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Charged Lepton Flavour Violation (phenomenology)

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The discovery of neutrino oscillations is the first laboratory evidence of New Physics beyond the Standard Model. Oscillating neutrinos necessarily imply that neutrinos are massive and that (neutral) lepton flavour is violated. A signal of charged lepton flavour violation however so far eludes experimental discovery. In this proceedings we review some phenomenological implications of current experimental bounds (and future sensitivities) on observables related to charged leptons, with a particular attention to charged lepton flavour violating processes. In connection to models aiming at providing an origin to neutrino masses, we also highlight some phenomenological implications of leptonic CP violation on charged lepton flavour violation observables.

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1. Introduction

The Standard Model of Particle Physics (SM) provides an extraordinarily successful and yet simple description of Nature at its smallest scales. Despite its exceptional success, it is now firmly established that the SM cannot account for a certain number of observations and one must thus envisage extensions of the model capable of accommodating experimental data. Moreover, a strong theoretical interest also fuels the study of "New Physics beyond the SM (BSM)", as the latter might provide a solution for some of the theoretical puzzles of the SM.

In its original formulation, the SM features strictly massless neutrinos which implies the conservation of total lepton number as well as individual lepton flavour. Furthermore, in stark contrast to the SM quark sector, the SM lepton sector does not have an intrinsic source of CP violation such that electric dipole moments (EDMs) of charged leptons are immensely suppressed (generated at the 4-loop order).

Up to the present moment, neutrino oscillations remain the only confirmed evidence for New Physics observed in a laboratory, implying that neutrinos have (tiny) masses and that neutral lepton flavour is violated in nature. Oscillating and massive neutrinos open the door to lepton flavour violation and new sources of CP violation.

In order to (indirectly) search for the New Physics that can potentially address the SM's observational and theoretical shortcomings, a useful approach is to test the (accidental) symmetries of the SM, which might not be present in New Physics models. In particular, lepton flavour conservation and lepton flavour universality (LFU) are broken by the presence of massive neutrinos. Thus, observables involving charged leptons, and muons in particular, offer uniquely versatile probes of various BSM constructions.

2. Lepton observables

In this section we briefly review the current status and future prospects of several muon-related observables well-suited to indirectly search for New Physics (NP). In particular, we focus on the muon anomalous magnetic moment, lepton flavour universality violation (LFUV) and charged lepton flavour violation (cLFV).

2.1 Lepton moments: $(g-2)_\ell$

The anomalous magnetic moment of a charged lepton ℓ ($a_\ell \equiv (g_\ell - 2)/2$) allows probing numerous aspects of the SM, and is also instrumental in determining some of its fundamental quantities.

Concerning muons, and following the release of the FNAL results [1], the experimental average and the latter SM prediction (following "Muon $g-2$ Theory Inititative [2]") lead to a tension between theory and observation, $\Delta a_\mu \equiv a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp}} = 251\,(59) \times 10^{-11}$ corresponding to a significance of $\sim 4.2\,\sigma$. Should this be confirmed[1], the need for NP capable of accounting for such a sizeable discrepancy is manifest.

[1]Recent lattice QCD calculations [3] of the leading-order hadronic vacuum polarisation contribution might suggest a much lower significance of the anomaly, around $\sim 1.5\sigma$.

The magnetic moment of the electron has also been at the origin of possible new tensions, upon comparison of the experimental value to the SM prediction (which depends on the value of α_e that is used for the computation of the latter): using α_e as extracted from measurements using Cs atoms one is led to $\Delta a_e^{\mathrm{Cs}} = -0.88\,(0.36) \times 10^{-12}$ corresponding to a deviation of $\sim -2.5\,\sigma$ [4, 5]. A more recent estimation of α_e was obtained, this time relying on Rubidium atoms, and the new determination of α_e (implying an overall deviation above the $5\,\sigma$ level for α_e) now suggests milder discrepancies between observation and theory prediction, $\Delta a_e^{\mathrm{Rb}} = 0.48\,(0.30){\times}10^{-12}$ corresponding to $\sim 1.7\,\sigma$ [6]. Other than (possibly) signalling deviations from the SM expectation, it is interesting to notice the potential impact of both Δa_e^{Cs} and Δa_μ: other than having an opposite sign, the ratio $\Delta a_\mu/\Delta a_e$ does not exhibit the naïve scaling $\sim m_\mu^2/m_e^2$ (expected from the magnetic dipole operator, in which a mass insertion of the SM lepton is responsible for the required chirality flip). Such a behaviour renders a common explanation of both tensions quite challenging, calling upon a departure from a minimal flavour violation hypothesis, or from single new particle extensions of the SM (coupling to charged leptons). Notice that the pattern in both Δa_e^{Cs} and Δa_μ can be also suggestive of a violation of LFU.

2.2 Lepton flavour universality

In the Standard Model, charged leptons are only distinguishable due to their masses. In particular, all electroweak couplings to gauge bosons are blind to lepton flavour, leading to an accidental symmetry called lepton flavour universality (LFU), whose validity has been determined to a very high accuracy for instance in $Z \to \ell^+\ell^-$ and $W^\pm \to \ell^\pm \nu$ $(\ell = e, \mu, \tau)$ decays [7].

Lepton flavour universality can also be tested in charged and neutral (semi-)leptonic meson decays, such as kaon and pion decays. In order to test LFU, one constructs ratios of the helicity suppressed widths $R_K^{e\mu} \equiv \frac{\Gamma(K\to e\bar\nu)}{\Gamma(K\to\mu\bar\nu)}$ with $R_K^{e\mu\,\mathrm{SM}} = (2.477 \pm 0.001) \times 10^{-5}$ [8] and $R_K^{e\mu\,\mathrm{exp}} = (2.488 \pm 0.009) \times 10^{-5}$ [7], thus also being highly consistent with the SM prediction. Equivalent ratios can be constructed for pion decays, with $R_\pi^{e\mu\,\mathrm{SM}} = (1.2354 \pm 0.0002) \times 10^{-4}$ [8] and $R_\pi^{e\mu\,\mathrm{exp}} = (1.230 \pm 0.004) \times 10^{-4}$ [7], which also confirm lepton universality as predicted by the SM.

However, during the last decade, hints on the violation of LFU in $b \to c\ell\nu$ and $b \to s\ell\ell$ decays have begun to emerge (and to fade away), in mounting tension with respect to the SM expectations. In particular, measurements of the "theoretically clean" ratios of branching ratios $R_{D^{(*)}} = \mathrm{BR}(B \to D^{(*)}\tau\nu)/\mathrm{BR}(B \to D^{(*)}\ell\nu)$ [9] and $R_{K^{(*)}} = \mathrm{BR}(B \to K^{(*)}\mu\mu)/\mathrm{BR}(B \to K^{(*)}ee)$ [10, 11] deviate around $2-3\,\sigma$ from their theoretical predictions, which are, up to phase space suppression, expected to be unity in the SM. A recent update to the measurement of R_K and R_{K^*} of the LHCb collaboration [12] is however consistent with the SM prediction.

A hint on LFUV, as currently present in the $b \to c\ell\nu$ system, can also be suggestive of lepton flavour violating New Physics [13].

2.3 Muon cLFV

Muons are possibly the best laboratory to look for cLFV, since they can be abundantly produced and have a comparatively long lifetime. Furthermore, due to their low mass, the number of kinematically allowed decay channels, flavour violating or not, is relatively small and the final states can be studied with great precision. Very high intensity muon beams are possible (obtained at meson factories and proton accelerators), allowing for a great variety of muon dedicated experiments with extremely high sensitivities. In view of this, it comes with no surprise that the best available experimental sensitivities, and consequently the best available bounds on cLFV processes, arise from rare muon processes.

The most minimal SM extension (adding three right-handed neutrinos ad-hoc) that accommodates neutrino oscillation data would in principle allow for cLFV transitions. However, due to the unitarity of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, and to the tiny differences of the neutrino masses, there is a strong GIM cancellation taking place, so that the expected rates are vanishingly small. For instance the prediction for the rate of $\mu \to e\gamma$ in this framework, using the current experimental constraints on neutrino mixing, is approximately given by[14, 15]

$$\text{BR}(\mu \to e\gamma) \simeq \frac{3\alpha_e}{32\pi}\left|\sum_{i=1}^{3} U_{ei}U_{\mu i}^{*}\frac{m_{\nu_i}^2}{M_W^2}\right|^2 \simeq O(10^{-55}), \tag{1}$$

clearly lying beyond the reach of any experimental sensitivity. Similar (extremely small) values are found for processes such as $\mu \to eee$ decays and the analogous lepton flavour violating τ decays. The observation of such cLFV signals would thus imply that more involved BSM extensions are needed in order to simultaneously explain the origin of neutrino masses and to interpret a possible cLFV signal. Any observation of cLFV would imply new degrees of freedom: the SM must be non-trivially extended.

In addition to the radiative and three-body decays ($\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$), several facilities are dedicated to studying muonic atoms. Muonic atoms are formed when a muon is "stopped" in some target material (N), usually very pure elements. The bound muon decays via interactions with the target nucleus, either exchanging a virtual photon, or, in the presence of New Physics, undergoing some non-electromagnetic interaction. In the SM, there are two possible outcomes. Either the muon decays in orbit (DIO) into an electron and two neutrinos, or it is captured by the target nucleus via inverse β decay. In the presence of New Physics, the exotic process of neutrinoless muon capture can occur, in which the electron is produced with sufficient kinetic energy to escape the Coulomb potential of the target nucleus, which can be left in the ground state, or in an excited one. Usually dominating, and from an experimental point of view the most advantageous, is the first case, called "coherent capture". This process is usually referred to as "$\mu - e$ conversion" and the associated observable is defined as

$$\text{CR}(\mu - e, \text{N}) = \frac{\Gamma(\mu^- + \text{N} \to e^- + \text{N})}{\Gamma(\mu^- + \text{N} \to \text{all captures})}, \tag{2}$$

which from a theoretical point of view has the additional advantage that most of the nuclear form factors cancel out, only the overlap integrals between the nuclear and leptonic wave function remain to be computed [16].

In the presence of lepton number violating interactions, another neutrinoless $\mu - e$ conversion can take place, given by

$$\mu^- \to (A, Z) \to e^+(A, Z-2)^{(*)}, \tag{3}$$

in which the the final state nucleus can be in its ground state or an excited one. Here, contrary to the $\mu^- - e^-$ conversion, no coherent enhancement is possible since the final and initial state nuclei are necessarily different from each other. Due to its LNV nature, this process is closely related to neutrinoless double-beta decay. From the theoretical perspective there is however a caveat; all but one of the nuclear form factors are presently unknown [17–19].

Another cLFV process in muonic atoms was proposed in [20]. It consists of a bound $1s$ muon and a bound $1s$ electron converting into a pair of electrons, and has been identified as potentially complementary to other cLFV muon processes:

$$\mu^- e^- \to e^- e^- . \tag{4}$$

As it has been pointed out in [20], it offers several experimental advantages. On the one hand, the experimental signal consists of two (almost) back-to-back emitted electrons with the same energy. On the other hand, this process is enhanced by the Coulomb potential of the nucleus, with respect to other observables in muonic atoms. So far, this process has not been experimentally investigated, but it would potentially offer complementary information of the flavour structure of lepton flavour violating NP.

Further interesting observables concern Muonium (Mu). Muonium is a Coulomb bound state consisting of an electron and an anti-muon ($e^-\mu^+$) which is formed when a μ^+ slows down inside matter and captures an electron. Being free of hadronic uncertainties, this hydrogen-like bound state is well described by electroweak interactions and is used to study fundamental constants of the SM, or search for deviations from the SM induced by the presence of possible New Physics interactions. Concerning cLFV transitions, one can study the spontaneous conversion of Muonium into anti-Muonium ($\overline{\mathrm{Mu}} = e^+\mu^-$) and the cLFV decay of Muonium, $\mathrm{Mu} \to e^+e^-$. An observation of these would again be a clear signal of New Physics.

2.4 More cLFV

Due to their large mass and consequently their large phase space, τ-leptons offer a vast array of cLFV signatures. Besides the radiative and three-body cLFV decays in full analogy to the muon sector, there are also numerous semi-leptonic cLFV decays into a lighter lepton and one or two mesons[2]. Studying cLFV decays across all lepton families is paramount to the understanding of the underlying New Physics flavour structures.

In addition to the radiative decays ($\tau \to e\gamma$ and $\tau \to \mu\gamma$) and same-lepton three-body decays ($\tau \to eee$ and $\tau \to \mu\mu\mu$), four other fully leptonic cLFV final states are possible:

$$\tau^- \to \mu^- e^+ e^- , \qquad \tau^- \to e^- \mu^+ \mu^- , \tag{5}$$

$$\tau^- \to \mu^- e^+ \mu^- , \qquad \tau^- \to e^- \mu^+ e^- , \tag{6}$$

in which the decays of the second row correspond to a “double” flavour violation. Thus, depending on the underlying New Physics framework, the different (charge) signatures can have very distinct amplitudes. Furthermore, the different semi-leptonic channels can offer very distinct probes of New Physics. Assuming there is only one meson in the final state, τ decays into $q\bar{q}$ and a lighter lepton are of particular interest, for instance $\tau \to \phi\mu$, because in this case there can be a resonant enhancement of the cLFV process. For instance, the Belle experiment has searched for 46 distinct cLFV τ decay modes, and Belle II is expected to significantly improve the obtained bounds [21].

[2] Searches for lepton and baryon number violating τ decays have also been conducted, for example $\tau \to p\mu^+\mu^-$.

Many experiments have searched for signals of cLFV in the decays of an extensive array of neutral and charged mesons. These processes probe $q \to q^{(\prime)} \ell_\alpha \ell_\beta$ contact interactions, possibly accompanied by another final state meson. The most stringent bounds have been obtained for K_L decays, but results for heavy meson decays have nevertheless reached an impressive level.

At higher energies, and in the presence of cLFV New Physics, also Z- and Higgs-bosons can undergo cLFV decays. Furthermore, cLFV transitions of charged leptons are often (depending on the underlying model) at least partly mediated via cLFV Z-penguin (also Higgs-penguin) diagrams, so that studying cLFV Z and Higgs decays offer important complementary information, and might help disentangling New Physics scenarios.

Currently, there is a vast world-wide array of dedicated experiments and searches, at different energy scales, aiming at discovering cLFV transitions. In Table 1 we list current experimental bounds and future sensitivities for some of the muon observables (see also [38]).

Observable	Current bound	Future Sensitivity
$\mathrm{BR}(\mu \to e\gamma)$	$< 4.2 \times 10^{-13}$ (MEG [22])	6×10^{-14} (MEG II [23])
$\mathrm{BR}(\tau \to e\gamma)$	$< 3.3 \times 10^{-8}$ (BaBar [24])	3×10^{-9} (Belle II [25])
$\mathrm{BR}(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$ (BaBar [24])	10^{-9} (Belle II [25])
$\mathrm{BR}(\mu \to 3e)$	$< 1.0 \times 10^{-12}$ (SINDRUM [26])	$10^{-15(-16)}$ (Mu3e [27])
$\mathrm{BR}(\tau \to 3e)$	$< 2.7 \times 10^{-8}$ (Belle [28])	5×10^{-10} (Belle II [25])
$\mathrm{BR}(\tau \to 3\mu)$	$< 3.3 \times 10^{-8}$ (Belle [28])	5×10^{-10} (Belle II [25])
$\mathrm{CR}(\mu - e, \mathrm{N})$	$< 7 \times 10^{-13}$ (Au, SINDRUM [29])	10^{-14} (SiC, DeeMe [30])
		2.6×10^{-17} (Al, COMET [31, 32])
		8×10^{-17} (Al, Mu2e [33])
$\mathrm{BR}(Z \to e^\pm\mu^\mp)$	$< 4.2 \times 10^{-7}$ (ATLAS [34])	$\mathcal{O}(10^{-10})$ (FCC-ee [35]
$\mathrm{BR}(Z \to e^\pm\tau^\mp)$	$< 5.2 \times 10^{-6}$ (OPAL [36])	$\mathcal{O}(10^{-10})$ (FCC-ee [35]
$\mathrm{BR}(Z \to \mu^\pm\tau^\mp)$	$< 5.4 \times 10^{-6}$ (OPAL [36])	$\mathcal{O}(10^{-10})$ (FCC-ee [35]
$\mathrm{BR}(h \to e^\pm\mu^\mp)$	$< 6.1 \times 10^{-5}$ [7]	1.2×10^{-5} (FCC-ee [37])
$\mathrm{BR}(h \to e^\pm\tau^\mp)$	$< 4.7 \times 10^{-3}$ [7]	1.5×10^{-4} (FCC-ee [37])
$\mathrm{BR}(h \to \mu^\pm\tau^\mp)$	$< 2.5 \times 10^{-3}$ [7]	1.5×10^{-4} (FCC-ee [37])

Table 1: Current experimental bounds and future sensitivities on some of the most relevant cLFV observables. All limits are given at 90% C.L., see also [38].

3. The probing power of cLFV

As discussed in the above, motivations to extend the SM are abundant and there are several reasons to believe that there is New Physics, in the form of new particles and/or interactions, somewhere between the electroweak and the Planck scale. The non-observation of cLFV signals and the implied experimental upper bounds on the associated processes, consequently lead to tight constraints on the parameters of New Physics models that could in principle predict sizeable rates

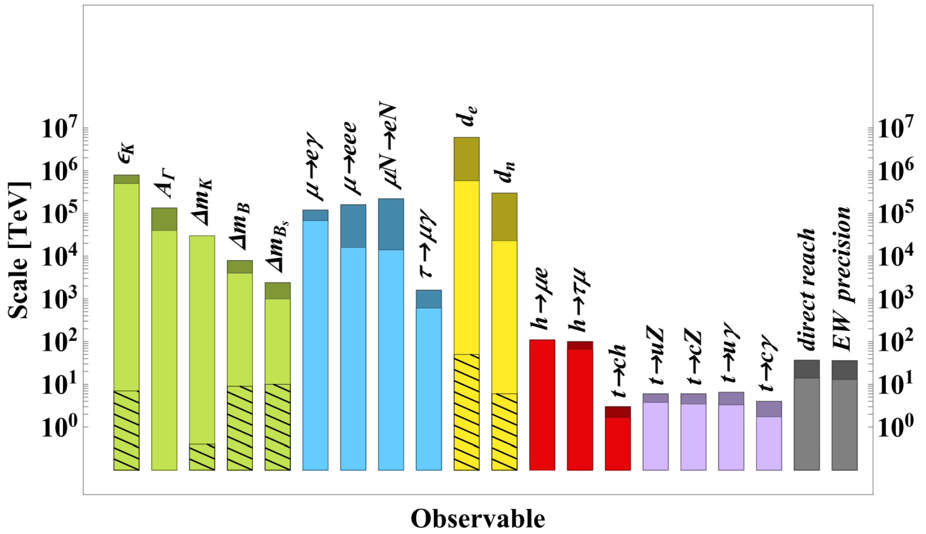

Figure 1: New Physics scales to be indirectly probed by the indicates observables. The darkend areas are the "naïve" New Physics scales by assuming the Wilson coefficients of order one, the coloured bars indicate the inherent New Physics scales assuming weak interaction strengths, while the hatched areas account for loop-suppression due to higher order effects. Figure taken from Ref. [39].

for cLFV transitions. While detailed comprehensive studies of well-motivated scenarios for New Physics must be carried out, a first approach in order to constrain generic classes of NP models relies in studies making use of effective field theory (EFT). In the EFT approach one extends the SM lagrangian via non-renormalisable operators $O^{(d)}$ (with mass dimension $d > 4$), which are suppressed by powers of the New Physics scale Λ^{d-4}. The resulting Lagrangian can be schematically written as a power series

$$\mathcal{L} \supseteq \mathcal{L}_{\text{SM}} + \frac{C^{(5)}O^{(5)}}{\Lambda} + \frac{C^{(6)}O^{(6)}}{\Lambda^2} + \frac{C^{(7)}O^{(7)}}{\Lambda^3} + \dots, \qquad (7)$$

in which $C^{(d)}$ denote the effective coupling constants (the so-called Wilson Coefficients) that depend on the New Physics parameters of fields that are dynamical above the matching scale Λ. Below the matching scale Λ fields with a mass larger than Λ are "integrated out" and their effects are encoded in the effective coupling constants of the $d > 4$ contact interactions. These (new) contact interactions can then be probed by low-energy observables, for instance cLFV transitions.

From a model-independent perspective, one can argue that the inherent scale of New Physics (setting the associated Wilson Coefficient(s) $C^{(d)} = 1$) that can be probed with current and future cLFV dedicated experiments is up to thousands of TeV, far beyond the direct reach of current and future colliders [39]. An overview of this is shown in Fig. 1, where one has the inherent New Physics scales to be indirectly probed by measurements of (or searches for) several flavour observables.

3.1 Correlations matter

As extensively argued, the observation of one (or several) cLFV processes would be a clear unambiguous sign of New Physics. It is however important to stress that although neutrino oscillations imply that lepton flavour is violated in Nature, a possible observation of charged lepton flavour violating processes is not necessarily associated with neutrino oscillation phenomena; cLFV can emerge as an independent process, without any connection to the mechanism of neutrino mass generation.

In order to disentangle the origin of cLFV and to constrain the flavour structure of the New Physics responsible for it, it is often useful to study correlations between different cLFV transitions. While for radiative decays the dipole operator at its origin is necessarily realised at loop level, $\mu \to 3e$ decays and neutrinoless $\mu - e$ conversion in Nuclei can stem from both higher order (photon-, Z- or Higgs-mediated diagrams, and boxes) or even from tree-level processes! Depending on the "nature" of the New Physics mediator(s) (scalar, vector, fermion, ...), certain operators might be enhanced with respect to others, while depending on the flavour structure of the couplings certain flavour transitions can be enhanced. On the one hand, the synergy of identical transitions between different flavours (e.g. the comparison of $\tau \to \mu\gamma$ with $\mu \to e\gamma$ decays) offers insight on the flavour structure of a common cLFV interactions; on the other hand, the comparison of different processes requiring similar flavour violation (e.g. $\mu \to e\gamma$, $\mu \to 3e$ and $\mu - e$ conversion) could shed light on the nature of the new mediator.

To illustrate this, we focus on neutrino mass models. Several mechanisms of neutrino mass generation are known to be associated with peculiar cLFV signatures, which have been extensively investigated as powerful tools to disentangle (and further give insight) certain mechanisms of neutrino mass generation. As an example, recall that while in type I seesaw constructions one typically finds $\text{BR}(\mu \to e\gamma)/\text{BR}(\mu \to 3e) \sim 5 - 10$ (for masses of the propagators around the TeV-scale), for a type III seesaw one has $\text{BR}(\mu \to e\gamma)/\text{BR}(\mu \to 3e) \sim 10^{-3}$. This is a consequence of having the cLFV 3-body decay occurring at the tree-level (see e.g. [40]). However, the presence of CP violating phases (Dirac and/or Majorana) in association to the new lepton mixings can strongly impact such predictive scenarios, as we will discuss in the next section.

An illustrative example of the probing power of cLFV observables (and their correlations) can be found in scotogenic models (in which the SM is extended via inert scalar doublets and right-handed neutrinos [41]). For instance, as shown in [42], cLFV rates, in particular $\text{BR}(\mu \to 3e)$ and $\text{BR}(\mu \to e\gamma)$, could shed light on the nature of the dark matter candidate (in this case, the lightest neutral component of the inert scalar). A future measurement of the ratio of rates for the latter cLFV observables could further hint on the absolute neutrino mass scale (see [42] for details).

Beyond leading order in EFT, i.e. the one-loop matching and running of Wilson Coefficients due to renormalisation group evolution, the contributions of different operators mix and lead to important synergies between observables of different flavour sectors; furthermore, semi-leptonic operators can give contributions to purely leptonic observables and vice-versa. Examples of this are shown in Figure 2 where two operators at a time are constrained by different cLFV observables [43, 44]. From the bottom panel of Figure 2 it is further evident that stringent indirect limits of certain observables, in this case cLFV $\Upsilon(1S)$ decays, can be derived from bounds on seemingly unrelated cLFV τ decays [44]. For limits on Wilson Coefficients beyond "two-at-a-time", also including higher-dimensional operators up to dimension 8, we refer the reader to e.g. [45, 46].

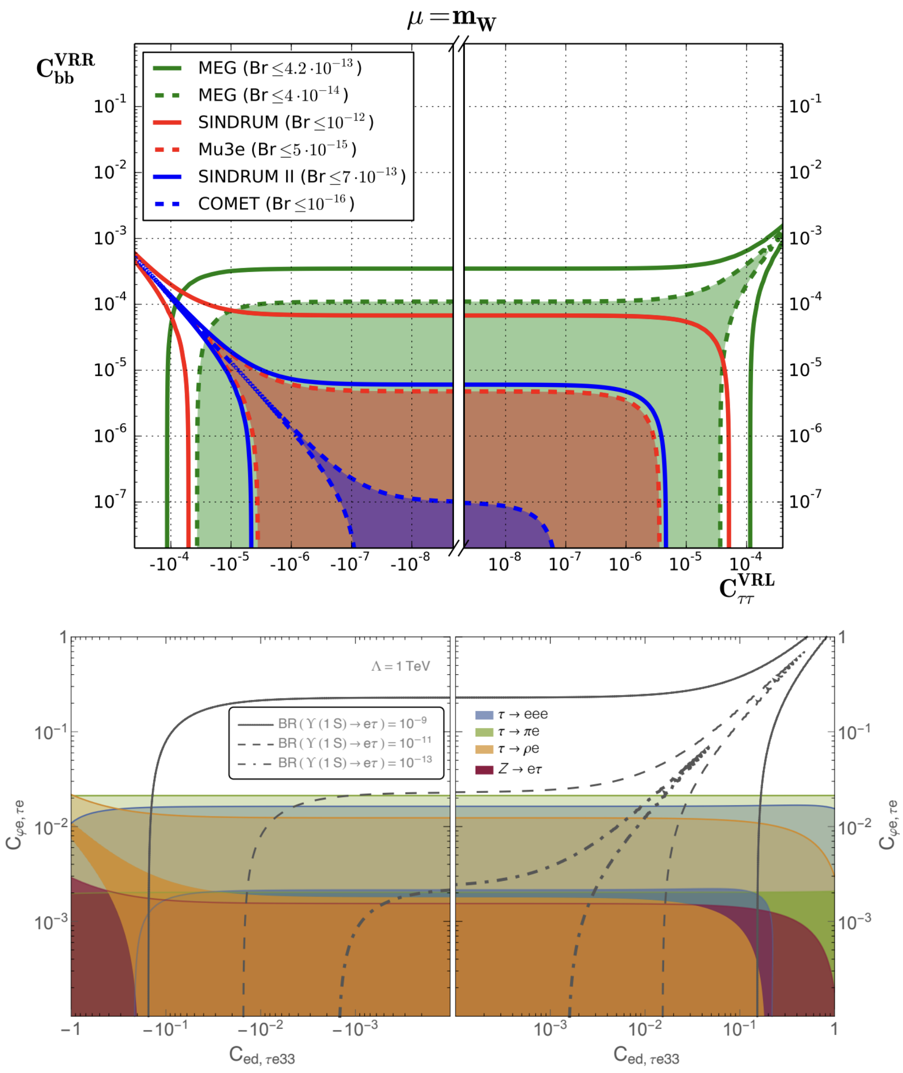

Figure 2: Combined limits on flavour violating Wilson Coefficients from the synergy of low-energy cLFV observables. **Top:** Semi-leptonic and semi-tauonic Wilson Coefficients are constrained by experimental bounds on $\mu - e$ flavour violating processes. See [43] for further details. **Bottom:** Low-energy $\tau - e$ flavour violating observables lead to stringent indirect upper limits on $\Upsilon(1S) \to e\tau$ decays. See [44] for further details. Figures are taken from [43, 44] respectively.

3.2 The role of CP violation

Numerous SM extensions have been proposed to explain neutrino masses and leptonic mixings. Models in which right-handed neutrinos are added to the SM so that Dirac neutrino masses generated from the Higgs mechanism successfully accommodate oscillation data; however, these extensions are plagued by naturality issues (smallness of the Yukawa couplings, Y^ν) and are very hard to test (for example, associated predictions for cLFV processes lying beyond any future experimental sensitivity). Other (more appealing) possibilities include the different realisations of the seesaw mechanism. In particular, models calling upon heavy Majorana neutral fermions (sterile states under the SM gauge group), as is the case of the type I seesaw [47] and its low-scale variants

variants (such as the inverse seesaw [48–50], can be realised at low energies - close to the TeV -, leading to a very rich phenomenology, which encompasses cLFV and lepton number violation (LNV) processes.

Several lepton number violating processes (including neutrinoless double beta decays, or (semi)leptonic meson decays) are known to exhibit a strong dependence on leptonic CP violating (CPV) phases [51]. In [52], a thorough study of the effects of Dirac and Majorana phases on leptonic cLFV transitions and decays was carried, and in what follows we highlight the most relevant results.

In our study, we have considered an effective "3+2 toy model", in which 2 heavy neutral leptons (HNL) are added to the SM content, called upon by many extensions of the SM. No assumption is made on the actual mechanism of neutrino mass generation. The spectrum contains 5 massive Majorana states, and leptonic mixings are encoded in a 5×5 matrix, parametrised via 10 mixing angles $\theta_{\alpha j}$ and 10 CPV phases - 6 Dirac $\delta_{\alpha j}$ and 4 Majorana φ_j. Within the limit of small mixing angles, the active-sterile mixings are given by

$$\mathcal{U}_{\alpha(4,5)} \approx \begin{pmatrix} s_{14}e^{-i(\delta_{14}-\varphi_4)} & s_{15}e^{-i(\delta_{15}-\varphi_5)} \\ s_{24}e^{-i(\delta_{24}-\varphi_4)} & s_{25}e^{-i(\delta_{25}-\varphi_5)} \\ s_{34}e^{-i(\delta_{34}-\varphi_4)} & s_{35}e^{-i(\delta_{35}-\varphi_5)} \end{pmatrix}, \tag{8}$$

with $s_{\alpha i} = \sin\theta_{\alpha i}$ Notice that the would-be PMNS matrix is no longer unitary, which leads to modified charged and neutral lepton currents, and hence (at least potentially) to significant contributions to several SM-forbidden observables. In order to illustrate the role of CPV phases regarding cLFV observables, let us consider the case of $\mu \to e\gamma$ decays, mediated by W bosons and both light and heavy neutrinos. The associated branching fraction (see [52] and references therein) is given by

$$\mathrm{BR}(\mu \to e\gamma) \propto |G_\gamma^{\mu e}|^2\,, \text{with} \quad G_\gamma^{\mu e} = \sum_{i=4,5} \mathcal{U}_{ei}\,\mathcal{U}^*_{\mu i}\,G_\gamma(m^2_{N_i}/M^2_W)\,, \tag{9}$$

in which $G_\gamma(x_i)$, with $x_i = m^2_{N_i}/m^2_W$, is a dimensionless loop function (see [52]). In the limit $m_4 \approx m_5$ and for $\sin\theta_{\alpha 4} \approx \sin\theta_{\alpha 5} \ll 1$ the form factor is given by

$$|G_\gamma^{\mu e}|^2 \approx 4 s^2_{14} s^2_{24} \cos^2\left(\frac{\delta_{14}+\delta_{25}-\delta_{15}-\delta_{24}}{2}\right) G^2_\gamma(x_{4,5})\,. \tag{10}$$

The cLFV rate clearly depends on the Dirac phases, with full cancellation obtained in the case $\delta_{14}+\delta_{25}-\delta_{15}-\delta_{24} = \pi$. Other form factors (for instance Z-penguins and boxes, relevant for three-body decays and muon-electron conversion, for example) also depend on the phases (both Dirac and Majorana phases), but have more involved associated expressions. The dependence of several $\mu - e$ cLFV observables on the Dirac phases is shown on the left plot of Fig. 3, illustrated for δ_{14}; under the simple hypothesis $\sin\theta_{\alpha 4} = \sin\theta_{\alpha 5}$, and for $m_4 = m_5 = 1$ TeV, one finds the above identified behaviour (and cancellation, for $\delta_{14} = \pi$), present for $\mu \to e\gamma$, $\mu \to 3e$ and $Z \to e\mu$ decays. A similar dependence is found for Majorana phases in the considered observables (except for radiative decays, to which the Majorana CPV phases do not contribute). This is shown on the right panel of Figure 3, for the same set of observables and underlying hypotheses.

Following the above mentioned first simple approach, we now carry out a realistic study of the impact of CPV phases on cLFV observables; comprehensive scans of the parameter space are

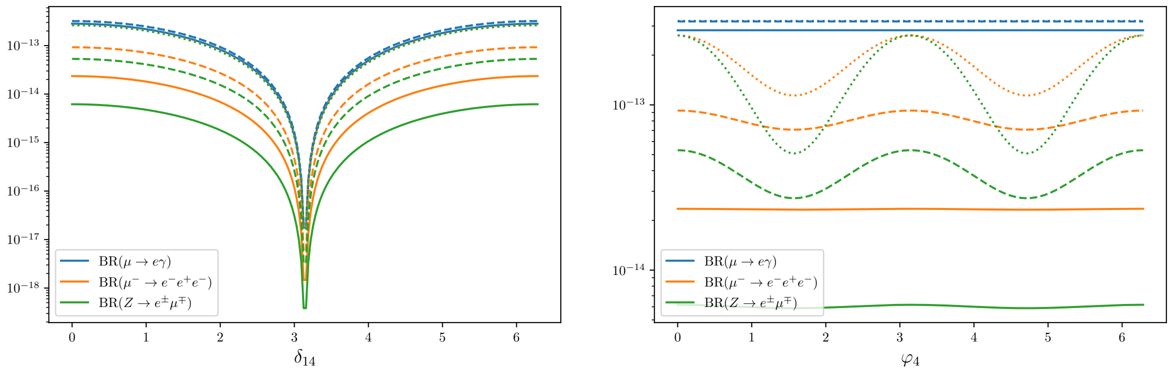

Figure 3: Dependence of cLFV observables on the CP violating Dirac phase δ_{14} (on the left) and Majorana phase φ_4 (on the right). Solid, dashed and dotted lines correspond to $m_4 = m_5 = 1, 5, 10$ TeV. From [52].

conducted (both for the mixing angles and all phases), and all available (relevant) constraints are applied. Concerning the latter, and in addition to the several cLFV constraints, we take into account experimental results and limits on SM extensions via TeV-scale HNL[3].

On the left plot of Figure 4, we display the effects of the CPV phases on the correlation between the rates of two $\mu - e$ sector observables, CR($\mu - e$, N) and BR($\mu \to 3e$). Leading to the results, a random scan was performed over a semi-constrained parameter space: in particular, one now only imposes $\theta_{\alpha 4} \approx \pm\theta_{\alpha 5}$. We have taken degenerate heavy states ($m_4 = m_5 = 1$ TeV), and for each point the CPV phases $\delta_{\alpha 4}$ and φ_4 were set to zero (blue points), randomly varied (orange) and further varied on a grid (green), the latter possibility aiming at ensuring that the special "cancellation" cases were included. Since in the present HNL mass regime both observables receive dominant contributions from Z-penguins, one expects that the associated rates be correlated, as is indeed observed - cf. thick blue line of the CR($\mu - e$, N) vs. BR($\mu \to 3e$) plot. However, and once CPV phases are non-zero, one observes a loss of correlation, all the most striking for the "special" values of the phases $\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi\}$ - corresponding to the green points. In view of this behaviour, it is important to emphasise that HNL extensions of the SM should not be disfavoured upon observation of a single cLFV signal; for example, should future collider searches strongly hint for the presence of sterile states with masses close to 1 TeV, and should BR($\mu \to 3e$)$\approx 10^{-15}$ be measured, one need not expect the observation of CR($\mu - e$, Al). While for vanishing CP phases the latter would be $\approx \mathcal{O}(10^{-14})$, in the presence of CPV phases, the expected range is now vast, with CR($\mu - e$, Al) potentially as low as 10^{-18}.

Additionally, we have also considered CP asymmetries in cLFV Z-boson decays [53],

$$\mathcal{A}_{CP}(Z \to \ell_\alpha \ell_\beta) \equiv \frac{\mathrm{BR}(Z \to \ell_\alpha^+ \ell_\beta^-) - \mathrm{BR}(Z \to \ell_\alpha^- \ell_\beta^+)}{\mathrm{BR}(Z \to \ell_\alpha^+ \ell_\beta^-) + \mathrm{BR}(Z \to \ell_\alpha^- \ell_\beta^+)} \,. \tag{11}$$

Sizeable CP asymmetries in Z decays turn out to be a generic feature of HNL extensions encompassing *at least two* heavy states. For final states composed of a tau and a light charged lepton, with

[3]We consider constraints from electroweak precision observables (M_W, G_F, invisible Z width, ...), lepton universality tests, (leptonic W and Z decays, ratios of leptonic meson decays, ratios of (semi)leptonic tau decays, ...), neutrinoless double beta decays, and finally perturbative unitarity constraints ($\Gamma_{N_{4,5}}/m_{4,5} \leq 1/2$); for a detailed description, and corresponding references, see [52].

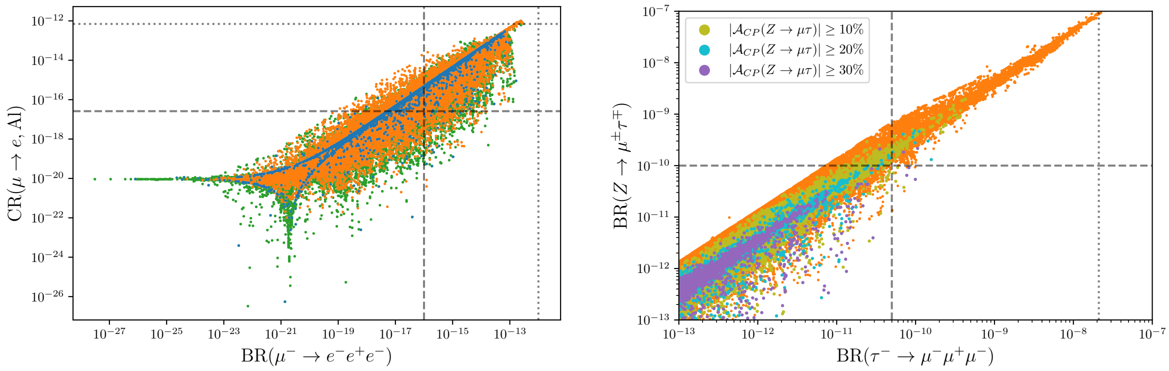

Figure 4: Left: Correlation of $\mu - e$ cLFV observables, for varying values of the CPV Dirac and Majorana phases: vanishing values (blue), non-vanishing (orange), "special grid" (green), cf. description in text. **Right:** Correlation of $Z \to \mu\tau$ and $\tau \to 3\mu$ decays. The colour code indicates the value of the CP-asymmetry in the Z-decay. In both panels, the dotted lines denote the current experimental bounds, while the dashed lines denote the future sensitivities (see Table 1). From [52, 53].

decay rates potentially within future sensitivity, the CP asymmetries can be as large as 20–30% for the case of $\mathcal{A}_{CP}(Z \to \mu\tau)$, interestingly in association with sizeable rates for $\tau \to 3\mu$ (also within future sensitivity). This is illustrated in the right plot of Figure 4.

If on the one hand it is clear that CP violating phases should be in general taken into account upon comparison between prediction and observation in the context of cLFV HNL extensions of the SM, $\mathcal{A}_{CP}(Z \to \ell_\alpha \ell_\beta)$ might hold the key to clearly establishing the presence of leptonic CP violation [53]; in turn, this might have strong implications regarding leptogenesis (relying on complete models including heavy sterile states). Whenever possible, data from individual channels (i.e. $\ell_\alpha^+ \ell_\beta^-$ and $\ell_\alpha^- \ell_\beta^+$) should thus be separately analysed and compared.

In summary, the presence of leptonic CPV phases (both Dirac and Majorana) should be consistently included in phenomenological analysis of the prospects of HNL extensions of the SM in what concerns cLFV.

4. Conclusion

Being the first laboratory evidence for New Physics, neutrino oscillations urgently call for extensions of the SM, in order to offer a viable mechanism of neutrino mass generation. Interestingly, due to offering a new source of CP violation and calling upon weakly interacting states, New Physics extensions aiming at providing an explanation for neutrino masses can often be connected to the baryon asymmetry of the universe and the dark matter problem. Consequently, the interest in high-intensity searches dedicated to the lepton sector has steadily increased.

The violation of accidental (lepton) symmetries of the SM, such as charged lepton flavour conservation and lepton flavour universality (both violated due to the presence of neutrino masses) opens many possible paths to search for New Physics. While massive neutrinos consist of only one possible source of lepton flavour and lepton flavour universality violation, indirect signals indicating the breaking of these symmetries in synergy with possible other indirect signals of New Physics will provide crucial guidelines for both experimental direct searches and theoretical efforts to describe

New Physics interactions. Furthermore, it is known that important synergies between different (flavour-) sectors of observables can arise that should be exploited; in order to obtain a thorough understanding of low-energy lepton flavour physics and constrain various classes of New Physics models, it is of paramount importance to leave no flavoured stone unturned. As we have pointed out, concerning HNL extensions of the SM, the possible presence of the leptonic phases (Dirac and/or Majorana) - which are a generic feature of mechanisms of neutrino mass generation - can have a strong impact on the rates of cLFV observables, leading to a suppression or enhancement of the latter, and should be taken into account upon interpretation of future data. CPV phases play a crucial role in the assessment of viability of (regimes of) SM extensions via HNL.

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