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Numerical investigation of automatic fine-tuning in the Schwinger model

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We perform a numerical investigation of the fine splitting of the pseudoscalar meson mass in the Schwinger model. We use overlap fermions at a single lattice spacing and in a mass range, where the analytic prediction $M_\pi = 2.008 \ldots m_f^{2/3}$ is satisfied. We then check the prediction of an exponential suppression of the fine splitting in the fermion mass. We generically find behavior compatible with exponential suppression, but with a prefactor that seems to substantially differ from a leading order expansion.

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1. Introduction

In 2020, Georgi demonstrated, that conformal coalescence in the Schwinger model [1, 2] leads to an automatic fine tuning [3]. Specifically, he predicted isospin breaking effects in the pseudoscalar meson mass to be suppressed exponentially in the average fermion mass m_f for the non-degenerate two flavor theory. A detailed analytical investigation of this effect that includes leading order prefactors can be found in the companion proceeding [4]. Here we would like to check this prediction numerically in a suitable range of fermion masses and splittings on a single lattice spacing.

2. Short review of analytical results

The leading order relation between the pseudoscalar meson mass M_π and the degenerate fermion mass m_f for the two flavour Schwinger model

$$M_\pi = 2.008\ldots m_f^{2/3}. \tag{1}$$

was found by Smilga some time ago [5]. For small, non-degenerate fermion masses with an average mass $m_f \ll \mu$ and a mass difference $\delta m \lesssim m_f$, Georgi has found an exponential suppression of the pseudoscalar meson mass splitting proportional to a factor [3]

$$e^{-(\frac{\mu}{m_f})^{\frac{2}{3}}} \tag{2}$$

where μ is the Schwinger mass in the two flavor case $\mu^2 = e^2\frac{\pi}{2}$. As detailed in companion proceedings [4], the leading order estimate of the meson mass splitting $\Delta M = M_{\pi^0} - M_{\pi^\pm}$, including prefactors, is expected to be

$$\frac{\Delta M}{M_{\pi^\pm}} = \frac{2}{3}\left(\frac{\pi}{2}\right)^{\frac{1}{4}} \mu^{-\frac{1}{6}} \frac{\delta m}{m_f^{\frac{5}{6}}} e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}. \tag{3}$$

Defining, for convenience, a parameter

$$k = \frac{3}{2}\left(\frac{2}{\pi}\right)^{\frac{1}{4}} \mu^{\frac{1}{6}} \frac{m_f^{\frac{5}{6}}}{\delta m} \tag{4}$$

we may rewrite this relation as

$$\log\left(k\frac{\Delta M}{M_{\pi^\pm}}\right) = -\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}. \tag{5}$$

We will check this relation numerically, keeping the parameter k fixed. Note that, according to (4), this implies that the ratio $\delta m/m_f^{5/6}$ is being kept fixed.

3. Technical Details

For our numerical investigation we employ overlap fermions, which have the distinct advantage of a missing additive mass renormalization. Also, the computational overhead of using overlap fermions is relatively small in the Schwinger model, as we diagonalize the fermion matrix directly. Dynamical fermions are obtained via reweighting of quenched configurations, which is feasible in the Schwinger model. Exact diagonalization of the fermion matrix allows us to compute propagators for arbitrary masses without inverting the fermion matrix for different masses. Note that for this reason correlators for different fermion masses but with otherwise identical parameters are obtained from the same gauge ensemble and are therefore correlated. To forego the problem of topological freezing, our gauge field update algorithm includes instanton multiplications (see [6–8] for details). All simulations were carried out on a 22^4 lattice at $\beta = 1.8$. We computed propagators for the $N_f = 2$ theory with nine different average fermion masses m_f. For each m_f, we produced a set of degenerate propagators $\delta m = 0$, as well as two different sets of nondegenerate propagators with mass splittings $\delta m / m_f$ of roughly 0.1 and 0.8 respectively. Due to $\delta m / m_f^{5/6}$ being kept fixed, the actual splittings will vary somewhat and their exact values are given in table 1.

δm	$\frac{m_f^{\frac{5}{6}}}{\delta m}$	k
$0.1 m_f$	3.218	16.24
$0.8 m_f$	1.652	2.030

Table 1: Mass splittings for our nondegenerate propagators.

4. Numerical results

We first turn our attention to the degenerate case and check the Smilga relation eq. 1. Fig. 1 shows the mass plateau for one degenerate fermion mass, which is in the middle of our investigated mass range. As one can see, a plateau value for the mass can clearly be identified. In fig. 2, we plot the extracted meson masses vs. the degenerate quark mass. The relation very closely matches the correct power law, with the coefficient of proportionality also close to the Smilga value 2.008 . . .. Zooming in, we see some difference from the predicted behavior, which can probably be attributed to discretization, finite volume and higher order effects in the mass expansion. Since our focus in this proceedings is on a first qualitative look at the fine splittings, we do not investigate these small differences in detail, but rather note, that their effect on the fine splitting needs to be studied further in future investigations.

Having checked the Smilga relation, we can put reasonable bounds on the fermion masses used in the fine splitting investigation. In fig. 2, the gray area marks the fermion masses that produce pions, which are heavier than the η', i.e. $M_\pi \gtrsim \mu$. They do certainly not fulfill the small mass limit condition, which we will keep in mind in the further analysis.

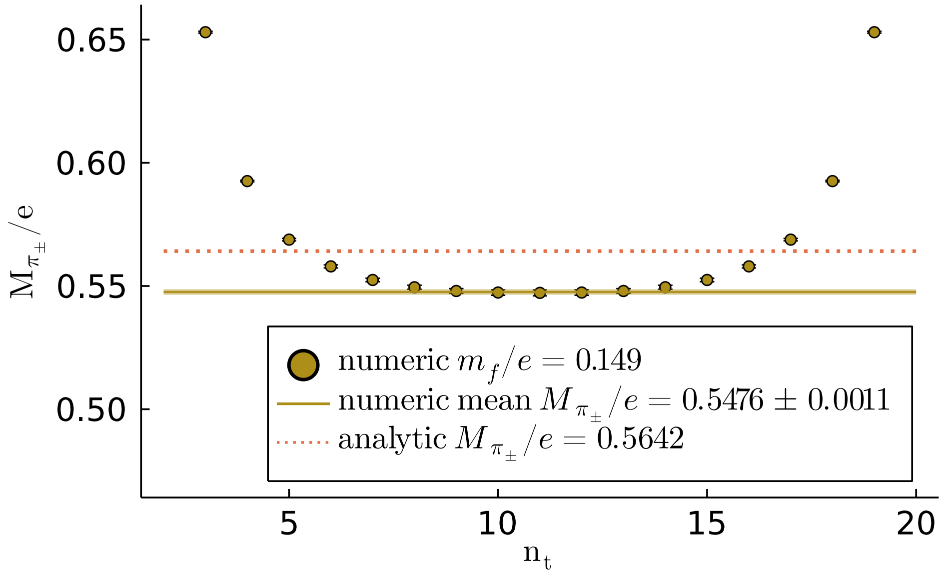

Figure 1: Mass plateau of a $\pi^{\pm}$-meson with fermion mass $m_f/e = 0.149$. The analytic value (dotted line) is the Smilga result from eq. 1.

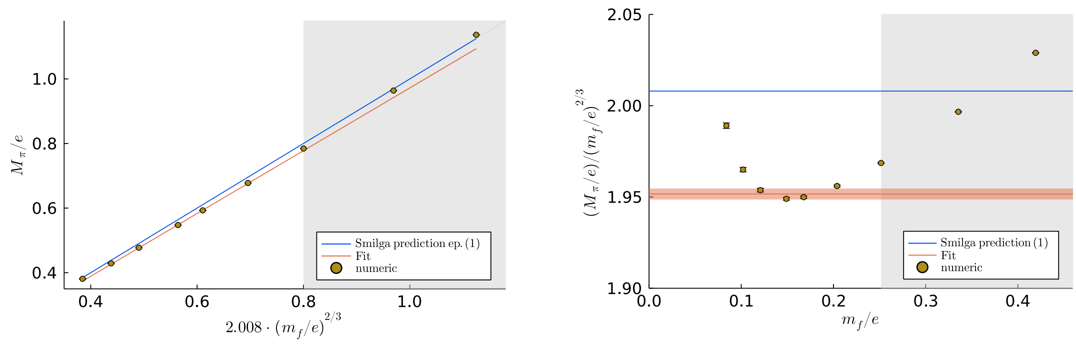

Figure 2: Degenerate pion mass for the $N_f = 2$ case vs. quark mass. The blue line represents the analytical result of eq. 1, while the orange line is a linear fit with zero offset to the three central poins. The fit results in a prefactor of 1.9522 ± 0.0028, compared to the Smilga prediction $2.008\ldots$. On the right hand side, the pion mass is divided by the appropriate power of the quark mass so that the small deviations can be seen more clearly.

We now turn our attention to the nondegenerate case. The first series of ensembles, with the constant factor of $k = 16.24$, corresponds to a relation between fermion mass and mass difference of $\delta m \approx 0.1 m_f$. In fig. 3, the pion mass splitting is plotted logarithmically vs. the quark mass splitting, given as the exponent appearing in eq. 3. In fact, eq. 3 gives a parameterless prediction of the fine splitting, which is also displayed in the figure and does not describe the data at all. The slope, and thus the general exponential behaviour, does however seem to be described rather well. This is borne out in fig. 4, where we plot the difference in the logarithmic fine splitting

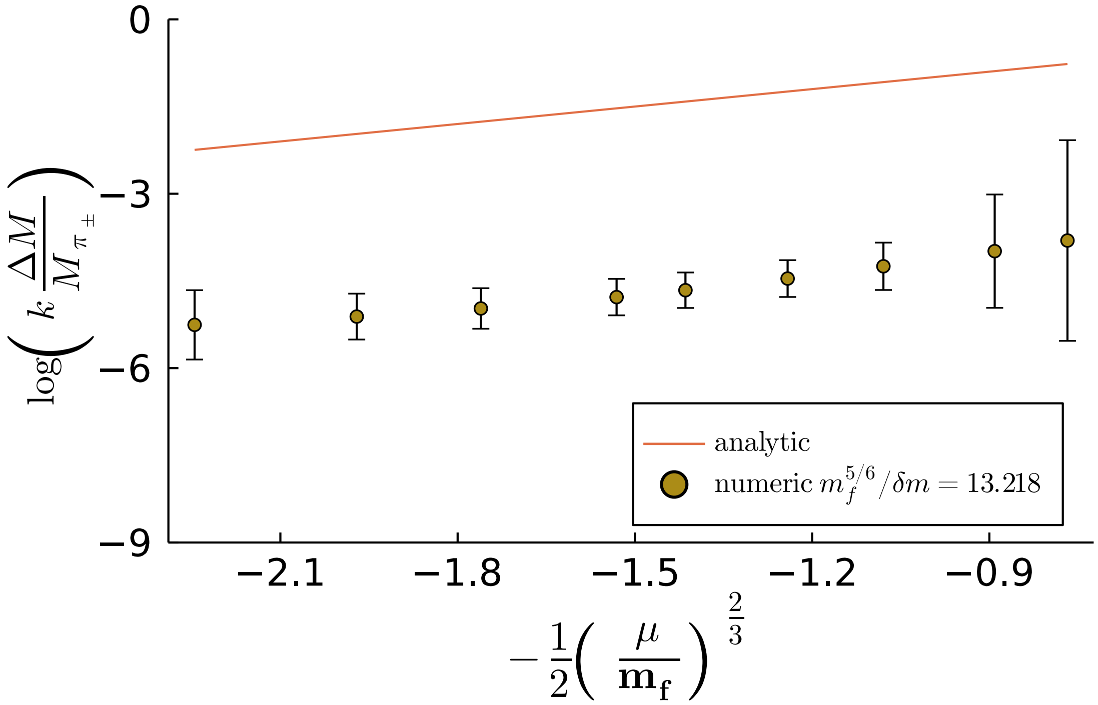

Figure 3: Isospin breaking effects in the pion mass vs. average quark mass for $\delta m \approx 0.1 m_f$ ($k = 16.24$). The continuous line is the prediction from eq. 3.

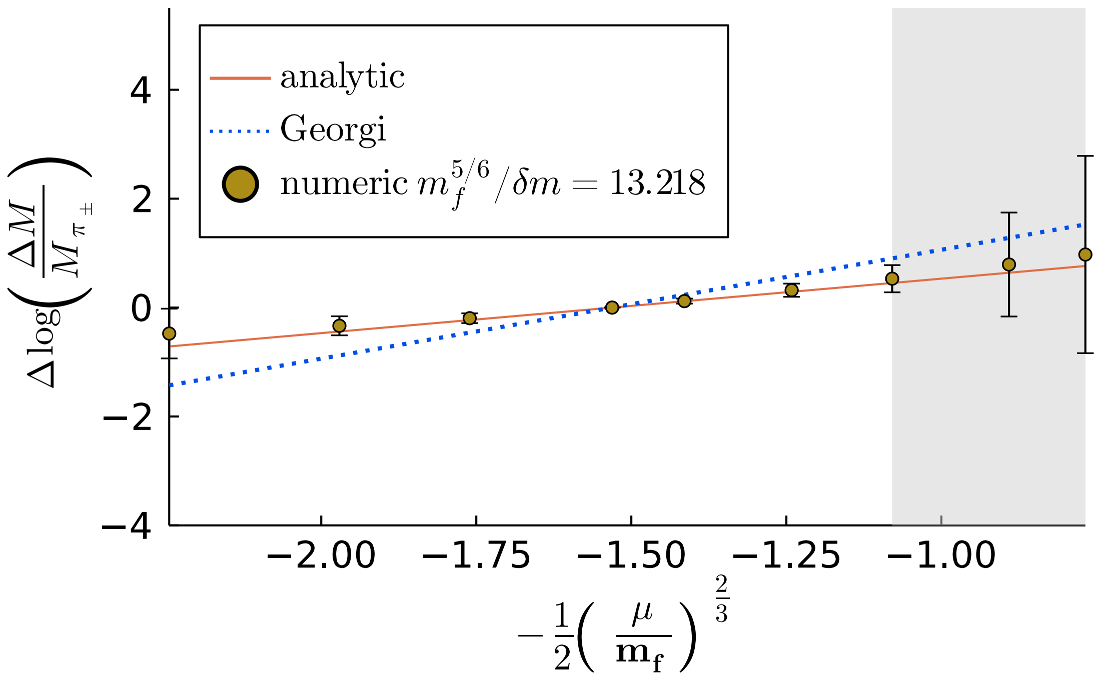

Figure 4: Difference in isospin breaking effects relative tho the fourth mass point. The orange line represents the analytic expectation from eq. 3, while the blue, dotted line is the exponent predicted by Georgi.

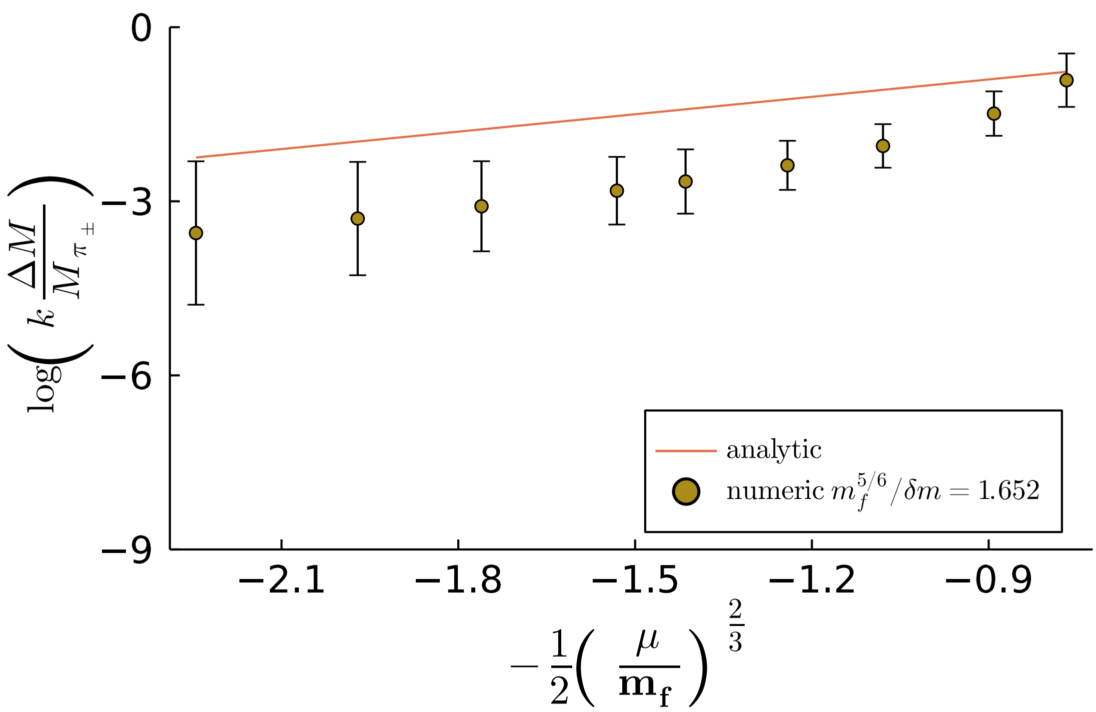

Figure 5: Same as fig. 3 but for the case of $\delta m \approx 0.8m_f$ ($k = 2.03$).

between any fermion mass and the fourth fermion mass in our ensemble. Note, that this procedure is also expected to substantially decrease the errors, since all masses stem from the same set of configurations and are therefore highly correlated. We choose to subtract the fourth mass since this lies in the middle of the mass regime and has a low statistical error. One can see, that the data are now described very well by the exponential prediction, including the factor $1/2$ from eq. 3. By comparison, the exponential without the factor $1/2$ seems to be too steep and does not reproduce the data well. Masses that do not fulfill the small mass limit condition are once more highlighted in gray.

The second set of nondegenerate masses that we explored utilises the same gauge ensemble and also identical m_f, but has a substantially larger quark mass splitting $\delta m/m_f \sim 0.8$, or, more precisely, a constant factor $k = 2.03$. This ensemble is meant to test whether indeed the exponential suppression can be seen even if $\delta m \sim m_f$ as Georgi predicted. Fig. 5 again compares the data and the parameterless prediction of eq. 3. As in the case of the smaller splitting, we observe that the slope seems to be correct, while the prefactor is not, albeit it seems to be closer in this case. This suggests a missing dependency in eq. 3 on m_f and δm that would account for the shift of the offset observed. We once more take the difference of the masses to the fourth mass, with the result displayed in fig. 6. We can see that, for relatively small fermion masses, the behaviour is again well described by the exponential from eq. 3. The precision, however, is worse than for small mass splitting, which is to be expected,because disconnected diagrams do contribute more. One can also see, that for larger fermion masses (notably in the grey band) there is some deviation from the simple exponential behavior. This is, however, to be expected, as the condition $m_f \ll \mu$ is being violated, and it is rather interesting that for small δm one does not see corrections to the exponential

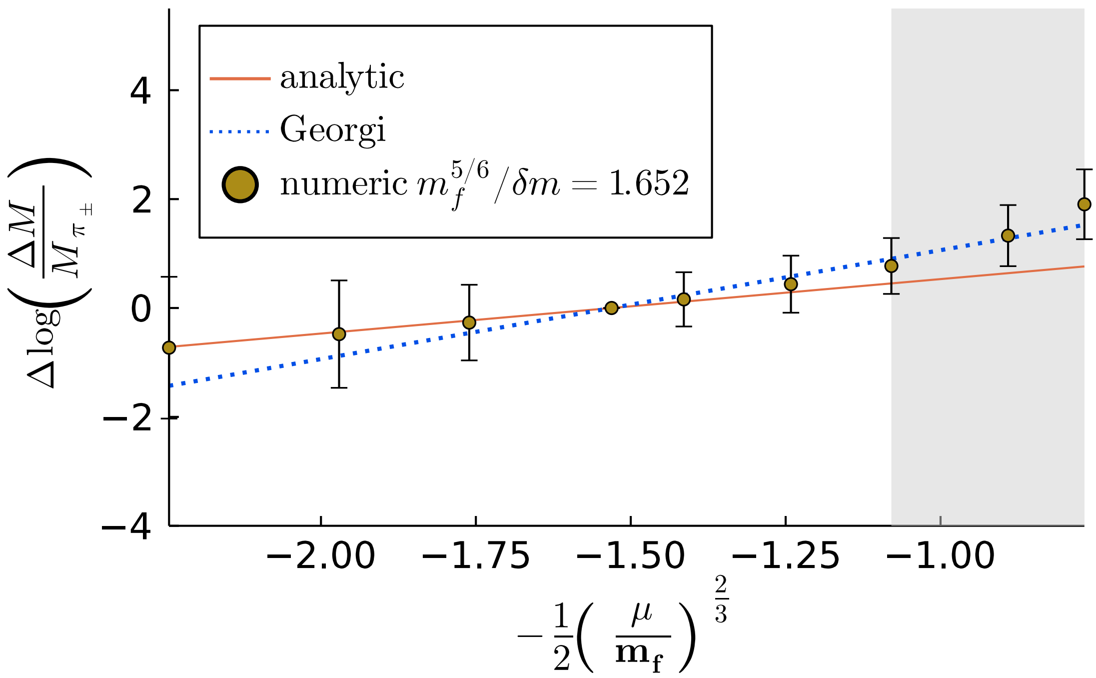

Figure 6: Same as fig. 4 but for the case of $\delta m \approx 0.8 m_f$ ($k = 2.03$).

behavior even in that region.

5. Conclusion and outlook

We performed a first numerical investigation of the predicted exponential fine splitting of meson masses in the Schwinger model. Our data are compatible with an exponential fine splitting in the $m_f \ll \mu$ region, even for large quark mass splittings $\delta m \sim m_f$. Furthermore, our data are compatible with the explicit exponent found in [4], although not with the prefactor.

The most pressing issue for further investigations is, of course, to understand and correct the current deviation in the prefactor. While it is very likely that one needs to refine the analytic result, further numerical investigations, especially for substantially lower fermion masses and towards the continuum limit, could also help clarifying this discrepancy.

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