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# Systematic uncertainties in integrated luminosity measurement at CEPC

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The very forward region is one of the most challenging regions to instrument at a future $e^+e^-$ collider. Machine-detector interface at CEPC will include a calorimeter dedicated for precision measurement of the integrated luminosity at a permill level or better. We discuss a feasibility of such precision from the point of view of systematic effects arising from luminometer mechanical precision and positioning, beam-related requirements and physics background. Additionally, a method of the beam energy spread determination, initially proposed for FCC, is discussed for the CEPC beams, as well as the impact of beam energy spread uncertainty on integrated luminosity measurement and the precision of electroweak observables determination at the $Z^0$ pole.

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## Contents





## 1. Introduction

The Circular Electron Positron Collider (CEPC) is a large international scientific facility proposed by the Chinese particle physics community in 2012 to test the validity scale of the Standard Model (SM) in precision measurements in the Higgs, BSM and EW sector. These measurements should provide critical tests of the underlying fundamental physics principles of the Standard Model and are vital in exploration of new physics beyond the SM. In $e^+e^-$ collisions, the CEPC is foreseen to operate at 91.2 GeV center-of-mass (CM) energy as a Z factory, at 160 GeV (WW production threshold) and at 240 GeV as a Higgs factory.

CEPC physics program requires relative uncertainty of the integrated luminosity measurement to be of order of $10^{-4}$ at the $Z^0$ pole and of order of $10^{-3}$ at 240 GeV CM energies. The method of integrated luminosity measurement at CEPC, as well as the machine parameters, detector concept, machine-detector interface (MDI) and physics performance, is described in [1].

Usual method of integrated luminosity measurement is counting of Bhabha scattering events at small polar angles, which is a well described QED process ($\delta\sigma_{Bh} \sim 10^{-4}$). However, there is an extensive list of systematic effects to be known with the same accuracy, such as detector related uncertainties, beam related uncertainties and uncertainties originating from physics and machine related interactions. In this paper we review the effects of detector and beam related uncertainties, namely uncertainties on the luminometer mechanical positioning and size and uncertainties on the beam energy, beam synchronization and interaction point (IP) position (Sections 3.1 and 3.2). Also, we review the uncertainty originating from the miscount of two-photon background (Section 3.3). Motivated by [2], in Section 4 we discuss the possibility of CEPC beam energy spread (BES) determination with the post-CDR beam parameters [3] and its impact on the integrated luminosity precision. In addition, we present the impact of the estimated BES precision on relevant electroweak observables at the $Z^0$ pole.

## 2. The very forward region at CEPC

The Machine Detector Interface (MDI) of CEPC covers the area of 6 m from the interaction point (IP) along the z-axis, in both directions. The two beams collide at the IP with a crossing angle of 33 mrad in the horizontal plane with the final focus length of 2.2 m. Layout of the interaction region at CEPC and a possible positioning of the luminometer are shown in figure 1 [4].

Luminometer at CEPC is proposed to cover the polar angle region between 30 mrad and 105 mrad (with the fiducial volume between 53 mrad and 79 mrad) corresponding to the luminometer aperture of 28.5 mm for the inner radius and 100 mm for the outer, at 950 mm distance from the interaction point. The accelerator components inside the detector are within a conical space with an opening angle of 118 mrad (figure 1 left). To ensure electron-photon separation and alignment of the device, a silicon disc will be positioned in front of the luminometer (figure 1 right).

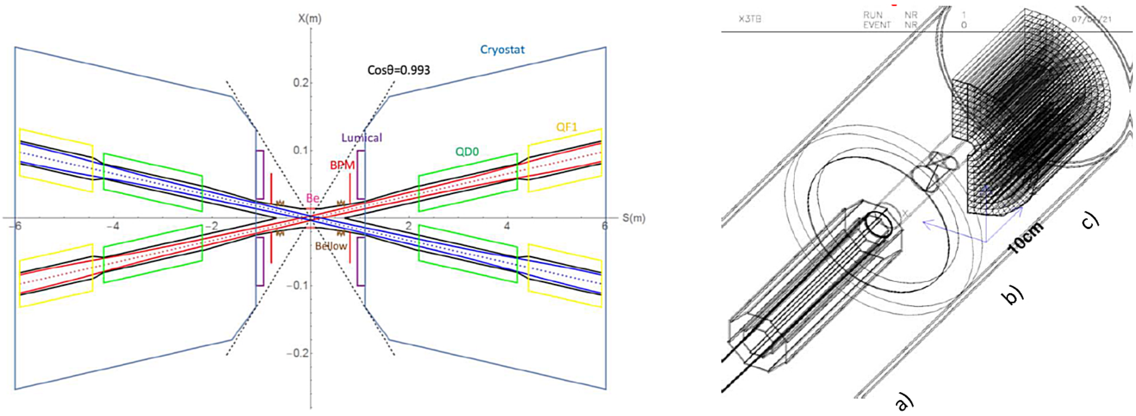


**Figure 1:** Layout of the MDI region at CEPC (left) and a possible positioning of octagon silicon layer surrounding the beam pipe (a), silicon tracking disc (b) in front of the LYSO luminometer (c) (right).

## 3. Integrated luminosity measurement and systematic uncertainties

Integrated luminosity measurement is a counting experiment based on Bhabha scattering, defined as $\mathcal{L} = N_{Bh}/\sigma_{Bh}$, where $N_{Bh}$ is Bhabha count in the certain phase space and within the detector acceptance (fiducial) region in the certain time interval and $\sigma_{Bh}$ is the theoretical cross-section in the same geometrical and phase space. In a real experiment, however, there are numerous systematic effects influencing Bhabha count. To control integrated luminosity at the required level of $10^{-4}$ ($10^{-3}$) at the $Z^0$ pole (240 GeV), both Bhabha count and theoretical cross-section should be known with the same precision. Further we discuss feasibility and requirements for such a precision of the Bhabha count, considering each systematic effect individually.

### 3.1 Uncertainties from mechanics and positioning

Systematic uncertainties from detector and machine-detector interface related effects have been quantified through a simulation study, assuming $10^7$ Bhabha scattering events generated using BHLUMI V4.04 Bhabha event generator [5], at two CEPC center-of-mass energies: 240 GeV and $Z^0$ production threshold. Final state particles are generated in the polar angle range from 45 mrad

**Table 1:** Required absolute precision of mechanical parameters individually contributing to the relative uncertainty of the integrated luminosity as $10^{-3}$ ($10^{-4}$) at 240 GeV CM energy ($Z^0$ pole).

| parameter | precision @ 240 GeV | precision @ 91 GeV |
|---|---|---|
| $\Delta r_{in}$ ($\mu$m) | 10 | 1 |
| $\sigma_r$ (mm) | 1.00 | 0.20 |
| $\Delta l$ (mm) | 1.00 | 0.08 |
| $\sigma_{x_{IP}}$ (mm) | 1.0 | 0.5 |
| $\sigma_{z_{IP}}$ (mm) | 10 | 7 |
| $\Delta\varphi$ (mrad) | 6.0 | 0.8 |



to 85 mrad that is within a few mrad margin outside of the detector fiducial volume, to allow the contribution of events with non-collinear final state radiation. The effective Bhabha cross-section in this angular range is of order of 5 nb at 240 GeV and 50 nb at the $Z^0$ pole. We assume the shower leakage from the luminometer is negligible. Furthermore, we used event selection in polar angle acceptance in a way it has been done at OPAL ([6], Chapter 1.3) - to be asymmetric between the left and right arms of the detector. It means that at one side we consider the full fiducial volume, while at the other side we shrink the inner radial acceptance by 1 mm. This has been done subsequently to the left (L) and right (R) sides of the luminometer, on event by event basis, resulting in cancelation of systematic uncertainties caused by the assumption of L-R symmetry of a Bhabha event.

Detector-related uncertainties arising from manufacturing, positioning and alignment that has been considered are:

- maximal uncertainty of the luminometer inner radius ($\Delta r_{in}$),
- RMS of the Gaussian spread of the measured radial shower position with respect to the true impact position in the luminometer front plane ($\sigma_r$),
- maximal absolute uncertainty of the longitudinal distance between left and right halves of the luminometer ($\Delta l$),
- RMS of the Gaussian distribution of mechanical fluctuations of the luminometer position with respect to the IP, caused by vibrations and thermal stress, radial and axial ($\sigma_{x_{IP}}, \sigma_{z_{IP}}$),
- maximal absolute angular twist of the calorimeters corresponding to different rotations of the left and right detector axis with respect to the outgoing beam ($\Delta\varphi$).

Considered deviations are maximal, as we assumed $10^{-3}$ and $10^{-4}$ contribution to the relative uncertainty of integrated luminosity from each individual effect, at 240 GeV and $Z^0$ pole respectively. Table 1 gives corresponding requirements of the listed parameters. Inner aperture of the luminometer is one of the most demanding mechanical parameters to control for run at the $Z^0$ pole, due to the $\sim 1/\theta^3$ dependence of the Bhabha cross-section on the polar angle.

### 3.2 MDI related uncertainties

Following uncertainties that may arise from the beam properties and its delivery to the interaction point are considered:

**Table 2:** Required absolute precision of MDI parameters contributing to the relative uncertainty of the integrated luminosity of $10^{-3}$ ($10^{-4}$) at 240 GeV CM energy ($Z^0$ pole). The average net center-of-mass energy uncertainty $\Delta E_{CM}$ limits are derived by error propagation from the Bhabha cross-section calculation.

| **parameter** | **precision @ 240 GeV** | **precision @ 91 GeV** |
|---|---|---|
| $\Delta E_{CM}$ (MeV) | 120 | 5 |
| $\Delta E$ (MeV) | 130 | 5 |
| $\Delta x_{IP}^{BS}$ (mm) | 1.0 | 0.5 |
| $\Delta z_{IP}^{SY}$ (mm) | 10 | 2 |
| $\Delta\tau$ (ps) | 15 | 3 |

- maximal deviation ($\Delta E$) of the individual beam energy from its nominal value, resulting in asymmetry in energy of the incoming $e^+$ and $e^-$ beams that may be caused by various effects, from the beam energy spread to beamstrahlung and initial state radiation. The former causes the longitudinal boost of the CM of interacting and, consequently, final state particles.
- maximal uncertainty of the average net CM energy ($\Delta E_{CM}$) from the Bhabha cross-section calculation based on $\sigma_{Bh} \sim 1/E_{CM}^2$ dependence,
- maximal radial ($\Delta x_{IP}^{BS}$) and axial ($\Delta z_{IP}^{SY}$) IP position displacements with respect to the luminometer, caused by the finite transverse beam sizes and beam synchronization respectively,
- maximal time shift in beam synchronization ($\Delta\tau$) leading to the IP longitudinal displacement $\Delta z_{IP}^{SY}$.

Table 2 gives absolute uncertainties of these parameters contributing to the relative uncertainty of integrated luminosity as $10^{-3}$ ($10^{-4}$) at 240 GeV CM energy ($Z^0$ pole). Figure 2 illustrates the counting loss in luminometer due to longitudinal boost of the CM frame ($\beta_z = 2 \cdot \Delta E/E_{CM}$) at both center-of-mass energies.

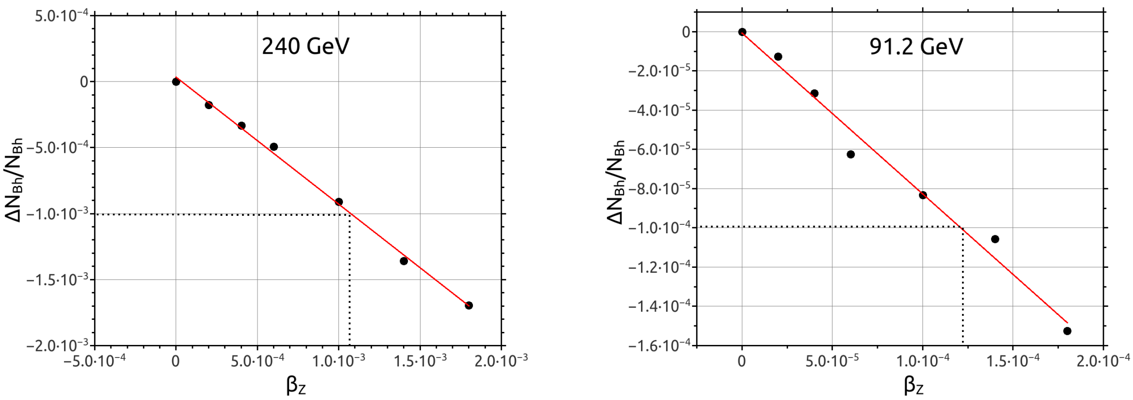


**Figure 2:** Loss of the Bhabha count in the luminometer due to the longitudinal boost of the CM frame $\beta_z$. Event selection asymmetric in polar angle, as described in Section 3.1, is applied. Dotted line indicates $10^{-3}$ and $10^{-4}$ relative uncertainty of the Bhabha count required at 240 GeV and $Z^0$ pole CEPC run, shown on the left and right respectively.

The biggest challenge at the $Z^0$ pole comes from the fact that the uncertainty of energy of individual beams needs to be controlled at the level of $\sim 10^{-4}$ with respect to the nominal beam energy, which is apparently a few times smaller than the foreseen BES at CEPC of 0.08%, corresponding to 36.5 MeV. The current value of BES at the $Z^0$ pole contributes to the relative uncertainty of the Bhabha count as $\sim 8 \cdot 10^{-4}$, through the asymmetry in beam energies equivalent to the longitudinal boost of the CM system of initial (final) states with respect to the laboratory frame and the consequent loss of the Bhabha coincidence due to accolinearity.

At 240 GeV CM energy conditions for precision integrated luminosity determination are more relaxed and the current BES of 0.134%, corresponding to the individual beam energy uncertainty of 161 MeV, contributes approximately as $1.3 \cdot 10^{-3}$ to the integrated luminosity uncertainty from the beam energy asymmetry. The impact of the BES uncertainty on the integrated luminosity precision will be separately discussed in Section 4.2, once the precision of BES measurement is estimated (Section 4.1).

### 3.3 Two-photon processes as a background

In $e^+e^-$ collisions there are several Feynman diagrams of four-fermion final state processes contributing to possible $e^+e^- f\bar{f}$ final state background for the Bhabha scattering. Contribution of multiperipheral (two-photon) processes (figure 3) is by far the biggest, due to the large cross-section (~nb) and the fact that spectator electrons (positrons) are emitted at very small polar angles, so some of them can be misidentified as Bhabha electrons. Here, we quantify the contribution from this source of physics background, assuming geometrical parameters as in Section 2.

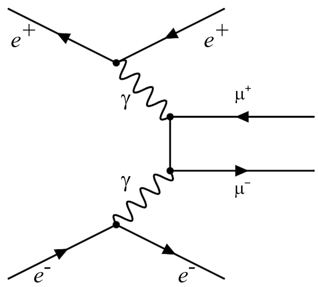


**Figure 3:** Feynman diagram of two-photon process producing $e^+e^-\mu^+\mu^-$ final state in $e^+e^-$ collisions.

To estimate the background to signal ratio at 240 GeV CEPC, we simulated $10^5$ $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ events using WHIZARD V2.8 [7], with the effective cross-section $\sigma_{\text{eff}} \sim 0.3$ pb in the fiducial volume of the luminometer. To illustrate cross-section dependence on the polar angle, $10^7$ Bhabha events are simulated using BHLUMI V4.04 in the polar angle range 20 mrad $< \theta <$ 200 mrad, with the effective cross-section of $\sim 3.3$ nb in the fiducial volume of the luminometer. We found that the contamination of signal with $e^+e^-$ pairs from the $e^+e^-\mu^+\mu^-$ final state is significantly below $10^{-4}$ in the luminometer's fiducial volume, even without any event selection. The total amount of background should be conservatively scaled by a factor 3 to account for lepton flavor integration.

## 4. Impact of the beam energy spread

Uncertainty of the beam energy spread determination will contribute to the overall systematic uncertainty of the integrated luminosity measurement by affecting the asymmetry of beam energies and consequently providing longitudinal boost $\beta_z$ of the colliding system in the laboratory frame.

### 4.1 Method of the beam energy spread determination

Motivated by the similar work done at FCCee [2] and having in mind that numerous precision observables, including integrated luminosity, depend on the precision of BES, we looked into possibility to measure it at CEPC using a well defined central process - di-muon production $e^+e^- \to \mu^+\mu^-$.

We argue that the effective CM energy ($\sqrt{s'}$) is sensitive to variation of the BES that consequently can be determined from the population of the peak of the $\sqrt{s'}$ distribution. To determine $\sqrt{s'}$ sensitivity to the BES, we generated a few hundred thousand $e^+e^- \to \mu^+\mu^-$ events at 91.2 GeV and 240 GeV CM energies. Events are generated using WHIZARD V2.8, in the polar angle range from $8^o$ to $172^o$, which corresponds to the angular acceptance of the central tracker (TPC) at CEPC. In the simulated events, the effects like the initial state radiation (ISR) and detector angular resolution are modeled and studied individually, to evaluate their impact on the $\sqrt{s'}$ distribution with respect to the concurrent BES. Detector energy resolution is simulated by performing Gaussian smearing of the muons' polar angles. Applying 0.1 mrad smearing corresponds to 100 $\mu$m position resolution foreseen for TPC at CEPC [1]. The effective CM energy squared, s', can be calculated from the reconstructed muons' polar angles [8], as:

$$\frac{s'}{s} = \frac{\sin\theta^+ + \sin\theta^- - |\sin\theta^+ + \theta^-|}{\sin\theta^+ + \sin\theta^- + |\sin\theta^+ + \theta^-|}, \tag{1}$$

relying on the excellent TPC spatial resolution. As illustrated in figure 4, BES dominates the $\sqrt{s'}$ shape at energies close to the nominal CM energy. We found that polar angle resolution in central tracker should not be larger than 0.5 mrad, corresponding to the 500 $\mu$m position resolution. The same holds for 240 GeV CEPC run.

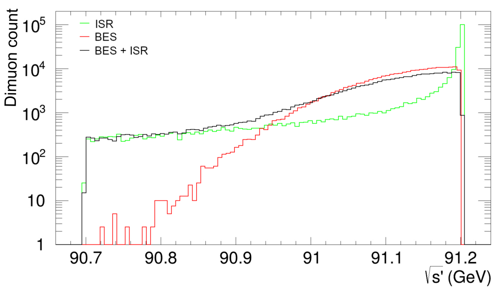


**Figure 4:** Count of di-muon events versus the effective CM energy (top part of the spectrum) at the $Z^0$ pole. BES is the dominant effect to reduce the number of events near the maximal CM energy.

To exploit $\sqrt{s'}$ peak sensitivity to the BES values, BES is varied around the nominal value, generating $10^5$ ($2.5\cdot10^5$) events per BES variation at 240 GeV (91.2 GeV). The observed dependence

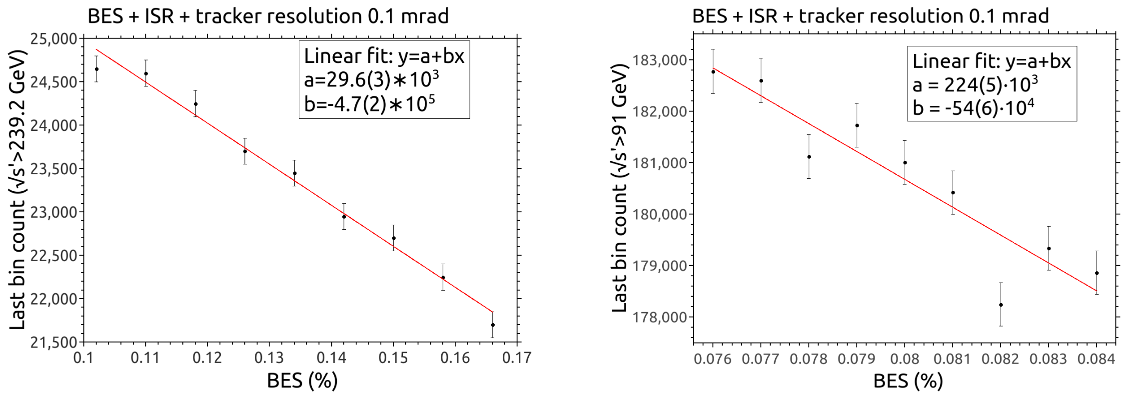


**Figure 5:** Number of di-muon events in the top 3‰ of the nominal CM energy at 240 GeV (left) and in the top 2‰ of the nominal CM energy at 91.2 GeV (right) for various BES values.

**Table 3:** BES relative variations experimentally accessible at CEPC. Values that are calculated or obtained from simulated BES measurement are bolded. Other entries in the table are taken from [1]. Uncertainty of BES determination contributes to the absolute uncertainty of the beam energy as $\Delta E_{BES}$.

| CEPC | $\mathcal{L}$ @ IP ($\mathrm{cm}^{-2}\mathrm{s}^{-1}$) | Nominal BES $\delta$ (%) | Number of events | Cross-section $e^+e^- \to \mu^+\mu^-$ | Collecting time | Relative statistical uncertainty of BES | Total relative uncertainty of BES | $\Delta E_{BES}$ (MeV) |
|---|---|---|---|---|---|---|---|---|
| $Z^0$ pole | $1.02 \cdot 10^{36}$ | 0.080 | $2.5 \cdot 10^5$ | **1.5 nb** | **3 min** | **1.2%** | **25%** | **9** |
| 240 GeV | $5.2 \cdot 10^{34}$ | 0.134 | $1.0 \cdot 10^5$ | **4.1 pb** | **5 days** | **2.3%** | **15%** | **24** |

is illustrated in figure 5, at the 240 GeV (left) and $Z^0$ pole (right). As expected, larger BES leads to the larger reduction of the number of di-muon events carrying near to maximal available energy from the collision. Knowing this dependence from simulation allows for determination of the effective BES (denoted as $\delta'$) once the count of di-muon events is known experimentally.

Table 3 shows that 1.2% relative statistical uncertainty of the BES measurement arises after only 3 minutes of data taking with $1.02 \cdot 10^{36}$ $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ instantaneous luminosity at the $Z^0$ pole. Statistical uncertainty is derived from the size of the error bars in figure 5 right, that is from the size of the di-muon sample. BES can be measured with the total relative uncertainty of 25%, where the systematic contribution comes from the uncertainty of fit parameters of the calibration curve. The total uncertainty is obtained by combining statistical and systematic components as uncorrelated, that is they are summed quadratically. Using the same approach, at 240 GeV center-of-mass energy BES can be measured with 15% total uncertainty (figure 5, left) and 2.3% relative statistical uncertainty in approximately 5 days of data taking with instantaneous luminosity of $5.2 \cdot 10^{34}$ $\mathrm{cm}^{-2}\mathrm{s}^{-1}$. Total uncertainty of the BES translates into maximal uncertainty of individual beam energies $\Delta E_{BES}$ of 9 MeV (24 MeV) at the $Z^0$ pole (240 GeV).

### 4.2 Impact on integrated luminosity measurement and precision electroweak observables

As already mentioned is Section 3.2, asymmetry in beam energies will give a rise to the longitudinal boost $\beta_z$ leading to the loss of coincidence of Bhabha hits in left and right arms of the luminometer. Considering BES uncertainty individually as a source of longitudinal boost $\sigma_{\beta_z}$ ($\sigma_{\beta_z} = 2 \cdot \Delta E_{BES}/E_{CM}$), achievable BES uncertainty of 9 MeV at the $Z^0$ pole will translate to $\sigma_{\beta_z} \sim 2 \cdot 10^{-4}$, contributing to the relative systematic uncertainty of the Bhabha count as $5 \cdot 10^{-4}$. The above holds for the symmetrical counting in the fiducial volume, while if asymmetric (LEP-style) selection described in Section 3.1 is applied, luminosity determination is a few times less sensitive to the precision of BES, as illustrated at figure 6. The above suggests that the luminometer should be positioned at the outgoing beams, in order to suppress this source of uncertainty more effectively.

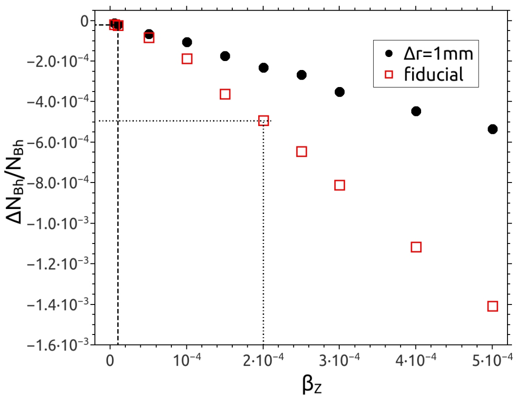


**Figure 6:** Sensitivity of the Bhabha count on the longitudinal boost $\beta_z$ caused by BES uncertainty, for counting in the symmetrical fiducial volume and LEP-style selection ($\Delta r_{cut}$ = 1mm) at the $Z^0$ pole. Referent values of $\beta_z$ correspond to the statistical ($1 \cdot 10^{-5}$, dashed line) and the total ($2 \cdot 10^{-4}$, dotted line) $\Delta E_{BES}$ contributions to the longitudinal boost.

As previously mentioned, several precision electroweak observables at the $Z^0$ pole depend on the BES uncertainty. Figures 7 and 8 illustrate that the cross-section for $Z^0$ production ($\sigma_Z$), $Z^0$ total width ($\Gamma_Z$) and mass ($m_Z$) will receive following contributions from the total BES uncertainty: $\delta(\sigma_Z) \sim 2.6 \cdot 10^{-3}$, $\Delta\Gamma_Z \sim 30$ MeV and $\Delta m_Z < 100$ keV, respectively. Naturally, uncertainties originated solely from the statistical uncertainty of the BES are significantly smaller, as indicated in figures 7 and 8, corresponding to $\delta(\sigma_Z) \sim 1.5 \cdot 10^{-3}$, $\Delta\Gamma_Z \sim 2$ MeV and $\Delta m_Z \sim 50$ keV. These results are summarized in Table 4, together with the BES precision impact on integrated luminosity uncertainty from figure 6.

## 5. Conclusion

In this paper we presented quantification of achievable luminosity precision for CEPC from the perspective of mechanical and MDI requirements, in parallel with the BES determination from the di-muon production and its impact on precision of integrated luminosity and precision electroweak observables at the $Z^0$ pole.

At the $Z^0$ pole, control of the luminometer inner radius at the micrometer level is posing the most demanding requirement regarding detector manufacturing and positioning precision. Uncertainty of the beam energy has to be known below the foreseen beam spread in order to contribute to the relative uncertainty of integrated luminosity as $1 \cdot 10^{-4}$. With the current beam design, BES as

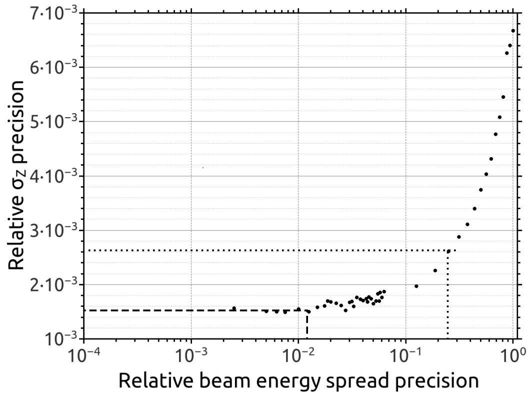


**Figure 7:** Impact of the relative precision of the BES on the $Z^0$ production cross-section $\sigma_Z$. Impact of the BES statistical and total uncertainties are indicated with dashed and dotted lines, respectively. Error bars on ordinate are within dots.

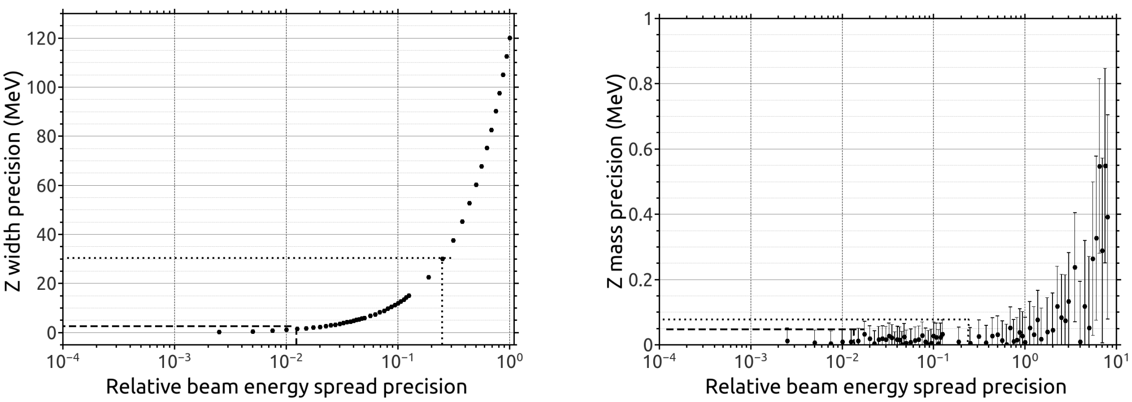


**Figure 8:** Impact of the relative precision of the BES on the $Z^0$ total width (left) and mass (right) absolute precisions. Impact of the BES statistical and total uncertainties are indicated with dashed and dotted lines, respectively. In the case of $Z^0$ mass, precision estimate is conservatively taken to include the error bars corresponding to the standard error of the mean obtained on the sample of one million di-muon events for each beam energy spread deviation. For the $Z^0$ total width precision, error bars on ordinate are within dots.

**Table 4:** Impact of the BES total and statistical uncertainties on precision observables at the $Z^0$ pole: relative statistical precision of the $Z^0$ production cross-section ($\sigma_Z$), absolute statistical precision of the $Z^0$ total width ($\Gamma_Z$) and mass ($m_Z$) and relaative systematic uncertainty of the integrated luminosity for counting in the fiducial volume and asymmetric counting.

| BES @ the $Z^0$ pole | $\delta(\sigma_Z)$ | $\Delta\Gamma_Z$ (MeV) | $\Delta m_Z$ (keV) | $(\Delta\mathcal{L}/\mathcal{L})_{fid}$ | $(\Delta\mathcal{L}/\mathcal{L})_{asym}$ |
|---|---|---|---|---|---|
| Total uncertainty ($\Delta E_{BES}$ = 9 MeV) | $2.6 \cdot 10^{-3}$ | 30 | <100 | $5 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ |
| Statistical uncertainty ($\Delta E_{BES}$ = 432 keV) | $1.5 \cdot 10^{-3}$ | 2 | 50 | $< 10^{-4}$ | $<< 10^{-4}$ |

a cause of the beam energy asymmetry contributes with $\sim 8 \cdot 10^{-4}$ to relative uncertainty of the integrated luminosity at the $Z^0$ pole. Per mill precision of the integrated luminosity measurement at 240 GeV CEPC seems to be feasible from the point of view of existing technologies and foreseen beam properties.

It is also shown that with the CEPC post-CDR instantenious luminosity upgrade, beam energy spread can be determined with the total accuracy corresponding to 9 MeV beam energy uncertainty in only 3 minutes of data-taking of $e^+e^- \rightarrow \mu^+\mu^-$ events at the $Z^0$ pole. It translates to the relative uncertainty of the $Z^0$ production cross-section of $2.6 \cdot 10^{-3}$ and absolute precision of the $Z^0$ mass and width below 100 keV and 30 MeV, respectively. Contribution to the integrated luminosity relative systematic uncertainty is $2.4 \cdot 10^{-4}$ for asymmetric Bhabha counting.

## Acknowledgments

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## References

[1] *CEPC Conceptual Design Report (CDR), Volume II Physics and Detector*, 2018, 381 pp, IHEP-CEPC-DR-2018-02.

[2] P. Janot, *Beam Energy Spread Measurement @ FCC-ee*, talk given at the FCC-ee Polarization Workshop, CERN, 2017.

[3] J. Gao, *CEPC Accelerator Status and TDR Progress*, talk given at the IAS Program on High Energy Physics (HEP 2021), High Energy Physics HKUST IAS (The Hong Kong University of Science and Technology), 2021.

[4] S. Hou, *Updates of the CEPC LumiCal Design*, talk given at the Joint Workshop of the CEPC Physics, Software and New Detector Concept in 2022, 2022.

[5] S. Jadach et al., *Upgrade of the Monte Carlo program BHLUMI for Bhabha scattering at low angles to version 4.04*, Comput. Phys. Commun. **102** (1997) 229–251.

[6] G. Abbiendi et al. [OPAL Collaboration], *Precision luminosity for Z0 line shape measurements with a silicon tungsten calorimeter*, Eur. Phys. J. C **14** (2000) 373–425.

[7] W. Kilian, T. Ohl, J. Reuter, WHIZARD: *Simulating Multi-Particle Processes at LHC and ILC*, Eur. Phys. J. C **71** (2011) 1742, arXiv: 0708.4233 [hep-ph].

[8] G. Abbiendi et al. [OPAL Collaboration, *Determination of the LEP Beam Energy using Radiative Fermion-pair Events*, CERN-PH-EP/2004-042 (2004), arXiv: 0408130 [hep-ex].