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Exotic hadrons from an effective field theory perspective

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The last two decades witness the discovery of tens of hadronic structures beyond the expectations of the traditional quark model. They are candidates of exotic hadrons. Many of these structures are close to the thresholds of a pair of hadrons, and thus allow for a treatment using effective field theories at the hadronic level. In this talk, I will give an overview of the understanding of such resonances from an effective field theory pespective, covering positive-parity heavy meson, hidden-charm and double-charm near-threshold hadrons.

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1. Introduction

The quest of rules underlying the meson and baryon spectrum in the 1950s and 1960s led to quark model [1, 2], which successfully classified the hadrons discovered thus far, predicted the existence of new hadrons and paved the way to quantum chromodynamics (QCD) in the 1970s. Various potential quark models before and after the inception of QCD were proposed to compute the meson (as a quark-antiquark bound state) and baryon (as a three-quark bound state) mass spectra as well as their decays, such as the Cornell model [3] and the Godfrey-Isgur (GI) quark model [4]. The agreement of the potential quark model predictions with the observed mass spectra is fairly good before the year of 2003 with a few exceptions (e.g. the lightest scalar mesons). Exotic hadrons are the hadrons with a composition different from quark-antiquark for a meson or three quarks for a baryon. Different types of exotic hadrons have been discussed: multiquark states (such as compact $qq\bar{q}\bar{q}$ tetraquarks and $qqqq\bar{q}$ pentaquarks), hybrid states ($q\bar{q}g$ and $qqqg$, where q and g denotes quark and gluon, respectively), glueballs (consisting of gluons) and hadronic molecules (composite systems of hadrons). While the former types of exotic states need to be studied with quark model notation, hadronic molecules, as extensions of the nucleus concept, can be tackled with effective field theory (EFT) techniques.

Since the dynamics of the formation of hadrons from quarks and gluons is nonperturbative, nowadays how the spectrum of the mesons and baryons observed experimentally can be understood is still of high interest, and a solution of this problem may furnish hints on the mechanism of color confinement. In the last two decades, numerous resonant or resonance-like hadron structures with properties at odds with potential quark model predictions have been observed. Many of them are generally regarded as candidates of exotic hadrons. In 2003, a few narrow resonances were discovered, including the scalar charm-strange meson $D^*_{s0}(2317)$ decaying to $D_s^+\pi^0$ by the BaBar Collaboration [5], the axial-vector charm-strange meson $D_{s1}(2460)$ decaying to $D_s^{*+}\pi^0$ by the CLEO Collaboration [6],[1] and the charmonium-like meson $X(3872)$, with quantum numbers $J^{PC} = 1^{++}$ [7], decaying to $J/\psi\pi^+\pi^-$ by the Belle Collaboration [8]. A common feature of these three particles is that their masses are much lower than the values predicted in potential quark models, such as the GI quark model [4]. Many more discoveries were made in the following years. In Fig. 1 and Fig. 2, we show the mass spectra of open-charm mesons and charmonium(-like) mesons observed by various experiments. The mesons observed before 2003 and since 2003 are depicted as red and blue lines, respectively. As a comparison, the predictions in the GI model are also shown as black lines. In Fig. 2, one sees a systematic deviation from the quark model predictions of charmonia for the structures above the lowest open-charm threshold ($D\bar{D}$).[2] Since the charm quark is heavy, one would naively expect quark models work well. However, the experimental facts show the opposite. The general agreement with quark model predictions below the open-charm threshold and disagreement above indicate the importance of excitations of light quark-antiquark $q\bar{q}$ pairs. Such $q\bar{q}$ pair excitations can be of two types, giving rise to meson-meson molecular or compact

[1]There is also a peak at around 2.46 GeV in the BaBar data of the $D_s^+\pi^0\gamma$ invariant mass distribution with $D_s^+\gamma$ in the D_s^{*+} signal region [5].

[2]The masses of the $\psi_2(3823)$ and $\psi_3(3842)$ are in fairly good agreement with the GI model predictions. Yet, the quantum numbers of these two states do not allow them to couple to the $D\bar{D}^{(*)}$ in S waves, and thus it is natural that these states get little influence from the $D\bar{D}^{(*)}$ channels.

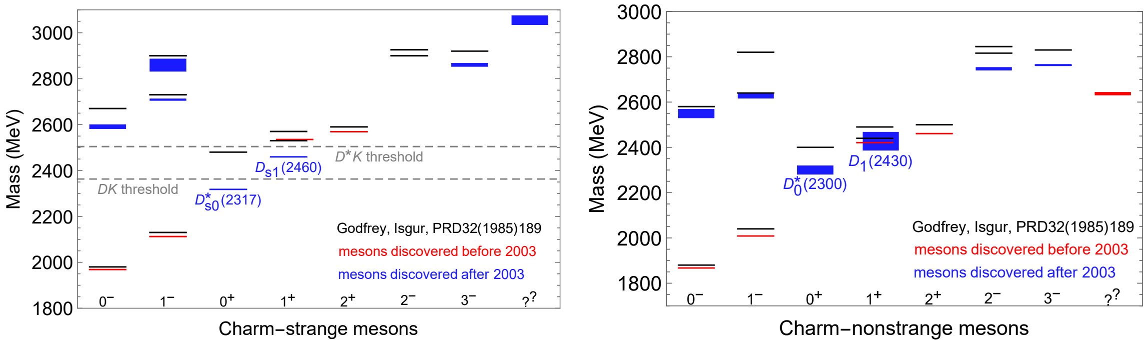

Figure 1: Mass spectra of the charm-strange and charm-nonstrange mesons discovered before 2003 (red) and since 2003 (blue), in comparison with the GI model predictions (black) [4].

tetraquark components to a physical charmonium-like state. For a few recent reviews of the relevant experimental discoveries and the corresponding theoretical studies, we refer to Refs. [9–15].

For hadrons with heavy quarks, the typical momentum exchange inside a hadron is of the order of $\Lambda_{\rm QCD} \sim 0.3$ GeV, when light quarks are involved, or $m_Q v$, where m_Q is the heavy quark mass and $v \ll 1$ is its velocity. Consequently, one can explore the expansions in powers of $\Lambda_{\rm QCD}/m_Q$ and v using the EFT techniques to study exotic hadrons with heavy quarks. For EFT studies of charmonium-like exotics at the quark level, we refer to a recent review [16]. In this paper, we will focus on the EFT studies at the hadronic level, which are particularly useful for hadronic molecules. Specifically, we will discuss the positive-parity charmed mesons in unitarized chiral perturbation theory (UChPT) and a few hidden-charm and double-charm near-threshold states in nonrelativistic effective field theory (NREFT). The use of EFT also enables one to perform a combined analysis of experimental measurements and lattice QCD calculations to reach a more complete scenario.

This paper is organized as follows. In Section 2, we will discuss the $D^*_{s0}(2317)$, $D_{s1}(2460)$, $D^*_0(2300)$ and $D_1(2430)$. With inputs from lattice QCD calculations and recent measurements of three-body B-meson decays by the LHCb experiment, we will show that an overall picture for these states has been achieved using UChPT. The picture strongly supports that the DK and D^*K molecules are the main components of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$, respectively, and each of the charm-nonstrange scalar meson $D^*_0(2300)$ and axial-vector meson $D_1(2430)$, currently listed as two entries in the Review of Particle Physics (RPP) [17], needs to be replaced by two different mesons. In Section 3, we will discuss the generalities of near-threshold structures and review very briefly the applications of NREFT (pionless and pionful) to the interactions between a pair of charmed hadrons. In particular, the hidden-charm pentaquark P_c states and the double-charm tetraquark T_{cc} are fully consistent with the hadronic molecular picture; in this picture, the P_c states are $\Sigma_c^{(*)}\bar{D}^{(*)}$ hadronic molecules and T_{cc} is a DD^* hadronic molecule. Section 4 is a brief summary.

2. Positive-parity heavy mesons

The masses and widths of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons have been measured to be [17]

$$\begin{aligned} M_{D^*_{s0}(2317)} &= (2317.7 \pm 0.6)\ \text{MeV}, \quad \Gamma_{D^*_{s0}(2317)} < 3.8\ \text{MeV}, \\ M_{D_{s1}(2460)} &= (2459.5 \pm 0.6)\ \text{MeV}, \quad \Gamma_{D_{s1}(2460)} < 3.5\ \text{MeV}. \end{aligned} \tag{1}$$

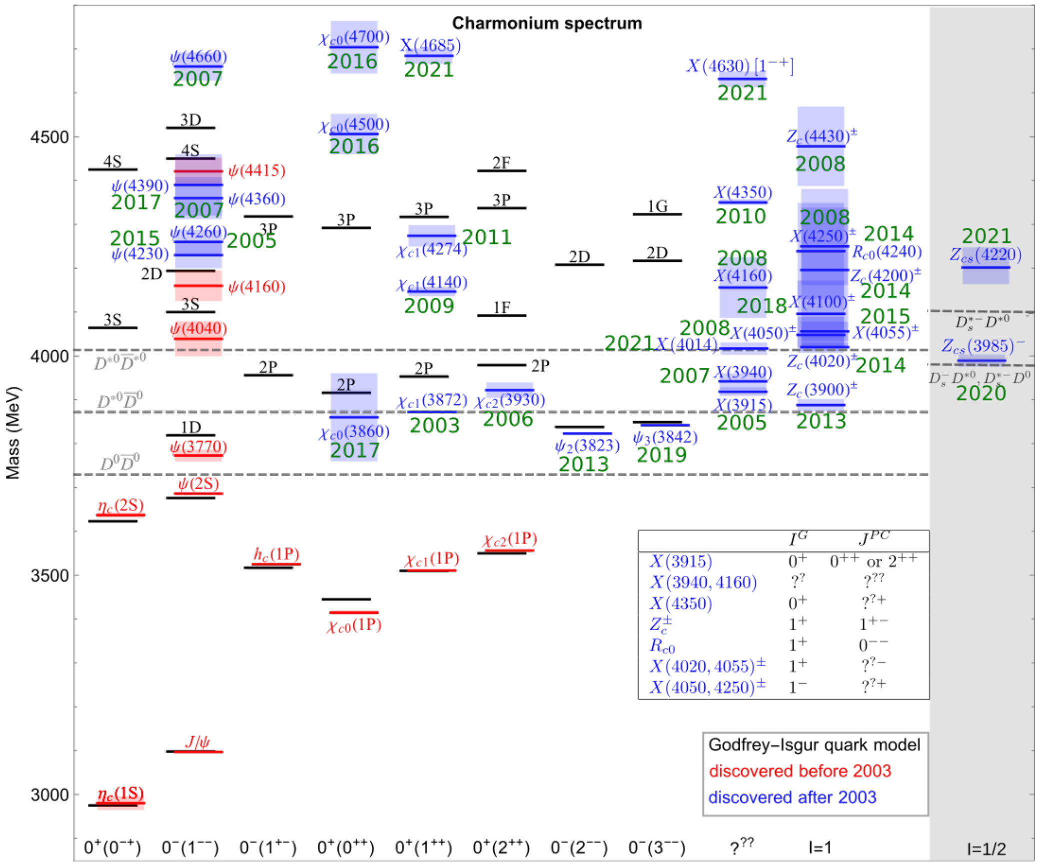

Figure 2: Mass spectra of charmonia and charmonium-like structures discovered before 2003 (red) and from 2003 to 2021 (blue), in comparison with the GI model predictions (black) [4]. The year is also listed when the first observation was made for each structure.

The masses are much lower than the values predicted for the lowest $J^P = 0^+$ and 1^+ charm-strange mesons in the GI quark model [4, 18]. No isospin partners were found, suggesting that these two charm-strange mesons are isospin-scalar states. One notices that the mass difference between the $D_{s1}(2460)$ and $D^*_{s0}(2317)$, (141.8 ± 0.8) MeV, is almost the same as that between the ground state $D^{*\pm}$ and $D^\pm$ mesons, (140.67 ± 0.08) MeV [17]. Such a coincidence requires a natural explanation.

The existence of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ was soon confirmed by the Belle Collaboration [19], who also reported broad structures in the $D^+\pi^-$ and $D^{*+}\pi^-$ invariant mass distributions in the $B^- \to D^{(*)+}\pi^-\pi^-$ decays [20]. The broad structures are denoted as $D^*_0(2300)$ and $D_1(2430)$, whose spin-parity quantum numbers are 0^+ and 1^+, respectively, in the current version of RPP [17].[3] Later, a few other experiments [21–24] reported a scalar charmed meson D^*_0 with similar masses (see Table 1), which were extracted from fitting to the data using the Breit-Wigner (BW) parametrization. The $D_1(2430)$ has been reported by the Belle [20], BaBar [25] and LHCb [26] Collaborations. Averaging the Belle and LHCb measurements gives [17]

$$M_{D_1(2430)} = (2412 \pm 9)\ \text{MeV}, \quad \Gamma_{D_1(2430)} = (314 \pm 29)\ \text{MeV}. \tag{2}$$

[3] The $D^*_0(2300)$ was known as $D^*_0(2400)$ in RPP before 2019.

Table 1: Mass and width of the lowest charm-nonstrange scalar meson D_0^* reported in experiments. The processes in which the measurements were performed are also listed. The last row gives the RPP weighted average of the Belle, BaBar and LHCb measurements.

Experiment	Mass (MeV)	Width (MeV)	Electric charge	Process
Belle [20]	$2308 \pm 17 \pm 32$	$276 \pm 21 \pm 63$	0	$B^- \to D^+\pi^-\pi^-$
FOCUS [21]	$2403 \pm 14 \pm 35$	$283 \pm 24 \pm 34$	+	γA
	$2407 \pm 21 \pm 35$	$240 \pm 55 \pm 59$	0	γA
BaBar [22]	$2297 \pm 8 \pm 20$	$273 \pm 12 \pm 48$	0	$B^- \to D^+\pi^-\pi^-$
LHCb [23]	$2349 \pm 6 \pm 4$	$217 \pm 13 \pm 13$	+	$B^0 \to \bar{D}^0\pi^+\pi^-$
LHCb [24]	$2360 \pm 15 \pm 30$	$255 \pm 26 \pm 51$	+	$B^0 \to \bar{D}^0 K^+\pi^-$
RPP [17]	2343 ± 10	229 ± 16		

The $D_0^*(2300)$ and $D_1(2430)$ are regarded as the SU(3) flavor partners of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$. However, this raises an issue: why are the masses of the $D_0^*(2300)$ and $D_1(2430)$ similar (or even larger, depending on the experiments, see Table 1) than those of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$? This is at odds with the expectation that the charm-strange meson needs to be sizeably heavier than the charm-nonstrange ones within the same SU(3) multiplet.

Different models were proposed for these positive-parity charm-strange and charm-nonstrange mesons (for reviews, see Refs. [11, 27]). Of particular interest is the hadronic molecular model [28–33], in which the main components of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are the S-wave DK and D^*K bound states, respectively. QCD has an approximate heavy quark spin symmetry (HQSS), and the D and D^* mesons are in the same spin doublet. At low energies relative to the DK and D^*K thresholds, the interaction between the D and K mesons is the same as that between the D^* and K mesons at the leading order (LO) in heavy quark expansion. Thus, if the DK pair forms a bound state, one would expect the D^*K pair forms a bound state, too, with a similar binding energy, and the experimental fact $M_{D_{s1}(2460)} - M_{D_0^*(2317)} \simeq M_{D^*} - M_{D^\pm}$ gets naturally explained in the hadronic molecular model.[4] Similarly, bottom partners of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ can be easily predicted as a consequence of heavy quark flavor symmetry [36],

$$\begin{aligned} M_{B_{s0}^*} &= \bar{M}_c + \Delta_{b-c} + \left(M_{D_{s0}^*} - \bar{M}_c\right)\frac{m_c}{m_b} \simeq 5.71 \text{ GeV}, \\ M_{B_{s1}} &= \bar{M}_c + \Delta_{b-c} + \left(M_{D_{s1}} - \bar{M}_c\right)\frac{m_c}{m_b} \simeq 5.76 \text{ GeV}, \end{aligned} \tag{3}$$

where $\bar{M}_c = \left[M_{D_{s0}^*(2317)} + 3M_{D_{s1}(2460)}\right]/4 \simeq 2.424$ GeV is the spin-averaged mass of the spin doublet $\left\{D_{s0}^*(2317), D_{s1}(2460)\right\}$, and $\Delta_{b-c} = m_b - m_c$ is the bottom and charm quark mass difference, which may be estimated as $\bar{M}_{B_s} - \bar{M}_{D_s} \simeq 3.33$ GeV, with $\bar{M}_{B_s} = 5.403$ GeV and $\bar{M}_{D_s} = 2.076$ GeV the spin-averaged masses of the $\left\{B_s, B_s^*\right\}$ and $\left\{D_s, D_s^*\right\}$ spin doublets. The

[4] It can also be explained in the chiral doublet model [34, 35].

simple predictions are in perfect agreement with the lattice QCD results [37],

$$\begin{aligned} M^{\text{lat.}}_{B^*_{s0}} &= (5.711 \pm 0.013 \pm 0.019)\ \text{GeV}, \\ M^{\text{lat.}}_{B_{s1}} &= (5.750 \pm 0.017 \pm 0.019)\ \text{GeV}. \end{aligned} \tag{4}$$

Because the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ couple to the DK and D^*K, respectively, in S waves, one can extract information of the internal structures of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons from the S-wave interactions between the ground state charmed mesons and the lightest pseudoscalar mesons, which are the pseudo-Nambu-Goldstone (pNG) bosons of spontaneous breaking of the approximate chiral symmetry of QCD, *i.e.*, $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, to its vector subgroup $\mathrm{SU}(3)_V$. At low energies, the interactions are constrained by chiral symmetry, and such constraints may be implemented by constructing chirally invariant effective Lagrangian. Treating the pseudoscalar charmed mesons as matter fields, the chiral Lagrangian up to the next-to-leading order (NLO), *i.e.*, $O(p^2)$ with p a typical soft momentum scale, reads [38][5]

$$\begin{aligned} \mathcal{L}^{(2)}_{\phi P} =& D_\mu P D^\mu P^\dagger - m^2 P P^\dagger \\ &+ P\left[-h_0\chi_+ - h_1\chi_+ + h_2 u_\mu u^\mu - h_3 u_\mu u^\mu\right] P^\dagger + D_\mu P\left[h_4 u_\mu u^\nu - h_5\left\{u^\mu, u^\nu\right\}\right] D_\nu P^\dagger, \end{aligned} \tag{5}$$

where $P = \left(D^0, D^+, D_s^+\right)$, m is the mass of the charmed mesons at LO, and h_i's ($i = 1, \ldots, 5$) are low-energy constants (LECs). The first two terms are of LO [31, 42–44]. The chiral covariant derivative is defined as

$$D_\mu P = \partial_\mu P + P\Gamma^\dagger_\mu, \quad D_\mu P^\dagger = \left(\partial_\mu + \Gamma_\mu\right) P^\dagger, \tag{6}$$

where $\Gamma_\mu = \left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right)/2$. The pNG boson ϕ_a ($a = 1, 2, \cdots, 8$) fields are collected in $u = \exp[\mathrm{i}\lambda_a\phi_a/(2F_0)]$, where λ_a's are the Gell-Mann matrices, F_0 is the pion decay constant in the chiral limit, which can be taken to be its physical value 92 MeV up to $O(p^2)$. In the NLO terms, $u_\mu = \mathrm{i}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right)$, $\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u$ with $\chi = 2B_0\,\mathrm{diag}\left(m_u, m_d, m_s\right)$ proportional to the light quark mass matrix that explicitly breaks chiral symmetry. Among the LECs h_i's, $h_1 = 0.42$ is fixed from reproducing the mass splitting between D_s^+ and D^+, while the others need to be fixed from fitting to results from lattice QCD calculations.

The scalar $D^*_{s0}(2317)$ and axial-vector $D_{s1}(2460)$ are below the DK and D^*K thresholds, respectively. Thus, in order to study the $D^{(*)}K$ interactions in the near-threshold region using chiral EFT, the scalar and axial-vector charmed mesons need to be either built in as explicit degrees of freedom or dynamically generated from the interactions between the ground state charmed mesons and the pNG bosons. The former approach is taken in Refs. [45–48], and the latter corresponds to the approach of UChPT in Refs. [31–33, 36, 38, 49–59]. In UChPT, the S-wave projected scattering amplitude $V(s)$ derived from the chiral Lagrangian is inserted into the Lippmann-Schwinger equation (LSE). Usually, the LSE is simplified to a factorized form, and the resulting T-matrix is [60, 61]

$$T(s) = V(s)\left[1 - G(s)V(s)\right]^{-1}, \tag{7}$$

[5] Using the path integral method, the one-loop renormalization of the chiral Lagrangian for spinless matter fields, such as the $D_{(s)}$ mesons, and the subtraction of power counting breaking terms have been worked out in Refs. [39, 40]. The chiral Lagrangian for charmed mesons at higher orders has been constructed in Ref. [41].

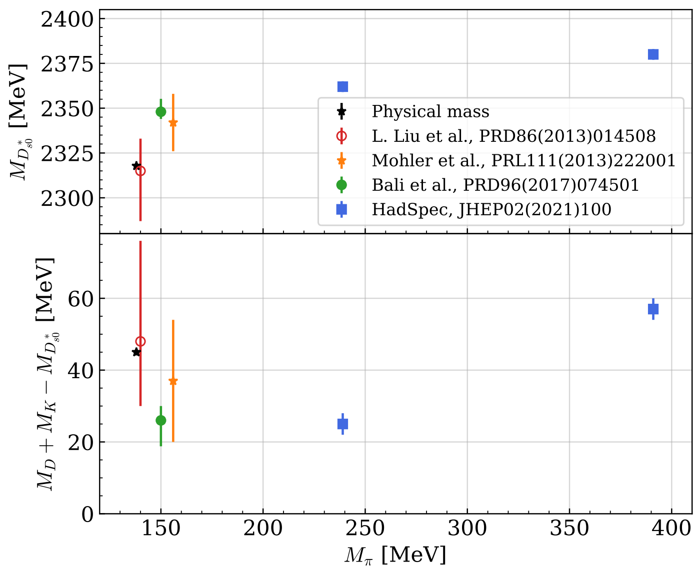

Figure 3: Mass and binding energy of the $D_{s0}^*(2317)$ at different pion masses from indirect [54] and direct lattice calculations [63–65] in comparison with the experimental values.

where $G(s)$ is a diagonal matrix in the channel space with the diagonal matrix element $G_i(s)$ given by the two-point scalar loop integral. $G_i(s)$ is ultraviolet divergent and needs to be regularized. UChPT furnishes a method of studying resonances which couple to pNG bosons and matter fields and have masses not much higher than the corresponding pNG-matter-field thresholds. It may also be regarded as a K-matrix parameterization of the T-matrix with the coupled-channel K-matrix constructed taking into account the chiral and SU(3) light-flavor symmetry constraints. When UChPT is implemented in a finite volume, unitarity ensures that the Lüscher formula can be recovered [62]. Because of the universality of the LECs, experimental data and lattice QCD results can be analyzed in the same framework of UChPT.

In Ref. [54], the S-wave scattering lengths are calculated in lattice QCD for the following channels: $D\bar{K}$ with $(S, I) = (-1, 1)$ and $(-1, 0)$, $D_s K$ with $(S, I) = (2, 1/2)$, $D\pi$ with $(S, I) = (0, 3/2)$, and $D_s\pi$ with $(S, I) = (1, 1)$, where S and I represent strangeness and isospin, respectively. The results at three different pion masses are fitted using UChPT with the h_i LECs as free parameters; an additional parameter is a subtraction constant introduced to regularize the scalar loop integral in UChPT [60, 61]. It turns out that the lattice results at pion masses up to about 500 MeV can be well described [54, 56]. With the LECs and subtraction constant fixed from the fitting, a charm-strange scalar meson D_{s0}^*, which couples to DK and $D_s\eta$ in S waves, is predicted, as a T-matrix pole, to have a mass of 2315^{+18}_{-28} MeV when the pion mass takes the physical value [54]. It is in full accordance with the measured mass of the $D_{s0}^*(2317)$. This indirect lattice result of the lowest-lying D_{s0}^* mass and those from more recent direct lattice calculations [63–65] are shown in Fig. 3. In the figure, results for the binding energy of the $D_{s0}^*(2317)$ as a DK bound state, $M_D + M_K - M_{D_{s0}^*}$, are also presented.

The probability of the $D_{s0}^*(2317)$ as an isoscalar DK hadronic molecule can be computed from the DK scattering length and effective range using the compositeness relations [66–68]. Results from various analyses of lattice energy levels [54, 64, 65, 69, 70] are shown in the left panel of

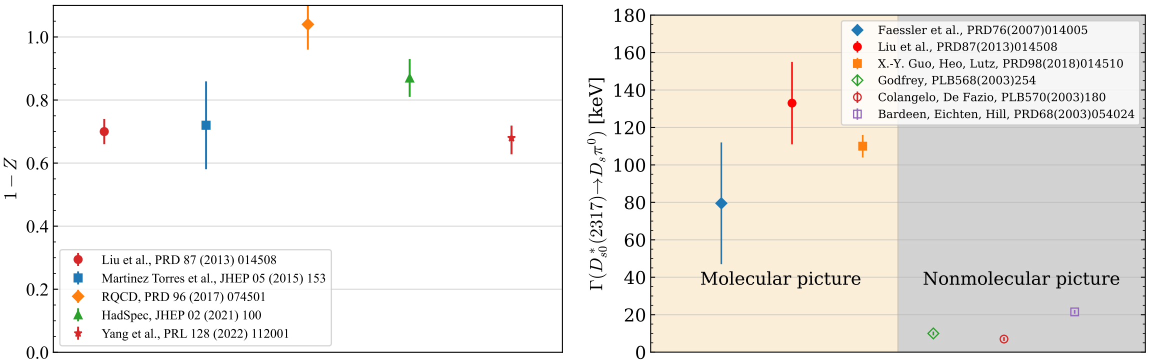

Figure 4: Left: compositeness $(1-Z)$ of the $D^*_{s0}(2317)$ as a DK hadronic molecule. Right: Predictions of the hadronic decay width of $D^*_{s0}(2317) \to D_s^+\pi^0$ in the hadronic molecular and nonmolecular pictures.

Table 2: Predictions on the scalar and axial-vector heavy mesons in UChPT [58, 59]. The left panel is for heavy-strange mesons, where the measured masses and lattice results are also shown for comparison. The right panel lists the poles predicted in UChPT for heavy-nonstrange mesons; for each of them, there are a pair of poles. The masses (poles) are given in units of MeV.

	UChPT	RPP [17]	Lattice		Lower pole	Higher pole
D^*_{s0}	2315^{+18}_{-28}	2317.7 ± 0.6	2348^{+7}_{-4} [64]	D^*_0	$2105^{+6}_{-8} - i\,102^{+10}_{-11}$	$2451^{+35}_{-26} - i\,134^{+7}_{-8}$
D_{s1}	2456^{+15}_{-21}	2459.5 ± 0.6	2451 ± 4 [64]	D_1	$2247^{+5}_{-6} - i\,107^{+11}_{-10}$	$2555^{+47}_{-30} - i\,203^{+8}_{-9}$
B^*_{s0}	5720^{+16}_{-23}	...	5711 ± 23 [37]	B^*_0	$5535^{+9}_{-11} - i\,113^{+15}_{-17}$	$5852^{+16}_{-19} - i\,36 \pm 5$
B_{s1}	5772^{+15}_{-21}	...	5750 ± 25 [37]	B_1	$5584^{+9}_{-11} - i\,119^{+14}_{-17}$	$5912^{+15}_{-18} - i\,42^{+5}_{-4}$

Fig. 3, supporting that the main component ($\gtrsim 70\%$) of $D^*_{s0}(2317)$ is the S-wave DK bound state. Being a hadronic molecular state, the $D^*_{s0}(2317)$ strongly couples to DK and the isospin splittings of the charged and neutral D and K mesons lead to more significant isospin breaking [54, 57, 71] than if the $D^*_{s0}(2317)$ is nonmolecular [34, 72, 73] (see the right plot in Fig. 3). A width of about 100 keV for the $D^*_{s0}(2317)$ is a smoking gun for the molecular nature, which may be measured at PANDA [74].

With heavy quark spin and flavor symmetries, the masses of D_{s1} and B^*_{s0}, B_{s1} as hadronic molecules can be predicted as well. The results, together with the predictions of the nonstrange positive-parity heavy mesons, using the LECs determined in Ref. [54] are listed in Table 2. One sees that the predicted D_{s1} mass is in accordance with the measured mass of the $D_{s1}(2460)$ and the predicted B^*_{s0} and B_{s1} masses are consistent with those in Eq. (3) and by lattice calculations [37], Eq. (4). More interestingly, UChPT predicts a pair of poles for each of the non-strange 0^+ and 1^+ heavy meson sectors, as listed in the right panel of Table 2. The double-pole structure arises because the interactions between the SU(3) flavor antitriplet charmed mesons and the octet light mesons can be decomposed as: $\overline{3} \otimes 8 = \overline{15} \oplus 6 \oplus \overline{3}$; see the left panel of Fig. 5. At LO of the chiral expansion, the interaction is dictated by chiral symmetry to be the Weinberg-Tomozawa term, which is repulsive for the $\overline{15}$ multiplet and attractive for both the sextet and antitriplet. The attraction in the antitriplet is stronger than that in the sextet. An analysis of the evolution of the poles from the SU(3) symmetric limit to the physical situation reveals that the lower D^*_0 (D_1) pole and the $D^*_{s0}(2317)$

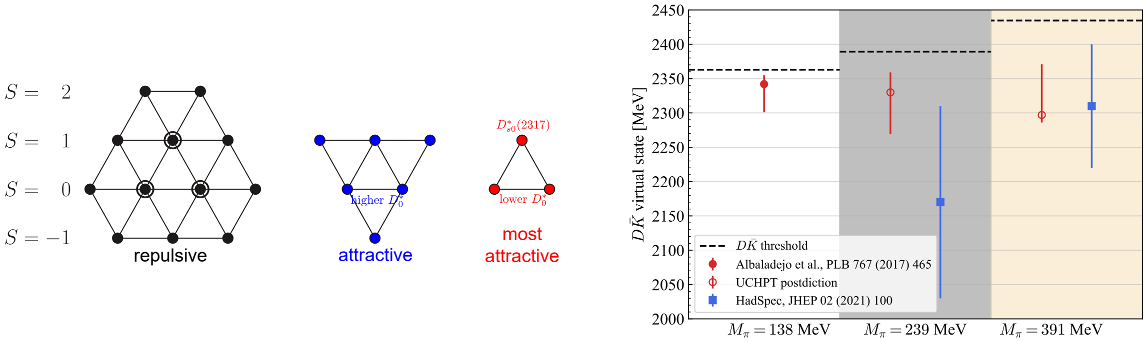

Figure 5: Left: Decomposition of the charmed-meson–light-meson pairs into different SU(3) flavor multiplets [58]. Right: Mass of the $D\bar{K}$ virtual state predicted in UChPT [54, 58] in comparison with the lattice results by the Hadron Spectrum Collaboration [65].

($D_{s1}(2460)$) form an antitriplet while the higher D_0^* (D_1) pole is in the sextet [58]. The lower D_0^* is lighter than its strange partner $D_{s0}^*(2317)$, in line with the expectation for the mass hierarchy within the same SU(3) multiplet. It is also significantly lighter than mass of $D_0^*(2300)$ listed in RPP [17]. The possibility of a low-mass D_0^* was first pointed out in Ref. [29] and supported in Refs. [31, 32]. UChPT predicts that the isospin singlet in the sextet corresponds to a $D\bar{K}$ virtual state [58, 59], which is supported by recent lattice calculations [65]. A comparison of the UChPT predictions of the $D\bar{K}$ isoscalar virtual state with the lattice results is shown in the right panel of Fig. 5.

The D_0^* and D_1 mesons have also been calculated in lattice QCD. Using operators of the $c\bar{q} + D^{(*)}\pi$ form, the $D^{(*)}\pi$ phase shifts were calculated in Ref. [75], and the D_0^* and D_1 masses were extracted using a BW parametrization. The pole of the D_0^* from that calculation was reported to be (2.12 ± 0.03) GeV only recently [76], consistent with the lower pole in Table 2. The first coupled-channel lattice calculation in the scalar sector was performed in Ref. [77] using operators of the $c\bar{q} + D\pi + D\eta + D_s\bar{K}$ form at a pion mass about 390 MeV. The energy levels were analyzed using coupled-channel T-matrix with a K-matrix parameterization, and a D_0^* state just below the $D\pi$ threshold was obtained. The energy levels obtained in Ref. [77] can also be well described with UChPT in a finite volume [56, 58], indicating that the existence of two D_0^* states is consistent with the lattice results. Recently, it was proposed that the higher D_0^* pole can be better extracted from lattice energy levels by imposing SU(3) symmetry constraints on the K matrix [78], and the extracted higher D_0^* pole is consistent with the one from UChPT. That the lowest-lying D_0^* and D_1 states are lighter than the masses listed in RPP receive support from recent lattice calculations [79, 80].

Further supports of the two-pole structure of the D_0^*, *i.e.*, the existence of two D_0^* states below 2.5 GeV, come from the fact that the LHCb data of the angular moments for the processes $B^- \to D^+\pi^-\pi^-$ [81], $B_s^0 \to \bar{D}^0K^-\pi^+$ [82], $B^0 \to \bar{D}^0\pi^-\pi^+$ [23], $B^- \to D^+\pi^-K^-$ [83], $B^0 \to \bar{D}^0\pi^-K^+$ [24] can all be well described using the UChPT amplitudes [59, 84] with the LECs determined in Ref. [54]. In addition, it was demonstrated in Ref. [85] that the BW parameterization of the D_0^* with resonance parameters in RPP [17] is inconsistent with the LHCb data for the decay $B^- \to D^+\pi^-\pi^-$ [81]. These strong evidence for the two-pole structure of the D_0^*, coming from analysis by the same group, has been recognized in the latest version of RPP [17]. However, in order to establish it, independent analysis of the data from other groups are necessary.

3. Hidden-charm and double-charm near-threshold states

The many charmonium-like structures shown in Fig. 2 are collectively called XYZ states. States like the $X(3872)$, $Z_c(3900)$, $Z_c(3985)$ and so on are rather close to the thresholds of a pair of charmed mesons. Milestones in recent years include the discovery of hidden-charm pentaquark P_c states [86, 87], which are just below the $\Sigma_c\bar{D}^{(*)}$ thresholds, and the double-charm tetraquark T_{cc}^+ just below the $D^{*+}D^0$ threshold [88, 89].

It is of particular interest to understand if there is a general rule underlying the near-threshold structures. This can be analyzed by constructing an EFT for coupled channels. Let us consider the energy region around the higher threshold for a two-channel system. The T-matrix at LO for S-wave scattering can be parametrized as [90]

$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1}, \tag{8}$$

where Σ_2 and μ_2 are the threshold and reduced mass of the higher channel, respectively, k_1 is the c.m. momentum of the lower channel,[6] $a_{11,12,22}$ are parameters, and E is the energy defined relative to Σ_2. While a_{22} can be understood as the scattering length in the higher channel when its coupling to the lower channel is switched off, a_{11} presents the interaction strength of the lower channel at Σ_2 (and thus is not a scattering length). The only term depending on E in the above expression has a square-root branch point at $E = 0$ and is the source of nontrivial behaviors around the higher threshold. It is straightforward to find

$$|T_{21}(E)|^2 \propto |T_{22}(E)|^2 \propto \begin{cases} \left[\left(\mathrm{Re}\,\frac{1}{a_{22,\,\mathrm{eff}}}\right)^2 + \left(\mathrm{Im}\,\frac{1}{a_{22,\,\mathrm{eff}}} - \sqrt{2\mu_2 E}\right)^2\right]^{-1} & \text{for } E \geq 0, \\ \left[\left(\mathrm{Im}\,\frac{1}{a_{22,\,\mathrm{eff}}}\right)^2 + \left(\mathrm{Re}\,\frac{1}{a_{22,\,\mathrm{eff}}} + \sqrt{-2\mu_2 E}\right)^2\right]^{-1} & \text{for } E < 0, \end{cases} \tag{9}$$

where

$$\frac{1}{a_{22,\,\mathrm{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2\left(1 + a_{11}^2 k_1^2\right)} - i\frac{a_{11}^2 k_1}{a_{12}^2\left(1 + a_{11}^2 k_1^2\right)} \tag{10}$$

with $a_{22,\,\mathrm{eff}}$ the scattering length of the higher channel with channel coupling effects. Unitarity ensures $\mathrm{Im}\, a_{22,\,\mathrm{eff}}^{-1} \geq 0$. Consequently, the line shape of $|T_{21}(E)|$ is maximized when $\mathrm{Re}\left(a_{22,\mathrm{eff}}\right) > 0$, corresponding to an attractive interaction with a virtual-state-like pole. In this case, the full width at half maximum of the peak is anti-proportional to μ_2; see the red and blue curves in Fig. 6 (a). If the interaction is stronger, the pole would be bound-state-like, and the line shape peaks below (but still close to) the higher threshold; see the green curve in Fig. 6 (a). If the interaction is repulsive, then there is no peak around the higher threshold. Therefore, one may conclude that there is a near-threshold peak in the line shapes of $|T_{21}(E)|$ and $|T_{22}(E)|$ as long as the S-wave interaction in higher channel is attractive, and such peaks are more prominent for heavier particles at given interaction strengths. Thus, it is natural that near-threshold structures appear more frequently in the charm sector than in the light-quark sectors. With a vector-meson exchange model, one can

[6] The lower channel can be either nonrelativistic or relativistic. In the former case, the EFT reduces to the one proposed in Ref. [91].

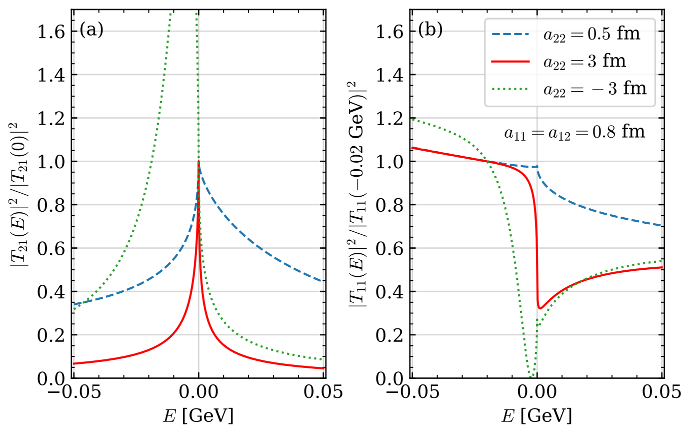

Figure 6: Threshold behaviors of $|T_{21}|$ and T_{11} for different sets of a_{ij} parameters [90]. For illustration, here we use the masses of π^- and J/ψ for the lower channel and those of D^0 and D^{*-} for the higher channel. (a) Line shapes of $|T_{21}|^2$ normalized at $E = 0$; (b) line shapes of $|T_{11}|^2$ normalized at $E = -0.02$ GeV.

find charmed hadron pairs which have attractive S-wave interactions [90]. These pairs include the isoscalar $D\bar{D}^*$, isospin-1/2 $\Sigma_c^{(*)}\bar{D}^{(*)}$ and so on. Certainly such results are model dependent, and reliable lattice conclusions would be valuable.

The line shape of $|T_{11}|$ near the higher threshold behaves differently from that of $|T_{21,22}|$ even though they have the same pole. From the expression of T_{11} [90]

$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - ik_1\right)\left[\frac{1}{a_{22,\,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}, \tag{11}$$

one sees that there is a zero in the denominator in addition to the pole. The competition between the zero and the pole determines the line shape of $|T_{11}|$ near the higher threshold. If the particles in the higher channel is strongly attractive (*i.e.*, $|a_{22}|$ is large), the zero is close to the higher threshold, winning the competition, and leads to a dip, as shown by the red and green curves in Fig. 6 (b). The behavior of a dip in $|T_{11}|$ and a peak in $|T_{21}|$ has been seen in experimental data: there is a narrow peak around the $K\bar{K}$ threshold, corresponding to the $f_0(980)$, in the $\pi\pi$ invariant mass distribution of $J/\psi \to \phi\pi^+\pi^-$ [92] and a dip around the $K\bar{K}$ threshold in the $\pi\pi$ invariant mass distribution of $J/\psi \to \omega\pi^+\pi^-$ [93].

For specific $XYZPT$ states, the closeness to the thresholds of a pair of charmed hadrons allows for an NREFT treatment with the charmed hadrons as the effective degrees of freedom. Depending on whether the pion exchange is explicitly included, the NREFT can be pionless (see, *e.g.*, Refs. [94–100]) or pionful (see, *e.g.*, Refs. [101–104]). Processes with the ultrasoft pions, such as the decay $X(3872) \to D^0\bar{D}^0\pi^0$ and the near-threshold $\pi X(3872)$ scattering, have also been studied in a framework treating both the charmed mesons and the pions nonrelativistically, called XEFT [105–111]. Discussions of the mixing of hadronic molecules with nearby $\bar{Q}Q$ states in the NREFT context can be found in Refs. [112, 113]. Here we only discuss the multiplet structures derived from HQSS with the P_c states as an example. For more thorough reviews of the applications of EFT techniques to the hidden-charm and double-charm systems, we refer to Refs. [11, 13, 114].

Let us consider the $\Sigma_c^{(*)}\bar{D}^{(*)}$ pairs in S waves. Organizing them in terms of spin and parity, there are seven pairs:

$$\begin{aligned} J^P &= \frac{1}{2}^- : \quad \Sigma_c\bar{D}, \Sigma_c\bar{D}^*, \Sigma_c^*\bar{D}^*, \\ J^P &= \frac{3}{2}^- : \quad \Sigma_c^*\bar{D}, \Sigma_c\bar{D}^*, \Sigma_c^*\bar{D}^*, \\ J^P &= \frac{5}{2}^- : \quad \Sigma_c^*\bar{D}^*. \end{aligned} \tag{12}$$

In the heavy quark limit, the heavy quark spin decouples, and the angular momentum of the light degrees of freedom s_ℓ, including the spin of light quarks and the orbital angular momentum, becomes a good quantum number. For $D^{(*)}$, $s_\ell = 1/2$, and for $\Sigma_c^{(*)}$, $s_\ell = 1$. We denote the total angular momentum of the light degrees of freedom for the $\Sigma_c^{(*)}\bar{D}^{(*)}$ pair as s_L. It takes two possible values: $1/2$ and $3/2$. Therefore, linear combinations of the seven pairs in Eq. (12) are grouped in two spin multiplets with $s_L = 1/2$ and $s_L = 3/2$, respectively, shown as follows:

$s_L^{P_\ell}$	$s_{c\bar{c}}^{P_{c\bar{c}}}$	J^P
$\frac{1}{2}^+$	0^-	$\frac{1}{2}^-$
$\frac{1}{2}^+$	1^-	$\frac{1}{2}^-, \frac{3}{2}^-$
$\frac{3}{2}^+$	0^-	$\frac{3}{2}^-$
$\frac{3}{2}^+$	1^-	$\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$

where P_ℓ and $P_{c\bar{c}}$ represent the parity of the light degrees of freedom and that of the $c\bar{c}$ pair, respectively, and $s_{c\bar{c}}$ is the total spin of the $c\bar{c}$ pair. Consequently, at LO of the nonrelativistic expansion, there are only two constant contact terms for the S-wave $\Sigma_c^{(*)}\bar{D}^{(*)}$ interactions for each total isospin ($I = 1/2$ or $3/2$). Since the $P_c(4312, 4440, 4457)$ are isospin-1/2 states, one may determine the two contact terms with masses of any two of the $P_c(4312, 4440, 4457)$ states as inputs. Because both the $P_c(4440, 4457)$ states are below the threshold of $\Sigma_c\bar{D}^*$, whose total spin can be either $1/2$ or $3/2$, there are two possibilities for assigning spins of these two states. It turns out that no matter what assignment is taken, seven $\Sigma_c^{(*)}\bar{D}^{(*)}$ hadronic molecules are always predicted [99, 115]. In addition to the three states reported by the LHCb Collaboration [87], there is a narrow $P_c(4380)$ state, which is a $\Sigma_c^*\bar{D}$ molecule, and three $\Sigma_c^*\bar{D}^*$ molecules.

The mass spectrum obtained in pionless NREFT gets only slightly modified when the pion exchange is taken into account nonperturbatively [102, 104]. The $J/\psi p$ invariant mass distribution measured by the LHCb Collaboration can be well reproduced in both possibilities of the spin assignments of $P_c(4440, 4457)$; however, the possibility of $J^P_{P_c(4440)} = 3/2^-$ and $J^P_{P_c(4457)} = 1/2^-$ is preferred since the results in this case are insensitive to the cutoff used in regulating the ultraviolet divergence in LSE [104]. Details of the analysis can be found in Ref. [104].

A similar NREFT analysis of the T_{cc}^+ with the full $DD\pi$ three-body effects has been performed in Ref. [116], which determines the pole of the T_{cc}^+ from the LHCb data [88, 89] to be $-356^{+39}_{-38} - i(28 \pm 1)$ keV relative to the $D^{*+}D^0$ threshold. The extracted scattering length and effective range for DD^* implies that the $T_{cc}(3885)^+$ is an isoscalar DD^* hadronic molecule with $J^P = 1^+$ using the compositeness relation [66, 117]. The $T_{cc}(3885)^+$ is expected to have a spin partner just below the D^*D^* threshold with $J^P = 1^+$ [100, 116]. A review of the double-charm tetraquarks can be found in Ref. [118].

4. Summary

From the discussions above, it is clear that EFT is an indispensable tool in understanding the exotic hadron candidates observed in experiments and in predicting new symmetry-related states. Experimental and lattice data need to be analyzed using EFT with symmetry constraints built in to reach a precision hadron spectroscopy whenever possible. With experimental data and lattice calculations in more processes and with higher statistics, a deeper understanding and a reliable classification of the exotic hadrons is foreseen.

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