

Theoretical and experimental challenges in quantum gravity phenomenology

J.M. Carmona^{a,b,*}

^aDepartamento de Física Teórica, Universidad de Zaragoza, Zaragoza 50009, Spain

E-mail: jcarmona@unizar.es

The experimental search of non-conventional effects predicted by bottom-up approaches and theoretical models of quantum gravity is a quite recent field of research. Generically, it requires to consider probes of very high energy and amplification mechanisms, conditions which are fulfilled by the propagation of the cosmic messengers. Advances in multi-messenger astronomy during the last decade has driven progress in the field, but a number of theoretical and experimental challenges still lie ahead. We will review them, making emphasis in two complementary lines of research: the study of time delays and the modification of interactions which appear in quantum-gravity motivated extensions of special relativity.

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^b Centro de Astropartículas y Física de Altas Energías (CAPA), Universidad de Zaragoza, Zaragoza 50009, Spain

^{*}Speaker

1. The development of quantum gravity phenomenology

After some initial ideas on the possibility to test quantum gravity by investigating departures from CPT invariance in systems of neutral mesons [1–5], the field of quantum gravity phenomenology started to develop in the last decade of the past century around proposals of observations in astrophysics that could provide some manifestation of physics at the Planck scale, such as Lorentz invariance violation (LIV). One of the first suggestions was the study of photon time delays [6–9], which could be produced in gamma-ray bursts and detected in first-generation Earth-based telescopes, like the HEGRA [10] or Whipple [11] telescopes, or satellite telescopes as EGRET [12].

At approximately the same time, the apparent violation of the GZK cutoff [13, 14] observed by the Akeno Giant Air Shower Array (AGASA) experiment [15] in Japan, was interpreted as a possible signature of the breaking of Lorentz invariance [16]; even if this result was incompatible with the observations of the HiRes (High Resolution Fly's Eye) in the USA [17], it was soon realized that the end of the spectrum of ultra high-energy cosmic rays (UHECR) constituted a sensitive probe of Planckian physics [18].

The second generation of UHECR (Auger) and high-energy gamma rays Earth-based (MAGIC, H.E.S.S., VERITAS) or space (Fermi) detectors, stimulated the notion of quantum gravity phenomenology, and the expectation that one could discover a footprint of quantum gravity rather soon.

Indeed, during the last decade a number of analyses on photon time delays have reached Planckian (or near-Planckian) sensitivity to linear (suppressed by the inverse of the Planck scale) modifications in the photon dispersion relation (see Sec. 5.1.4 in the review [19] and Table 1 of Ref. [20]). However, no evidence of a delay in the time of flight of photons from transient sources has been reported. The observation of GRB 090510 with the Large Area Telescope (LAT) on board the Fermi satellite is the most stringent constraint for the linear case in time-of-flight studies (constraints on the quantum gravity scale at the 10¹⁹ GeV level [21]), while observations of the active galactic nucleus Mrk 501 with the H.E.S.S. telescopes have given the most restrictive constraints for the quadratic case (at the 10¹⁰ GeV level [22]).

On the other hand, the study of the end of the spectrum of UHECR by the Pierre Auger Observatory in Argentina, the largest cosmic ray observatory in the world, completed in 2008 (but collecting data since 2004), has revealed a number of complexities. While a suppression of the spectrum at high energies is confirmed, its interpretation as a signature of the GZK cutoff is not so immediate; many unknowns about the distribution of astrophysical sources and about the energy spectrum and mass composition of cosmic rays influence the tests of violations of Lorentz invariance [23, 24].

The observation of high-energy astrophysical neutrinos in 2013 by the IceCube detector [25, 26] offered a new opportunity for quantum gravity phenomenology. In fact, the apparent ending of the spectrum at the PeV scale [27] could be explained as due to superluminal neutrinos [28]; such interpretation, however, weakens with the recent detection [29] of an event compatible with an energy at the Glashow resonance [30] of 6.3 PeV. The clarification of the situation will need to wait until future experiments extending the sensitivity of IceCube to the (10 PeV–1 EeV) energy range (see a list of proposals in the last section of Ref. [31]).

The detection of gravitational waves in 2015 [32] brought up completely new strategies in

multimessenger astronomy, and opened further possibilities for tests of fundamental physics, as with the strong constraints in the difference of velocities between photons and gravitational waves [33] that were derived from the multimessenger observation of a binary neutron star merger in 2017 [34].

Most of the quantum gravity phenomenology studies focus on modifications of space-time symmetries as a generic quantum gravity effect. In particular, some challenges may be identified in this approach: new strategies for time delays, the study of effects in the spectra of the cosmic messengers, and new dynamical calculations in the two big scenarios of physics beyond standard relativity that may incorporate a signature of quantum gravity: Lorentz invariance violation and its alternative, doubly special relativity (DSR). In the following we will present the main features of these two scenarios and will deepen into these theoretical and experimental challenges.

2. The LIV and DSR paradigms

DSR was formulated twenty years ago [9]. It does not exist at present a well-defined dynamic framework leading to DSR kinematics, and it presents some controversial issues, not only in its theoretical interpretation (soccer-ball and spectator problems), but also with respect to some of its predictions, such as time delays [35, 36]. However, it is a very attractive theoretical scenario (in contrast to LIV, it maintains a relativity principle, as in other revolutions in physics), it may solve fine-tuning problems of the LIV scenario, and may be the key of the interesting phenomenological scenario of an energy scale orders of magnitude below the Planck mass, without the constraints from time delay experiments.

2.1 Deformed relativistic kinematics

From a purely kinematical point of view, DSR is a *deformed relativistic kinematics*. Let us consider a 2-2 particle process,

$$A_1(p^{(1)}) + A_2(p^{(2)}) \to A_3(p^{(3)}) + A_4(p^{(4)}).$$
 (1)

A kinematical analysis of the previous process in special relativity (SR) involves the conservation law

$$p_{\mu}^{(1)} + p_{\mu}^{(2)} = p_{\mu}^{(3)} + p_{\mu}^{(4)} \tag{2}$$

and the dispersion relation for each particle,

$$p^{(i)2} = m_i^2, (3)$$

which are both invariant under linear Lorentz transformations

$$p_{\mu}^{(i)'} = L_{\mu}^{\nu} p_{\nu}^{(i)}. \tag{4}$$

These kinematical ingredients can be *deformed* by the introduction of a high-energy scale Λ , in the sense that both the (Λ dependent) modified composition law (MCL) of momenta \oplus , which defines a modified conservation law

$$\left[p^{(1)} \oplus p^{(2)}\right]_{\mu} = \left[p^{(3)} \oplus p^{(4)}\right]_{\mu},\tag{5}$$

and the modified dispersion relation (MDR)

$$C(p^{(i)}, \Lambda) = m_i^2, \tag{6}$$

recover their form of SR in the $\Lambda \to \infty$ limit.

DSR contains a relativity principle; that is, the deformed kinematics is invariant under Λ -deformed Lorentz transformations acting on a pair of momentum variables $(k, l) \to (k', l')$,

$$\left[p^{(1)'} \oplus p^{(2)'}\right]_{u} = \left[p^{(3)'} \oplus p^{(4)'}\right]_{u}, \qquad C(p^{(i)'}, \Lambda) = m_{i}^{2}. \tag{7}$$

Let us consider, as an example, a first-order DSR [37]; that is, we assume that the MDR and the MCL can be expanded in powers of momenta and the inverse of the ultraviolet scale Λ , and all the energies are much smaller than this scale, so that the dominant effect of the corrections to the kinematics of special relativity are on the first order terms in the Λ^{-1} expansion,

$$C(p) = p_0^2 - \vec{p}^2 + \frac{\alpha_1}{\Lambda} p_0^3 + \frac{\alpha_2}{\Lambda} p_0 \vec{p}^2 = m^2$$
 (8)

for the MDR, and

$$[p \oplus q]_0 = p_0 + q_0 + \frac{\beta_1}{\Lambda} p_0 q_0 + \frac{\beta_2}{\Lambda} \vec{p} \cdot \vec{q}$$
 (9)

$$[p \oplus q]_i = p_i + q_i + \frac{\gamma_1}{\Lambda} p_0 q_i + \frac{\gamma_2}{\Lambda} p_i q_0 \tag{10}$$

for the MCL.

It can be seen that the relativity principle imposes a relationship between the coefficients of the MCL and the MDR [37],

$$\alpha_1 = -\beta_1, \qquad \alpha_2 = \gamma_1 + \gamma_2 - \beta_2, \qquad (11)$$

a generalization of the *golden rule* that was derived in Ref. [38]. Because of these relations, the kinematics of the relativistic invariance (DSR) and non-invariance (LIV) cases is very different, as we will explicitly see in the following example.

2.2 Example: pair production in a photon background

The universe has some degree of opacity to the propagation of high-energy photons, which may interact with several backgrounds of low-energy photons, such as the extragalactic background light (EBL), or the cosmic microwave background (CMB) [39],

$$\gamma(E) + \gamma_{\text{EBL/CMB}}(\varepsilon) \to e^- + e^+$$
 (12)

This process is only viable in SR for energies of the high-energy photon which are higher than the threshold value

$$E_{\rm th}^{\rm SR} = \frac{m_e^2}{\varepsilon} \,. \tag{13}$$

Beyond SR, the threshold equation gets Λ -dependent modified terms [40]

$$\frac{\gamma_1 + \gamma_2 - \beta_1 - \beta_2 - \alpha_1 - \alpha_2}{8\Lambda} E_{th}^3 + O\left(\frac{E^2 \varepsilon}{\Lambda}, \frac{E m_e^2}{\Lambda}\right) + E_{th} \varepsilon - m_e^2 = 0.$$
 (14)

In the LIV case, where typically one does not consider a modification of the linear composition laws ($\gamma_i = \beta_i = 0$), the threshold energy is modified to

$$E_{\rm th}^{\rm LIV} \approx \frac{m_e^2}{\varepsilon} \left[1 + \frac{(m_e^2)^2}{\varepsilon^3} \frac{\alpha_1 + \alpha_2}{8\Lambda} \right] \doteq \frac{m_e^2}{\varepsilon} \left[1 + \frac{m_e^2}{\varepsilon^2} \frac{m_e^2}{\varepsilon \Lambda_{\rm eff}} \right], \tag{15}$$

where we have introduced Λ_{eff} to reabsorb some constants. In the DSR case, however, the golden rules (11) imply that

$$\gamma_1 + \gamma_2 - \beta_1 - \beta_2 - \alpha_1 - \alpha_2 = 0, \tag{16}$$

so that the leading correction in Eq. (14) is zero and one needs to compute the subleading terms, obtaining [40]

$$E_{\rm th}^{\rm DSR} \approx \frac{m_e^2}{\varepsilon} \left[1 + \frac{\beta_1 + \beta_2 + 3\gamma_2 - \gamma_1}{4\Lambda} \frac{m_e^2}{\varepsilon} \right] = \frac{m_e^2}{\varepsilon} \left[1 + \frac{m_e^2}{\varepsilon \Lambda_{\rm eff}'} \right]. \tag{17}$$

Therefore, the DSR correction is of order E/Λ , which means that it is invisible if $E \ll \Lambda$, while LIV has an enhancement over the previous correction by a factor $(m_e/\varepsilon)^2$. As an example, for a value of ε which is typical for the CMB, and considering a 10% correction, the bound on Λ that one extracts in the LIV case is of the order of the Planck mass, while in the DSR case is of the order of the PeV. This is a generic feature: DSR effects in particle reactions are softer than LIV effects.

2.3 Relative locality

SR has the property of absolute locality: if one observer sees a local event (meeting of worldlines at a single point), this is true for any other observer. This is a reflection of translational invariance, which is a consequence of conservation of the total momentum of a system of particles. In SR, where the total momentum is a sum of momenta, $\mathcal{P} = \sum p_i$, translations are constant displacements $x_i \to x_i + a \ \forall i$. In a deformed relativistic kinematics, however, the total momentum is a nonlinear composition of the momenta, $\mathcal{P} = \bigoplus p_i$, and translations are momentum-dependent displacements, $x_i \to x_i + f(\{p\})$. As a consequence, locality becomes a relative property: local interactions are perceived so by close observers, but non-local for observers which are far away. This is the property known as *relative locality*.

The relative locality of an interaction can be described in a classical model of N incoming and N outgoing worldlines through a variational principle from the action [41]

$$S_{\text{total}} = S_{\text{free}}^{\text{in}} + S_{\text{free}}^{\text{out}} + S_{\text{int}},$$

$$S_{\text{free}}^{\text{in}} = \sum_{J=1}^{N} \int_{-\infty}^{0} d\tau \left(x_J^{\mu} \dot{p}_{\mu}^{J} + \mathcal{N}_J \left(C(p^J) - m_J^2 \right) \right),$$

$$S_{\text{free}}^{\text{out}} = \sum_{J=N+1}^{2N} \int_{0}^{\infty} d\tau \left(x_J^{\mu} \dot{p}_{\mu}^{J} + \mathcal{N}_J \left(C(p^J) - m_J^2 \right) \right),$$

$$S_{\text{int}} = \left(\bigoplus_{N+1 \le J \le 2N} p_{\nu}^{J}(0) - \bigoplus_{1 \le J \le N} p_{\nu}^{J}(0) \right) z^{\nu},$$
(18)

where the parameter τ along each worldline was chosen such that $\tau = 0$ corresponds to the interaction. Asking that $\delta S_{\text{total}} = 0$ for any δz^{μ} , δx_{J}^{μ} , δp_{μ}^{J} , one gets

$$x_J^{\mu}(0) = z^{\nu} \frac{\partial \mathcal{P}_{\nu}}{\partial p_{\mu}^J} \forall J, \tag{19}$$

where

$$\mathcal{P} = \bigoplus_{1 \le J \le N} p^J(0) = \bigoplus_{N+1 \le J \le 2N} p^J(0). \tag{20}$$

Therefore, the interaction is seen as local $(x_J^{\mu}(0) = 0 \,\forall J)$ only for the observer which establishes the origin of space-time coordinates at the interaction vertex (corresponding to $z^{\mu} = 0$). As we will see, this fact makes the computation of time delays in DSR theories more cumbersome than in the simpler LIV case.

3. Some challenges in quantum gravity phenomenology

3.1 Determination of time delays

As indicated in the Introduction, there are no at present firm evidences of delays in the time of flight of photons coming from transient sources. Many of the analyses, however, focus on single sources, and then do not exploit the redshift dependence of the phenomenon, and are more exposed to intrinsic source effects than in more complex analyses.

A combination of experiments as it is being promoted at present within the so-called LIV consortium [42] with Imaging Atmospheric Cerenkov Telescopes, or IACTs, the inclusion of propagation effects in the analyses, and new approaches to minimize the intrinsic effects at the sources, as in [43], are strategies that will allow to confirm or refute some controversial analyses [44, 45] performed on GRB photons detected at the Fermi Gamma-ray Burst Monitor.

Moreover, most of the analyses assume a LIV scenario. A better understanding of the predictions of DSR with respect to time delays is also a theoretical challenge. In fact, it is an open question whether one would expect time delays in the DSR framework. While there are several papers [46–49] affirming the existence of such an effect, some works point to the opposite [50–53]. The main difficulty in the analysis of this question is in the implementation of translations between different frames, which has a reflection on the relative locality property of DSR, as explained in Sec. 2.3.

As seen from the action (18), the emission and detection of a photon, described by the observers O and O', which are close to the source and the detector, respectively, is very complicated and depends on the four-momenta of all the particles involved in the interaction. It is then remarkable that the derivation of the time delay of a high-energy photon emitted by a source in a model describing its propagation from the source to the detector can be made consistent with the relative locality framework, as it was presented in Ref. [36]. In this model, the translation between observers O and O' is described using the most general first-order deformation of the Poincaré algebra acting on the canonical one-particle phase space (x, t, Π, Ω)

$$E = \Omega + \frac{a_1}{\Lambda} \Omega^2 + \frac{a_2}{\Lambda} \Pi^2, \quad P = \Pi + \frac{a_3}{\Lambda} \Omega \Pi,$$

$$N = x \Omega - t \Pi + \frac{a_4}{\Lambda} x \Omega^2 + \frac{a_5}{\Lambda} x \Pi^2 - \frac{a_6}{\Lambda} t \Omega \Pi.$$
(21)

The MDR for photons that is derived from the Casimir of the algebra is

$$\Omega = \Pi - \frac{(a_4 + a_5 - a_6)}{3\Lambda} \Pi^2. \tag{22}$$

The time delay δt (difference in arrival time of a high-energy photon with momentum Π with respect to a low-energy photon that were emitted simultaneously for the observer at the source) is obtained as

$$\delta t = L \left[\frac{2(a_4 + a_5 - a_6)}{3\Lambda} \Pi - \frac{2(a_1 + a_2 - a_3)}{\Lambda} \Pi \right]. \tag{23}$$

One can see that the result of the time delay itself is independent of the details of the emission and detection and depends only on the four-momentum of the high-energy photon. It is a sum of two contributions that can cancel, producing no observable time delays in specific setups, what makes certain DSR models parametrized by scales many orders of magnitude below the Planck mass compatible with the observational constraints coming from time delays. It can be shown that the previous expression for the time delay is invariant under the change of energy-momentum variables, and that the model is compatible with the relative locality description given by the action (18) [36].

3.2 Spectrum modifications

The study of modifications to standard interactions, such as those related to high-energy cosmic rays, has been usually made at a kinematic level, trying, for example, to look for the generation, suppression or modification of thresholds of reactions, such as the one associated with the GZK cutoff. This is of course a first step, but a more complete analysis of the expected effects on the spectra of the cosmic messengers will be necessary to reveal quantum gravity footprints. In order to properly analyze the end of the UHECR spectrum, to estimate correctly the presence of associated gamma-ray or neutrinos, the influence of the cosmic photon backgrounds on the propagation of high-energy gamma rays, or to adequately carry out simulations of atmospheric showers in the presence of quantum gravity effects, it will be necessary a better understanding of the dynamics of the LIV and DSR cases.

In 2021, the Large High Altitude Air Shower Observatory (LHAASO) announced the detection of ultrahigh-energy photons up to 1.4 PeV from 12 γ -ray Galactic sources (so-called PeVatrons) [54]. This discovery represents the opening of a new window at the PeV scale, which can already provide important tests of new physics.

At PeV energies, the dominant background photons responsible for the opacity of high-energy gamma rays are those of the CMB [39]. As mentioned in Sec. 2.2, the sensitivity to LIV parameters from a modification of the threshold of the pair production process $\gamma + \gamma_b \rightarrow e^+ + e^-$ is of the order of the Planck scale. In particular, LHAASO results severely constrain the superluminal case (a dispersion relation for the photon in which its energy is higher than the modulus of its momentum) because of pair emission ($\gamma \rightarrow e^+e^-$) and photon splitting ($\gamma \rightarrow 3\gamma$) processes, which become allowed in the LIV scenario, while the subluminal case could better explain the detection of gamma rays of the highest energies [55].

At low redshifts, the probability of survival of a very high-energy photon of energy E can be expressed in terms of the mean free path $\lambda_{\gamma}(E)$ and the distance D to the source as

$$P_{\gamma \to \gamma}(E, D) \approx \exp(-D/\lambda_{\gamma}(E))$$
. (24)

In SR, the mean free path can be computed from the formula [56]

$$\frac{1}{\lambda_{\gamma}(E)} = \frac{1}{8E^2} \int_{4m_{\sigma}^2}^{\infty} ds \, s \, \sigma_{\gamma\gamma}(s) \int_{s/(4E)}^{\infty} d\varepsilon \frac{1}{\varepsilon^2} n_{\gamma}(\varepsilon), \tag{25}$$

where s is the square of the invariant mass of the two photons, of energies E and ε , $s = 2E\varepsilon(1-\cos\theta)$, where θ is the collision angle. For the interaction with CMB photons, one takes $n_{\gamma}(\varepsilon) = (\varepsilon/\pi)^2 (e^{\varepsilon/kT_0} - 1)^{-1}$, with $T_0 = 2.73$ K, and $\sigma_{\gamma\gamma}(s)$ is the well-known Breit-Wheeler cross section [39, 57, 58].

As we saw earlier, DSR is in fact sensitive to the PeV scale, and there are models that are phenomenologically compatible with such a low value of the high-energy scale Λ . Although, as mentioned before, we have not yet a complete dynamical framework for DSR, a good approximation for the new cross section $\tilde{\sigma}$, exploiting the fact that the theory has to be relativistically invariant, turns out to be [56]

$$\tilde{\sigma}_{\gamma\gamma}(\tilde{s}, \tilde{s}/\Lambda^2) \approx \tilde{\sigma}_{\gamma\gamma}(\tilde{s}, 0) \doteq \tilde{\sigma}_{\gamma\gamma}(\tilde{s}) \approx \sigma_{\gamma\gamma}(\tilde{s}),$$
 (26)

where \tilde{s} is the modified invariant.

The dependence on the energy of the modification of the mean free path in the case of LIV differs from the case of a relativistic deformed kinematics. In fact, while a subluminal LIV produces necessarily a greater transparency of the Universe than in SR, the mean free path in DSR can be above or below the SR value for different energy ranges [56]. Then, in case future data would allow us to identify a modification in the transparency of the Universe expected in SR, the spectrum of very high-energy gamma rays could allow us to distinguish if the origin of the modification is a LIV or a deformed relativistic kinematics.

3.3 Dynamical calculations

In contrast with the case of DSR, effective field theory provides a well-defined dynamical framework for the LIV scenario, the so-called Standard Model Extension (SME) [59], for which many studies and tests exist, especially for LIV modifications coming from operators in the Lagrangian of dimension less or equal than $4 (d \le 4)$, such as for the case of the decay of superluminal neutrinos that was considered in the context of the initial claim on superluminal propagation by the OPERA experiment in 2011 [60–62]. However, there are limited results for decay widths and cross section computations in LIV scenarios of relevance for quantum gravity phenomenology, which involve d > 4 operators. In the case of the decays of superluminal neutrinos, with interest in the determination of effects in the spectrum of cosmic neutrinos, they were treated theoretically [63], and applied phenomenologically [28], with certain restrictions. An update of the theoretical computations have recently been given in Ref. [64], including for the first time the charged weak current and the neutrino splitting contributions to the decay width for LIV corrections which are suppressed by a high-energy scale.

Another important example for quantum gravity phenomenology is the proper calculation of the cross section that modifies the Breit-Wheeler result, affecting the opacity of the universe to gamma rays in the LIV scenario, which up to now has only been computed in some contexts and under specific assumptions and approximations [65]. A full program of similar calculations for other processes that are relevant in the production, propagation and detection of the cosmic messengers

will be necessary to better estimate the modifications that the LIV scenario introduces in the spectra of these messengers, whose determination at the highest energies will be dramatically improved in the coming years. Besides the LIV case, the formulation of a consistent framework for dynamical calculations in DSR is, of course, a top-priority theoretical challenge.

4. Conclusions

Quantum gravity phenomenology has shown to be more elusive and subtle than initially thought. The first twenty years of this field has allowed us to put into practice smart theoretical and experimental ideas, but the expected effects have turned out to be not as direct as they could have been foreseen and further efforts will be needed to discover quantum gravity footprints. This peculiarity is not exclusive to quantum gravity phenomenology; it is also shared by other fields of physics, such as supersymmetry (or, in general, new physics) searches in the LHC, direct dark matter detection, or axion hunts.

Fortunately, a new generation of experiments (IceCube Gen-2, the upgrades of the Pierre Auger observatory, CTA, LHAASO, and others) is going to produce new results, at higher energies and sensitivities, in the coming years, offering a bright future for very high-energy astroparticle physics in the short term from the experimental side, which will need to be accompanied by improvements in the theoretical models used to describe this physics. This represents an outstanding opportunity to develop new strategies and approaches able to lead to a maturity of the young field of quantum gravity phenomenology, and allow for the transition from an exploratory stage to a precision era, favored by the experimental advances in the detection of the cosmic messengers in their high-energy regime.

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