PROCEEDINGS OF SCIENCE

Phase Plane Rotation as Variance Reduction Method in Monte Carlo Simulations of Axial-Symmetric Radiation Sources

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Monte Carlo algorithms often require large computing resources, primarily in the form of processing time. Knowledge of the underlying principles governing the simulated problem under specific circumstances allows the use of certain variance reduction methods. This work introduces a novel approach to variance reduction based on the partial axial symmetry of the clinical accelerator. The development of the title method was performed using an accelerator simulation that was divided into two phases: The first phase involves simulating the fluence through the symmetrical part of the accelerator, while the second phase involves fluence through the non-symmetrical part. Simulation outputs of the first phase in the form of phase space files were manipulated using in-house software for rotation of the intermediate phase plane. This way, multiple simulations of particle fluence from the source to the last rotationally symmetric component were effectively replaced by multiple random rotations of the phase plane. Analysis of phase plane rotation effects and usability of this approach as a variance reduction method is further investigated in the EGSnrc code. The usage of phase plane rotation increased the number of particles by the number of rotations factor, resulting in less variance of the estimated dose. However, rotation introduces artifacts that can be treated as a systematic error and these artifacts are discussed throughout this work. Qualitative analysis of the dose profiles undoubtedly shows the superiority of phase space rotations in comparison to particle recycling variance reduction. Also, notable improvement is achieved in terms of the efficiency of the simulation. The presented results demonstrate improvement in the overall accuracy of the simulation. Given the geometry-independent software solution, it is to be expected that the presented method can be applied with equal success to a variety of axial-symmetric radiation sources, and further extended to other packages for Monte Carlo simulations of radiation transport.

The 11th Conference of the Balkan Physical Union (BPU11 Congress)
28 August - 01 September, 2022
Belgrade, Serbia

*Speaker

PoS(BPU11)231

1. Introduction

The use of Monte Carlo methods in radiation dosimetry and general medical physics has increased almost exponentially since the 1970s [1]. Several codes have been developed for this purpose out of which EGSnrc [2] is one of the most popular choices in medical physics practice. When estimating quantities of interest by Monte Carlo code, simulation efficiency is the usual problem, which leads to the development of variance reduction methods. By reducing the CPU time required to obtain a sufficiently small variance in the quantity of interest, one can increase the overall efficiency of the simulation.

Various variance reduction methods have been presented in the literature [3], out of which only a few discuss simulation geometry as the primary motivator behind variance reduction. The radiation source, such as a linear accelerator, provides an opportunity to utilize the geometry of its component assemblies in an attempt to make simulation more efficient. The specific geometry of linear accelerator assemblies indicates a certain form of axial symmetry. In the photon mode, all the components of the accelerator, from the source to the jaws, have axial symmetry regarding the z-axis. It follows that the majority of the accelerator segment is axial-symmetric, thereby movement of particles through this segment is rotationally invariant. This observation was used by Bush et al. [4] to demonstrate that particle redistribution in the axial-symmetric plane can be used as a variance reduction method for linear accelerator simulations. The presented work builds on previous ideas utilizing symmetry inside the simulation, but presents a solution independent of the simulation setup and/or Monte Carlo code used. The general idea is to manipulate radiation source modeling outputs in form of a phase space file. This file represents an axial symmetric field from which particle coordinates are extracted and additional particles are artificially produced by successive random rotations around the central axis. This way, population of particles inside a given phase plane is increased without affecting radiation field symmetry. Further simulation can be continued with a greater population of particles than input, which results in less variance of the final estimated quantity.

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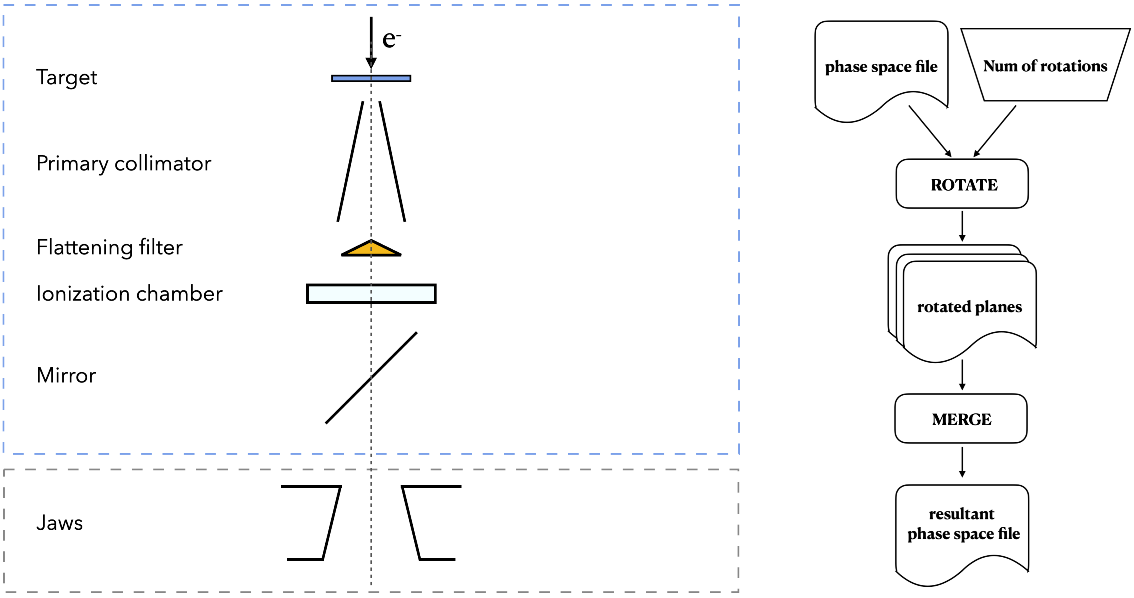
2. Implementation

2.1 Simulation setup

Development of the title method was performed using a linear accelerator simulation in BEAMnrc source modeling environment from EGSnrc code system [5], with the component modules defined as depicted in Figure 1a. The first phase of the simulation produces an axial symmetric phase plane, while the second phase produces a rectangular field phase plane. Simulation outputs of the first phase in the form of phase space files were used as input for the title method. Software manipulation of the axial-symmetrical phase plane was split into two functions: The first function takes the input phase plane and rotates the plane multiple times by a random angle. This was done by sampling a random angle and applying the standard two-dimensional rotation matrix to the particle x and y coordinates. After multiple rotations, the second function takes rotated planes and merges them into one output phase plane. An in-house software for the manipulation of .egsphsp files was written for this purpose. A flowchart of the described procedure is presented in Figure 1b.

2.2 Processing results

The resulting axial-symmetric phase plane is used as an input for the second phase of the simulation. The final rectangular field phase plane is used as a source in the DOSXYZnrc package for the estimation of the dose distribution in a water phantom. Dose distribution is obtained in a 500mm cubic phantom with a 5mm voxel size. All simulations are performed on a 16 x 2.27 GHz multiprocessor system, using the g77 compiler provided within the EGSnrc package. Results were presented using MATLAB (ver. R2020b) routines written for the analysis of phase space (.egsphsp) and dose distribution (.3ddose) outputs.



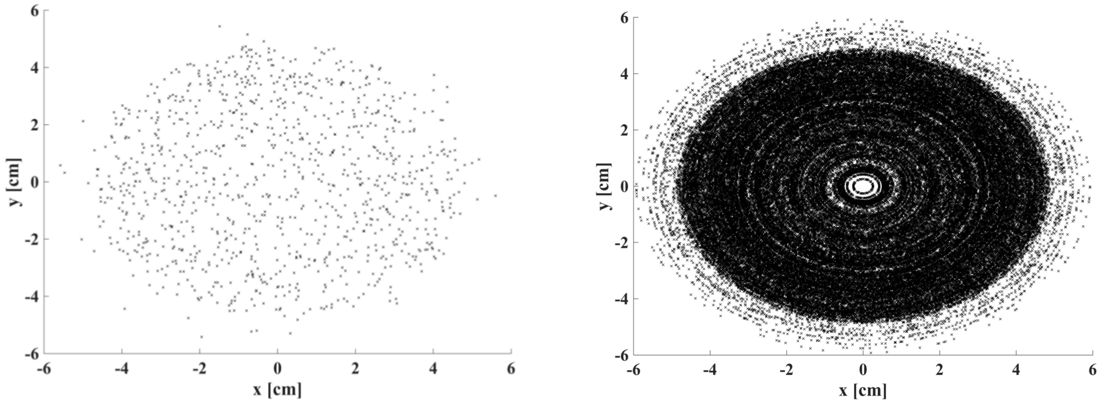
(a) Linear accelerator simulation setup **(b)** Phase plane rotation

Figure 1: Simulation setup

3. Rotation artifacts

The effectiveness and utility of a variance reduction method is largely determined by the artifacts introduced. Considering that the presented method is based on the geometrical manipulation of the phase plane, the appearance of artifacts reflecting regular structures caused by rotation is expected. When the initial population of particles is small, concentric circles artifact representing multiple rotations of the initial phase plane is introduced, as shown in Figure 2. Visual indicators of these artifacts are histograms of particle density for the x and y axis shown in Figure 3. Ripples on these histograms are a specific indicator of these artifacts and the amount of ripple is the direct consequence of the insufficient initial population of particles or excessive use of rotation. Thereby, this simple analysis serves as a sanity check for using the title method.

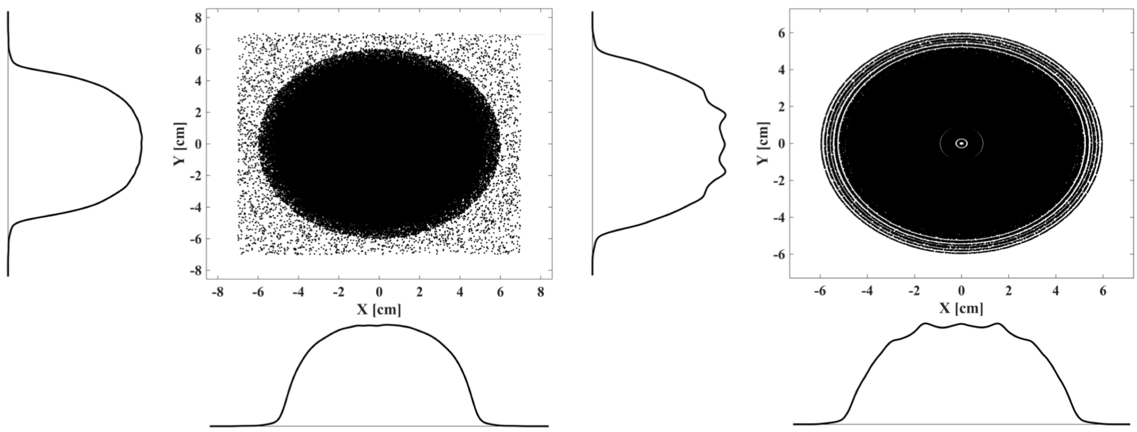
Concentric circle artifacts can be sufficiently minimized by taking a large initial population of particles to rotate. For the presented method, a population with more than 10^6 particles can be considered large enough. Furthermore, by successive attempts the threshold number of rotations after which the artifact becomes noticeable can be found.



(a) Initial phase plane P1

(b) P1 rotated 100 times

Figure 2: Concentric circle artifact as a result of the exorbitant rotation of scarce populated initial phase plane



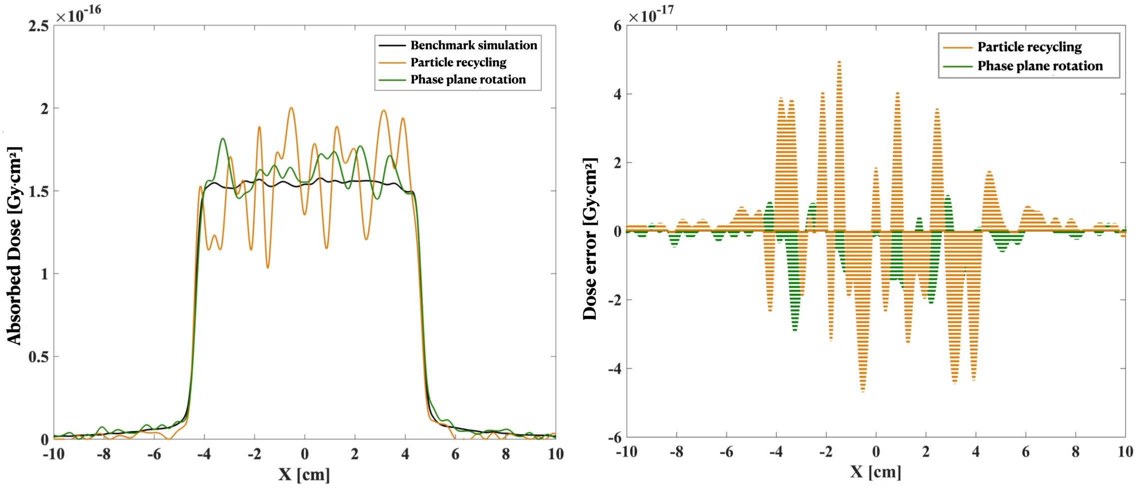
(a) Analog simulation

(b) Phase plane rotation

Figure 3: Particle density histograms for phase planes containing 10^6 particles, obtained using analog simulation and phase plane rotation

4. Benchmark comparisson

Justification of the proposed method as a viable variance reduction method is done by benchmarking the phase plane rotation and particle recycling with reduction-less analog simulation. Metric used for comparison is dose profile representing standard 6 MV 10×10 cm^2 field which is produced as the result of the accelerator simulation. The benchmark analog simulation phase plane consists of approximately 10^9 particles, which is a large enough population to obtain smooth dose profile. As presented on Figure 4a, phase plane rotation produces less fluctuations on the dose profile than particle recycling. Also, Figure 4b shows less deviation from the benchmark profile when using phase plane rotation. By carefully choosing the initial phase plane and the number of rotations, contribution of the phase plane rotation to the variance of the final result can be equated to the contribution of the latent variance of the phase plane used.



(a) Profile comparisson

(b) Deviation from the benchmark profile

Figure 4: Analysis of profiles obtained by particle recycling and phase plane rotation

5. Increase in efficiency

5.1 Measurement

The practical usefulness of the proposed method is best observed when simulation efficiency is estimated. For this purpose, a standard definition of Monte Carlo simulation efficiency, as the inverse product of variance and CPU time, was adopted. The variance was calculated as the sum of the relative errors of the dose in the phantom, accounting for the contribution of voxels in which the dose was greater than or equal to half the maximum dose in the phantom. Dose error sum is weighted by the total number of voxels inside the phantom. Comparison is done for simulations with phase plane rotation and with particle recycling, which produces similar dose variance value of the final dose distribution obtained. Simulations were then analyzed for three axial-symmetric phase planes containing 10^3, 10^4 and 10^5 particles, respectively.

5.2 Efficiency disscussion

Results presented in Figure 5 are indicating that for a similar magnitude of variance similar efficiency can be achieved using rotation and recycling, with particle recycling achieving slightly better efficiency. This result could mislead the conclusion that particle recycling has an advantage over rotation in terms of efficiency, which may be true when considering the numerical result. It can be shown, however, that the slight advantage of particle recycling comes at the expense of a systematic error in the final result. The used definition for variance estimates the uncertainty of the quantity of interest's mean value without considering the systematic error introduced by variance reduction. Recycling of the particles produces clouds of increased particle density which have a much better-defined average dose, and that effect is what reduces the variance and therefore increases the overall simulation efficiency. Therefore, reliable estimation of how useful the variance reduction technique is would need to consider both systematic error and the statistical uncertainty of the estimated quantity in the final result.

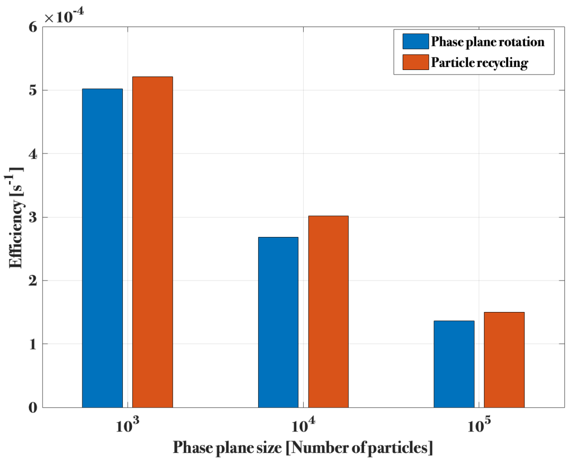


Figure 5: Efficiency comparison

6. Conclusions

In this work, the utility value of the phase plane rotation method was confirmed and the potential limitations of its use were noted. The systematic error introduced by phase plane rotation is convincingly smaller than the one introduced by particle recycling, indicating the presented method's applicability. One of the few advantages of the phase plane rotation method in comparison to similar variance reduction approaches is that once processed phase space file could be used in multiple different simulations. Also, the presented method is package independent solution, which means that it could be used in other packages for Monte Carlo simulations of radiation transport, limited only by the structure of the simulation output files. Finally, although this work describes a variance reduction solution for the linear accelerator Monte Carlo simulations, the same approach can be applied when simulating any other axial-symmetric radiation source.

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