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# On DBI Lagrangian Dynamics and its Mechanical and Cosmological Realization

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This work has been motivated by application of DBI Lagrangians, and their dynamics, in cosmology and the theory of inflation. The focus is on the study of the dynamics of a scalar tachyon field with a non-standard Lagrangian of a DBI type. For some relevant analyses, it is suitable to use lower-dimensional models, including a zero-dimensional classical mechanical analogue. Original calculations for several specific and important (tachyonic) potentials are presented. These potentials are also exactly solvable in the framework of Friedmann cosmology, and they have physical motivation in inflationary cosmology. For the purpose of quantization of the (tachyonic) dynamical models, the so-called locally-equivalent Lagrangians of the standard type are considered. As one example, the classical and quantum formalism for an inverted harmonic oscillator with time-dependent frequency is given in more details.

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## 1. Introduction to DBI lagrangians and tachyonic FLRW cosmology

The DBI Langrangian (or a family of DBI Lagrangians) has been known and widely considered in many different fields of theoretical physics. Classical and quantum dynamics of tachyon systems have previously been examined, describing spatially homogeneous scalar fields, in the limit of classical and quantum mechanics. Understanding and modeling these systems are of particular importance in the development of field and string theory. The presented work has been motivated by its application in cosmology and in the theory of inflation.

Quantization of such models is not straightforward and as of yet still poorly understood. Because of this, we apply an approach based on the idea of locally-equivalent Lagrangians. Here, it is suitable to use lower-dimensional models, including a zero-dimensional classical mechanical analogue. As mentioned above, for the purpose of quantization of the (tachyonic) dynamical models, the so-called locally-equivalent Lagrangians of the standard type are considered. We offer two specific examples in some details.

Origins of the Born-Infeld theory can be tracked back to 1934, followed by an important contribution by Dirac in 1962. A deformation of the theory of electromagnetism, which coincides with ordinary electromagnetism for small excitations [1] of the electromagnetic field, has been known as "DBI theory", described by a DBI Lagrangian.

It became evident that this theory can describe a gauge field on single D-branes at low energy, also. The DBI-action is referred to as the low-energy effective action on D-branes. A significant interest in DBI Lagrangian application in cosmological inflation was triggered by A. Sen, when a classical time dependent solution in open string field theory was found (see reference [2] and references therein).

Let us consider a tachyon field with a DBI-type Lagrangian density:

$$\mathcal{L}_T = -V(T)\sqrt{1+(\partial T)^2}, \tag{1}$$

where $T$ is the tachyon field, $V(T)$ is the tachyonic potential, $(\partial T)^2 = g_{\mu\nu}\partial^\mu T\partial^\nu T$ and $g_{\mu\nu}$ are the components of the metric tensor $\hat{g}$, with "mostly positive" signature. The action of a tachyon field in a curved spacetime is given by:

$$S_T = \int d^4x\sqrt{-g}\mathcal{L}_T\,. \tag{2}$$

By plugging the Lagrangian density (1) into the Euler-Lagrange equation:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\frac{\partial(\sqrt{-g}\mathcal{L}_T)}{\partial(\partial^\mu T)}\right) - \frac{\partial\mathcal{L}_T}{\partial T} = 0\,, \tag{3}$$

one obtains the following equation of motion [3]:

$$D_\mu\partial^\mu T - \frac{D_\mu\partial^\nu T}{1-(\partial T)^2}\partial_\mu T\partial_\nu T - \frac{1}{V(T)}\frac{dV(T)}{dT} = 0\,, \tag{4}$$

where $D_\mu$ is the covariant derivative with respect to the metric $\hat{g}$.

The ambient spacetime we consider is of a flat FLRW type, given by the first quadratic form:

$$ds^2 = -dt^2 + a^2(t)\left(dx^2+dy^2+dz^2\right)\,, \tag{5}$$



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where $a(t)$ is the scale factor. In the case of a spatially homogeneous tachyon field, the Lagrangian density (1) then simplifies to:

$$\mathcal{L}_T = -V(T)\sqrt{1-\dot{T}^2}, \tag{6}$$

where $\dot{T} = dT/dt$ and the equation of motion (4) reduces to:

$$\ddot{T} + 3H(t)\dot{T}(1-\dot{T}^2) + \frac{1}{V(T)}\frac{dV(T)}{dT}(1-\dot{T}^2) = 0\,, \tag{7}$$

where $H(t) = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter.

For future convenience, let us reformulate the problem a bit, by taking the following form of the action $S_T$, considering the fact that the spacetime is of a flat FLRW type:

$$\begin{aligned} S_T &= \int d^4x\sqrt{-g}\mathcal{L}_T \equiv \int dt \int dx \int dy \int dz\, a^3(t)\, \mathcal{L}_T \\ &= \int dt \int dV\, a^3(t)\mathcal{L}_T \equiv \int dt\, a^3(t)\mathcal{L}_T \equiv \int dt\, L_T\,, \end{aligned}$$

where we normalised the spatial volume $\int dV \equiv 1$ and introduced the Lagrangian[1]:

$$L_T \equiv a^3(t)\mathcal{L}_T = a^3(t)\left(-V(T)\sqrt{1-\dot{T}^2}\right)\,, \tag{8}$$

The equation of motion (7) can now be recovered using an Euler-Lagrange equation of the form:

$$\frac{d}{dt}\left(\frac{\partial L_T}{\partial \dot{T}}\right) - \frac{\partial L_T}{\partial T} = 0\,. \tag{9}$$

For any quantum approach to this dynamical model and its cosmological application, the goal would be to try to introduce a new field $\phi(T(t))$ and potential $U(\phi)$. Then, the Lagrangian (8) might be rewritten in a general classically-equivalent standard-type form:

$$L_S = f(t)\left(\frac{1}{2}\dot{\phi}^2 - U(\phi)\right), \tag{10}$$

with separated kinetic and potential terms, and the time-dependent function $f(t)$ appearing in (10) would ideally be $f(t) = a^3(t)$. We explore if this "ideal" case is possible to obtain or not.

The rest of the paper is organized as follows: we will first remind on the method of Darboux, as a crucial tool in exploring the possible existence of the (local) equivalence of a DBI and a standard-type (preferably quadratic) Lagrangian. In the following section, we will define and explore the existence and form of an equivalent standard-type Lagrangian in a cosmological framework. The central part of the paper and most of the original results are in the section considering "A problem in classical and quantum mechanics". Special attention is devoted to the inverted harmonic oscillator with time-dependent mass and frequency, as an equivalent of the dynamics of a tachyon field in non-static space-time, i.e. Universe. The dynamics of the field in an exponentially expanding Universe is briefly discussed, and correlated to an inverted Caldirola-Kanai oscillator. The paper is concluded by some suggestions for further work, comments on the possibility for a $p$-adic and adelic generalization of the quantum-mechanical dynamics, as well as general comments.

[1]We will maintain an explicit distinction between the Lagrangian density and the Lagrangian itself.



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### 1.1 The method of Darboux

The problem of reconstructing an adequate Lagrangian, starting from equation(s) of motion, is known as the inverse problem of the calculus of variations. It is well known in the literature [4] that the procedure of constructing a Lagrangian is greatly simplified when the equation of motion is of the form:

$$\ddot{q} + A(q, \dot{q}, t) = 0\,, \tag{11}$$

where $A(q, \dot{q}, t)$ is an arbitrary function of its arguments. The method of Darboux [5, 6] states that a Lagrangian satisfying the equation of motion (11) is given by:

$$L = \int_{\dot{q}_0}^{\dot{q}} (\dot{q} - \omega)\Lambda(q, \omega, t)d\omega - \int_{q_0}^{q} A(\xi, \dot{q}_0, t)\Lambda(\xi, \dot{q}_0, t)d\xi + \frac{dF(q,t)}{dt}\,, \tag{12}$$

where the lower boundaries of integration $q_0, \dot{q}_0$ are arbitrary, as well as the function $F(q,t)$, and the Jacobi last multiplier $\Lambda$ [7] is given by:

$$\Lambda(q, \dot{q}, t) = \exp\left(\int \frac{\partial A(q, \dot{q}, t)}{\partial \dot{q}} dt\right)\,. \tag{13}$$

The proof that (12) is indeed a correct Lagrangian can be found in [8].

In this work, the method of Darboux will give us a "head start" on the road to obtaining a suitable standard-type Lagrangian. Taking $q \equiv T$, the "Darboux Lagrangian" for the tachyon field takes the form[2]:

$$L = \int_{\dot{T}_0}^{\dot{T}} (\dot{T} - \omega)\Lambda(T, \omega, t)d\omega - \int_{T_0}^{T} A(\xi, \dot{T}_0, t)\Lambda(\xi, \dot{T}_0, t)d\xi\,. \tag{14}$$

## 2. Equivalent standard-type Lagrangian in a cosmological framework

First, we note that the task of obtaining a desired type of Lagrangian was already performed in the case of a static Universe, by utilizing classical canonical transformations [9, 10]. In this simplified scenario, the authors were able to reduce the original tachyonic Lagrangian to a standard-type one, where the potential is time-independent[3]. We also note that the same results can be obtained by a straightforward application of the method of Darboux.

In the case of an expanding Universe, however, because of the explicit time dependence, the situation is more complex, both mathematically and conceptually. If we were to apply the method of Darboux directly to equation (7), we would run into a problem - the cubic term disables the method from producing meaningful results. Fortunately, equation (7) can be rewritten so that we can indeed obtain useful results.

In order to write equation (7) in a more useful form, we will need the equation of motion of the tachyon field (7), as well as the first Friedmann equation in a flat spacetime and the continuity equation, respectively:

$$H^2 = \frac{1}{3M_P^2}\rho\,, \tag{15}$$

$$\dot{\rho} = -3H(\rho + p)\,, \tag{16}$$

[2] We have conveniently set the function $F(q,t) = 0$.

[3] The abscence of time-dependence is natural here, since the original framework is a static one.



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where $\rho$ and $p$ are the density and pressure of an ideal (cosmological) fluid described by the tachion field.

Taking the time derivative of the first Friedmann equation (15) and using the continuity equation (16), we find:

$$\dot{H} = -\frac{1}{2M_P^2}(\rho + p)\,, \tag{17}$$

which can be combined with (15) to obtain:

$$\dot{H} + \frac{3}{2}H^2 = -\frac{1}{2M_P^2}p\,. \tag{18}$$

Dividing (18) with (15) then gives:

$$-\frac{p}{\rho} = 1 + \frac{2}{3}\frac{\dot{H}}{H^2}\,, \tag{19}$$

which, considering that:

$$p = -V(T)\sqrt{1-\dot{T}^2}, \tag{20}$$

$$\rho = \frac{V(T)}{\sqrt{1-\dot{T}^2}}\,, \tag{21}$$

finally implies that:

$$1 - \dot{T}^2 = 1 + \frac{2}{3}\frac{\dot{H}}{H^2}\,. \tag{22}$$

A useful result can be obtained by plugging (22) into the second term of equation (7) (expressing the third term in the same manner yields a less appealing final result). Equation (7) now takes the following form:

$$\ddot{T} + 3H(t)\left(1 + \frac{2}{3}\frac{\dot{H}(t)}{H^2(t)}\right)\dot{T} + \frac{V'(T)}{V(T)}(1-\dot{T}^2) = 0\,. \tag{23}$$

If we apply the method of Darboux to equation (23), we find:

$$A(T,\dot{T},t) = 3H(t)\left(1 + \frac{2}{3}\frac{\dot{H}(t)}{H^2(t)}\right)\dot{T} + \frac{V'(T)}{V(T)}(1-\dot{T}^2), \tag{24}$$

$$\Lambda(T,\dot{T},t) = \frac{a^3(t)H^2(t)}{V^2(T)}\,, \tag{25}$$

which give the following Lagrangian:

$$L = a^3(t)H^2(t)\left[\frac{1}{2}\left(\frac{\dot{T}}{V(T)}\right)^2 + \frac{1}{2}\frac{1}{V^2(T)}\right]. \tag{26}$$

Note: we placed the "first" lower boundary of integration in (14) at $\dot{T}_0 = 0$, as this is both allowed and, obviously, the most convinient choice. We have also discarded a constant term of $-\frac{1}{2V^2(T_0)}$, as it does not affect the equation(s) of motion in any way. Pay special consideration: we could not have chosen such a $T_0$ so that the second (indefinite) integral in (14) gives zero when evaluated at $T_0$, since tachyonic potentials have the following properties [11]:

$$V(0) = \lambda, \quad V'(T > 0) < 0, \quad V(|T| \to \infty) \to 0\,. \tag{27}$$



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Thus, only a $V(T_0) \longrightarrow \infty$ would produce a zero-valued term of $-\frac{1}{2V^2(T_0)}$. Such a value $T_0$, considering (27), is not guaranteed to exist.

Given the form of the Lagrangian (26), a useful result can be obtained by introducing a "new" field $\phi$ in the following manner:

$$\dot{\phi} = \frac{\dot{T}}{V(T)}, \tag{28}$$

which implies that:

$$\phi = \int^{T} \frac{dT}{V(T)}. \tag{29}$$

The lower boundary in (29) is unimportant, as the original defining relation (28) is invariant under shifts $\phi \longrightarrow \phi + C,\ C = const.$

The "new" Lagrangian, taking into account that $H = \frac{\dot{a}}{a}$, now takes the form:

$$L = a(t)\dot{a}^2(t)\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2V^2(T(\phi))}\right], \tag{30}$$

where $T(\phi)$ is to be calculated by inverting the function $\phi(T)$ given by (29), and the potential $U(\phi)$ is defined as:

$$U(\phi) = -\frac{1}{2V^2(T(\phi))}. \tag{31}$$

We have thus managed to obtain an equivalent standard-type Lagrangian, however it now describes a standard-type scalar field with a time-independent potential in a space-time which is no longer the same starting FLRW one - the time-dependent function appearing in (30) is $a(t)\dot{a}^2(t)$, in contrast to the FLRW case of $a^3(t)$.

In a cosmological framework, the Lagrangian (30) can be interpreted as describing a standard-type scalar field of with a time-independent potential (31) in a Bianchi type I spacetime, given by:

$$ds^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2, \tag{32}$$

with $a_1(t) = a(t)$ and $a_2(t) \equiv a_3(t) = \dot{a}(t)$. The Bianchi type I spacetime in which two of the scale factors are the same is labelled as a locally rotationally-symmetric Bianchi type I spacetime [12].

Another possible approach could be to introduce a "new" scale factor $A(t) = (a(t)\dot{a}^2(t))^{1/3}$ and again deal with a FLRW spacetime (albeit not identical to the starting one).

## 3. A problem in classical and quantum mechanics

In this section, we will step away from the field theory/cosmological framework, and consider the Lagrangian (30) in the context of classical mechanics. We will also take the concrete case of an exponential tachyonic potential:

$$V(T) = V_0 e^{-\omega T}, \quad V_0 = const,\ \omega = const, \tag{33}$$

as it bears some interesting features. Taking $\phi \equiv x$ and $m(t) = a(t)\dot{a}^2(t)$, the Lagrangian (30) takes the form:

$$L = \frac{1}{2}m(t)\dot{x}^2 + \frac{1}{2}m(t)\omega^2 x^2. \tag{34}$$



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We now deal with a purely dynamical problem - of an inverted harmonic oscillator with a time-dependent mass[4].

### 3.1 Inverted harmonic oscillator with time-dependent mass and frequency

Let us take a step "backwards" before considering the concrete case of (34), and first deal with an even more general case of an inverted harmonic oscillator with time-dependent mass and frequency (the problem of a (non-inverted) harmonic oscillator with time-dependent mass and frequency was previously studied in [13]), given by the Lagrangian:

$$L = \frac{1}{2}m(t)\dot{x}^2 + \frac{1}{2}m(t)\omega^2(t)x^2\,, \tag{35}$$

with the corresponding equation of motion being:

$$\ddot{x} + 2\frac{\dot{\eta}(t)}{\eta(t)}\dot{x} - \omega^2(t) = 0\,. \tag{36}$$

where $\eta(t) = \sqrt{m(t)}$. The analogus problem of treating a general harmonic oscillator with time-dependent mass and frequency was considered in [14]. We have followed the same approach here, which consists of considering a solution of the form:

$$x(t) = \frac{\alpha(t)}{\eta(t)}\left(Ae^{\gamma(t)} + Be^{-\gamma(t)}\right)\,. \tag{37}$$

The choice of (37) is motivated by the exact forms of solutions for some particular types of time dependencies of $m(t)$ and $\omega(t)$. By plugging (37) into (36), we obtain:

$$\left(\frac{\ddot{\alpha}}{\eta} - \frac{\alpha\ddot{\eta}}{\eta^2} + \frac{\alpha\dot{\gamma}^2}{\eta} - \frac{\alpha\omega^2}{\eta}\right)(Ae^{\gamma} + Be^{-\gamma}) + \left(\frac{\alpha\ddot{\gamma}}{\eta} + 2\frac{\dot{\alpha}\dot{\gamma}}{\eta}\right)(Ae^{\gamma} - Be^{-\gamma}) = 0\,. \tag{38}$$

Since constants $A$ and $B$ cannot vanish simultaneously, it follows that:

$$\ddot{\alpha} - \left(\omega^2(t) + \frac{\ddot{\eta}}{\eta} - \dot{\gamma}^2\right)\alpha = 0, \tag{39}$$

$$\ddot{\gamma} + 2\frac{\dot{\alpha}\dot{\gamma}}{\alpha} = 0\,, \tag{40}$$

and we also note a useful direct consequence of equation (40):

$$\alpha^2\dot{\gamma} = const\,, \tag{41}$$

since $\alpha \neq 0$, obviously.

By imposing the boundary conditions $x(t') = x'$ and $x(t'') = x''$ and plugging them into (37), we obtain the constants $A$ and $B$, which then give:

$$x(t) = \frac{\alpha}{\eta\sinh(\gamma'' - \gamma')}\left(\frac{\eta' x'}{\alpha'}\sinh(\gamma'' - \gamma) - \frac{\eta'' x''}{\alpha''}\sinh(\gamma' - \gamma)\right)\,. \tag{42}$$

[4] Take note that in the context of field theory, the multiplicative time-dependent factor cannot be interpreted as the mass of the field - the mass of the field is contained in the potential term. Here, in the classical mechanics context, the situation is different.



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Evaluating the classical action:

$$S_{cl} = \int_{t'}^{t''} L(\dot{x}, x, t)dt = \frac{1}{2}m''x''\dot{x}'' - \frac{1}{2}m'x'\dot{x}' \,, \tag{43}$$

using (42) gives:

$$\begin{aligned} S_{cl}(x'', t''; x', t') &= \frac{1}{2}m''x''^2 \left( \frac{\dot{\alpha}''}{\alpha''} - \frac{\dot{\eta}''''}{\eta} \right) - \frac{1}{2}m'x'^2 \left( \frac{\dot{\alpha}'}{\alpha'} - \frac{\dot{\eta}''}{\eta} \right) \coth(\gamma'' - \gamma') \\ &+ \frac{1}{2}(m''\dot{\gamma}''x''^2 + m'\dot{\gamma}'x'^2) - \frac{\eta'\eta''\sqrt{\dot{\gamma}'\dot{\gamma}''}x'x''}{\sinh(\gamma'' - \gamma')} \,, \end{aligned} \tag{44}$$

where we have used (41). Since the propagator for a quadratic Lagrangian can be calculated using the formula [15]:

$$K(x'', t''; x', t') = F(t'', t')e^{iS_{cl}(x'', t''; x', t')/\hbar} \,, \tag{45}$$

where:

$$F(t'', t') = \left[ \frac{i}{2\pi\hbar} \frac{\partial^2}{\partial x' \partial x''} S_{cl}(x'', t''; x', t') \right]^{\frac{1}{2}} \,, \tag{46}$$

our task to obtain the correct propagator is completed.

### 3.2 Inverted Caldirola-Kanai oscillator

Let us take a brief look at the case that is of significant importance in cosmological inflation. If one takes the scale factor to be exponentially expanding, we find that:

$$m(t) = me^{rt} \,, \tag{47}$$

where $r = const$. The inverted harmonic oscillator with constant frequency and exponentially increasing mass is well-known in the literature [16] as the inverted Caldirola-Kanai oscillator.

For the case of the inverted Caldirola-Kanai oscillator, we find that $\alpha = const$ and:

$$\gamma(t) = \Omega t, \quad \Omega^2 = \sqrt{\omega^2 + \frac{r^2}{4}} \,, \tag{48}$$

and the propagator (45) takes the following explicit form:

$$\begin{aligned} K(x'', t''; x', t') &= \left( \frac{m\Omega}{2\pi i\hbar \sinh \Omega t} \right)^{\frac{1}{2}} \times e^{\frac{1}{4}rt} \times \exp \left( \frac{im}{4\hbar}(x'^2 - e^{rt}x''^2) \right) \\ &\times \exp \left( \frac{im\Omega}{2\hbar \sinh \Omega t} \left[ (e^{rt}x''^2 + x'^2) \cosh \Omega t - 2x'x''e^{\frac{1}{2}rt} \right] \right) . \end{aligned} \tag{49}$$

Details of the calculation and a disagreement we have noted in comparison to the literature [16] will be given elsewhere.



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## 4. Conclusion

We extended the consideration presented in [9], where local equivalence was studied in (one-dimensional) Euclidean space, to FLRW space-time, suitable for application in cosmology and inflation theory. The existence of a (classical) canonical transformation utilized in [9] has been a solid base for current and future research of local equivalence between DBI and standard-type Lagrangians on a Riemann (FLRW) space-time.

A procedure of transforming a DBI Lagrangian to a standard-type one was presented. We applied the procedure to DBI Lagrangians with tachyonic potentials. Using the Friedman equations we obtained a dynamical (Klein-Gordon) equation for the tachyon field. The equivalent standard-type Lagrangian, that generates the same equation of motion, was obtained, using a technique related to the Darboux method. This is a promising point for the consideration of a quantum origin of inflation driven by a "tachyon" inflaton.

As a continuation and generalization of the previous results [9, 10, 17], in the zero-dimensional limit of DBI dynamics, we obtain a model of a time-dependent inverted harmonic oscillator. This allows us to consider the dynamics of tachyon fields and corresponding inflation driven by numerous different potentials, and calculate the time dependence of the field mass, frequency and other relevant physical quantities. Let us note that the most common - exponential tachyonic potential leads to the inverted Caldirola-Kanai oscillator with very interesting dynamics. We underline that quantum propagators were analytically calculated for numerous potentials, while a few of them were explicitly presented in the paper.

Regarding the possibility for further research, we specifically point out the generalization of the quantum mechanical approach and form of the propagators from the real number field to a $p$-adic and adelic case [9, 18, 19]. The relevance of non-Archimedean geometry and ultrametric spaces (which are intimately related to the $p$-adic numbers) for exploring the origin of inflation close to the Planck scale was discussed in details in the above-listed references.

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