublets. Preliminary results show that this can be marginally achieved at tree-level, but the one-loop corrections do not cooperate well: the mini-tuning in M_{126}^2 at one loop cannot be suppressed down to the estimated two-loop level in perturbative regions, so the obstruction

becomes worse at one-loop compared to tree-level. This analysis required a proper assessment of one-loop corrections also from hitherto ignored parameters in V_{10} and goes beyond the scope of these proceedings.

4. Conclusion

We summarized in these proceedings the progress made in analyzing the minimal $SO(10)$ GUT model with the scalar sector consisting of $\mathbf{45} \oplus \mathbf{126} \oplus \mathbf{10}_C$. Emphasizing the most recent developments, we report on our recent work [6] as well as progress in ongoing research.

We explicitly showed in [6] by numerical calculation that in some parameter-space regions the tree-level tachyonic instabilities in the scalar potential are indeed cured by one-loop corrections, as anticipated. The model is thus quantum in nature, i.e. the first consistent perturbative calculation is at one-loop. It was also shown in the same work that the breaking pattern resulting in a $SU(4)_C \times SU(2)_L \times U(1)_R$ intermediate symmetry is the only phenomenologically viable option.

Our ongoing research focuses on an observation relevant for the Yukawa sector: the fine-tuning required for a realistic SM Higgs doublet is obstructed in perturbative regions of the parameter space by other scalar states becoming tachyonic. Preliminary results show that this problem most probably cannot be overcome at any perturbative order, strongly indicating that the minimal $SO(10)$ GUT model is in fact not perturbatively viable.

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References

- [1] S. Bertolini, L. Di Luzio and M. Malinsky, *Phys. Rev. D* **80** (2009), 015013 [arXiv:0903.4049 [hep-ph]].
- [2] S. Bertolini, L. Di Luzio and M. Malinsky, *Phys. Rev. D* **81** (2010), 035015 [arXiv:0912.1796 [hep-ph]].
- [3] S. Bertolini, L. Di Luzio and M. Malinsky, *Phys. Rev. D* **85** (2012), 095014 [arXiv:1202.0807 [hep-ph]].
- [4] L. Gráf, M. Malinský, T. Mede and V. Susič, *Phys. Rev. D* **95** (2017) no.7, 075007 [arXiv:1611.01021 [hep-ph]].
- [5] H. Kolešová and M. Malinský, *Phys. Rev. D* **90** (2014) no.11, 115001 [arXiv:1409.4961 [hep-ph]].
- [6] K. Jarkovská, M. Malinský, T. Mede and V. Susič, *Phys. Rev. D* **105** (2022) no.9, 095003 [arXiv:2109.06784 [hep-ph]].
- [7] T. Ohlsson and M. Pernow, *JHEP* **06** (2019), 085 [arXiv:1903.08241 [hep-ph]].
- [8] N. G. Deshpande, E. Keith and P. B. Pal, *Phys. Rev. D* **47** (1993), 2892-2896 [arXiv:hep-ph/9211232 [hep-ph]].
- [9] C. S. Aulakh, B. Bajc, A. Melfo, A. Rasin and G. Senjanovic, *Phys. Lett. B* **460** (1999), 325-332 [arXiv:hep-ph/9904352 [hep-ph]].
- [10] S. Bertolini, L. Di Luzio and M. Malinsky, *Phys. Rev. D* **87** (2013) no.8, 085020 [arXiv:1302.3401 [hep-ph]].