PROCEEDINGS OF SCIENCE

COR System Error Estimation

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We present error estimation for the COR system dedicated to the cosmic rays trajectories in magnetosphere tracing available at https://cor.crmodels.org and on GitHub https://github.com/COR-Cut-off-rigidity/Trajectories_IGRF_T04_C. The used numerical method is analyzed and model dependence on crucial parameters is shown. The base error criterium to evaluate/determine model precision is defined.

**** 27th European Cosmic Ray Symposium - ECRS ****
**** 25-29 July 2022 ****
**** Nijmegen, the Netherlands ****

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PoS(ECRS)043

1. Introduction

Cosmic ray trajectories have been the subject of research since the mid-1930s, starting with the work of Størmer [1] and [2], where the first attempts to integrate equations of particle motion were made. The trajectory calculation process requires solving equations that have no closed-form solution. Therefore the only viable way to solve these equations is to use the numerical "brute-force" approach, as stated in [3]. This, in the context of evaluating cut-off rigidities of the cosmic ray trajectories in Earth's magnetosphere, has been computationally quite challenging in the past. However nowadays, as the computational power is increasing, we can calculate cut-off rigidities of cosmic ray trajectories more effectively.

Over the years a few tools were developed for evaluating cut-off rigidities of cosmic ray trajectories in the Earth's magnetosphere. For example tool "Magnetocosmic" described in [4], which is not maintained anymore or tool "Izmiran" described in [5]. There are however little to no mentions of error estimations of these models.

This article describes the COR model and presents its error estimation of cut-off rigidity evaluation as the function of the rigidity step size. In section 2 we briefly present the COR system, in section 3 the COR model as itself is described, and at last, section 4 is focused on error estimation of the COR model.

2. The COR System

The COR (Cut-Off Rigidity) tool/system provides models for simulating cosmic ray trajectories in the Earth's magnetosphere. It was built as a replacement for the obsolete solution MAGNETOCOSMIC described in [4]. COR offers a web service to run the calculations as well as tools and an environment for preliminary analysis and visualizations. System COR and its features were described in article [6] and in [7]. COR system components and models are open-source, available on GitHub: `https://github.com/COR-Cut-off-rigidity`. The web interface of the system is publicly available at `https://cor.crmodels.org`.

3. The COR Model

The COR model is a standalone software that evaluates cosmic ray trajectories in Earth's magnetosphere. It produces calculated values of cut-off rigidities for cosmic ray trajectories, which describe a spectrum of allowed and forbidden rigidities. It is a command-line application, which is used by the COR system internally to run the simulations.

The COR model algorithm for trajectory tracing was validated in several articles in previous decades [8–14]. The program calculates the geomagnetic field vector using the Tsyganenko-Sitnov model, introduced in [15] and IGRF model [16]. It solves the Lorentz force equation by the Runge-Kutta scheme of 6th order on each step of the particle's trajectory. The trajectory is tracked backwards in the geomagnetic field, starting at the specified altitude up to the magnetopause.

COR model was originally written in Fortran language at IEP SAS[1] and was later rewritten into C language, together with both T05 and IGRF models. This was mainly done because of performance

[1] `https://websrv.saske.sk/uef/en/`

and maintenance reasons. The new version of the model supports parallel execution so that a single particle trajectory calculation can be run on multiple CPU cores. It is also measurably faster on single-threaded execution, thanks to implemented code optimizations. Results produced by the new implementation of the model were compared against results from the original model to ensure its correctness. The source code is available on https://github.com/COR-Cut-off-rigidity/Trajectories_IGRF_T04_C.

4. COR Error Estimation

To evaluate calculated cut-off rigidity values, it is necessary to calculate several particle trajectories each with a different rigidity value. The rigidity difference between 2 trajectory calculations is called rigidity step size. This value is part of the input configuration for the simulation and it has a significant impact on the simulation from the computation time point of view. The smaller the step, the more trajectories have to be calculated.

Choosing smaller rigidity step sizes should cause higher accuracy of the calculated cut-off rigidities. This was our logical assumption since with a smaller rigidity step size used in the iteration over the range of rigidity values, it is possible to more precisely detect the first allowed particle trajectory - the cut-off rigidity.

We have developed a method that allows us to estimate results errors in connection with the rigidity step size used in calculations. Our goal was to quantify accuracy changes of cut-off rigidities with different rigidity step sizes used in the simulation. This required running several calculations for the same location, each with a different rigidity step size.

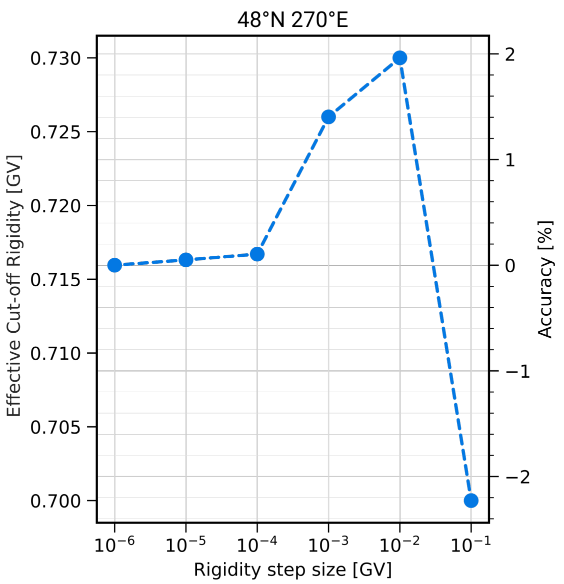

Figure 1: Example of effective cut-off rigidity dependency on the size of rigidity step size for location 48°N 270°E.

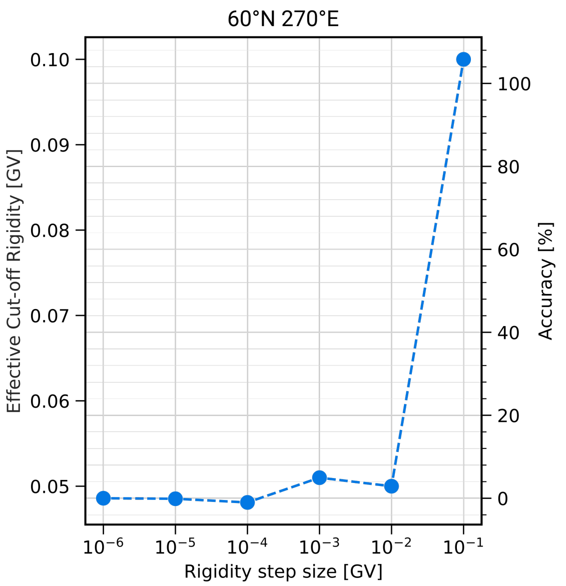

Figure 2: Example of effective cut-off rigidity dependency on the size of rigidity step size for location 60°N 270°E.

Figures 1 and 2 show effective cut-off rigidity dependency on the rigidity step size. There were 6 cut-off rigidity calculations for the same location, each with a logarithmically scaled rigidity step size. In both figures, we can see the effective rigidity convergence with the decreasing rigidity step size. The accuracy shown in percentage on the right side of both figures is calculated from the results of the evaluation of effective cut-off rigidity with the smallest rigidity step size.

The next step was to find a maximum rigidity step size that will guarantee, that with further decrease of the step size, cut-off rigidity values do not change more than the selected threshold. We chose a threshold with 1% difference of cut-off rigidity evaluated with the smallest rigidity step size (10^{-6} GV).

To achieve an accuracy of at least 1%, we need to choose a rigidity step size of at least 10^{-4} GV in the case of figure 1. But in the case of figure 2, which was calculated for the higher latitude, we need rigidity step size at least 10^{-5} GV. Based on this it seemed like for the locations closer to the geomagnetic poles a smaller rigidity step size is needed to achieve the desired accuracy. This behavior was also present in lower and upper cut-off rigidity.

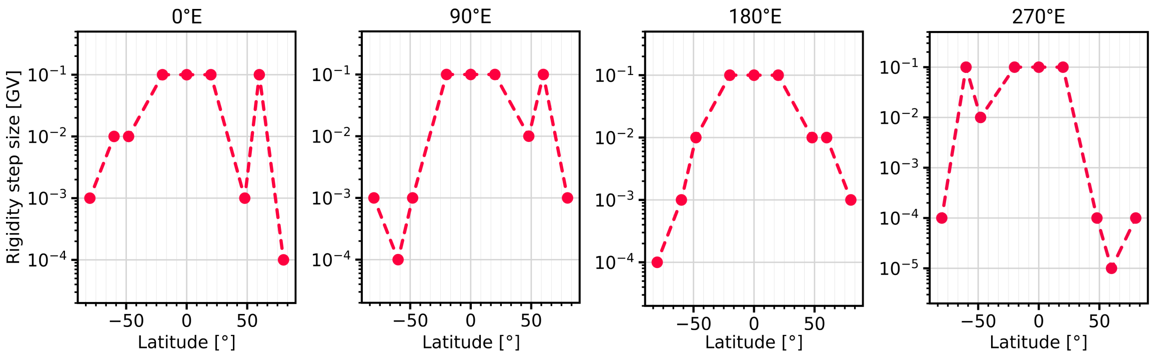

Figure 3: Dependencies of optimal rigidity step size (1% results accuracy) on latitude for selected longitudes.

Figure 3 shows rigidity step size dependencies on geographical latitude for selected geographical longitudes. In the process of running different calculations, we discovered, that for polar latitudes even a rigidity step size of 10^{-6} GV was not sufficient enough. However, running calculations with even smaller step sizes becomes very expensive, mainly from the computation time point of view.

Following figures: 4, 5 and 6 show heatmap of optimal (1% accuracy) rigidity step size for different geographical locations. In these visualizations, we can see signs of geomagnetic field structure similar to the shape of the geomagnetic equator. This indicates the rigidity step size dependency on the geomagnetic field.

For the locations closer to the equator, rigidity step sizes of 10^{-1} GV - 10^{-2} GV are sufficient enough, and for higher latitudes, the smaller step size is needed with values up to 10^{-5} GV for effective cut-off rigidity values and up to 10^{-6} GV for lower and upper cut-off rigidity values.

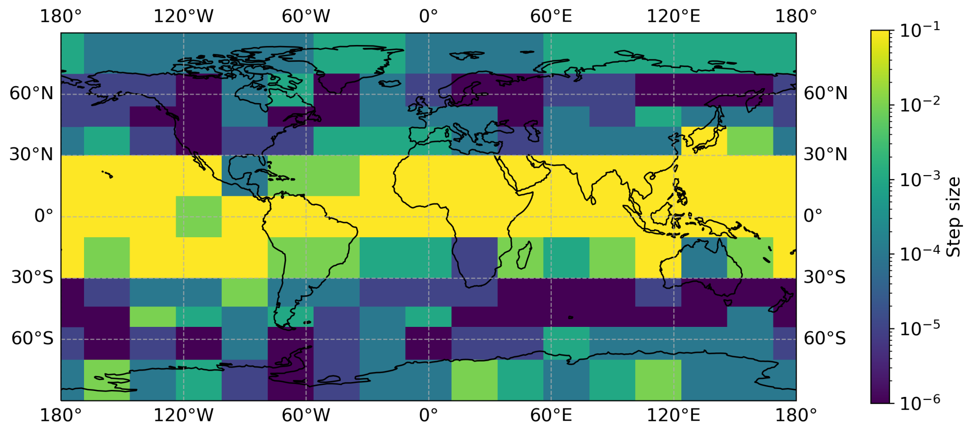

Figure 4: Rigidity step size dependency on the geomagnetic field for 1% accuracy of lower cut-off rigidity values.

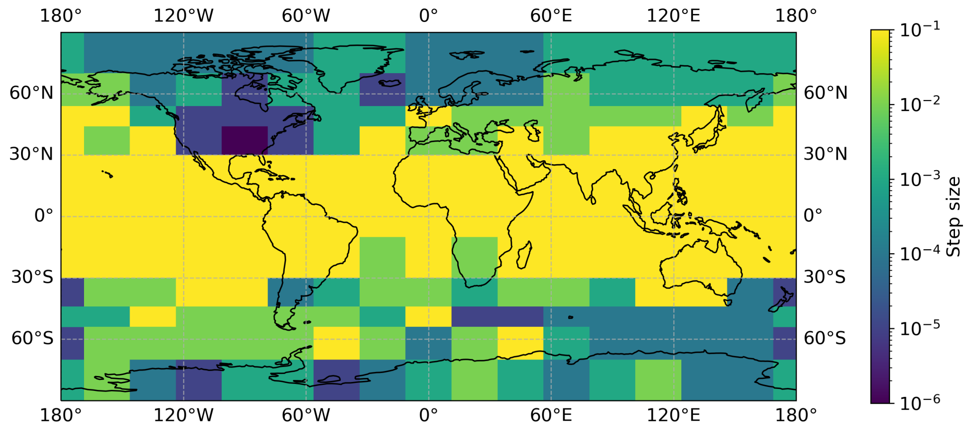

Figure 5: Rigidity step size dependency on the geomagnetic field for 1% accuracy of upper cut-off rigidity values.

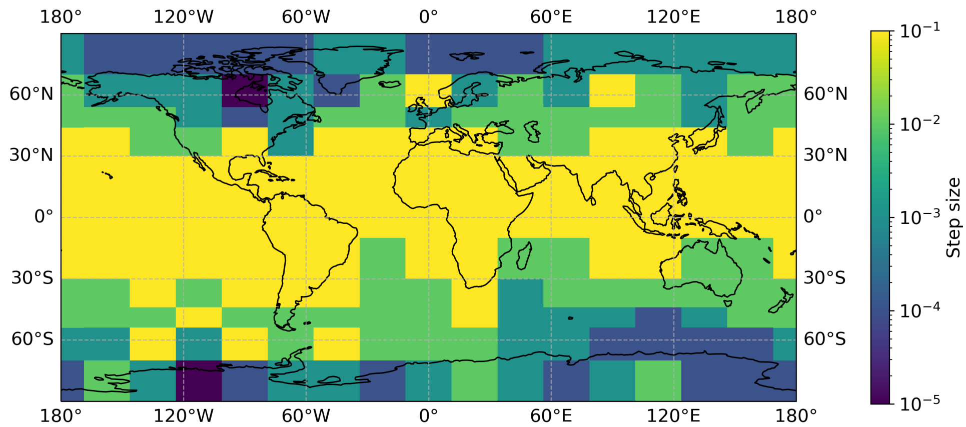

Figure 6: Rigidity step size dependency on geomagnetic field for 1% accuracy of effective cut-off rigidity values

Rig. Step Size	Area (Lower Cut-off Rigidity)	Area (Upper Cut-off Rigidity)	Area (Effective Cut-off Rigidity)
10^{-1} GV	35.25 %	58.22 %	56.78 %
10^{-2} GV	11.38 %	16.35 %	25.61 %
10^{-3} GV	9.81 %	10.48 %	11.61 %
10^{-4} GV	17.18 %	9.13 %	5.40 %
10^{-5} GV	12.80 %	5.21 %	0.60 %
10^{-6} GV	13.58 %	0.61 %	0.00 %

Table 1: Table shows for each rigidity step size percentage of Earth's surface for which given step size is sufficient enough to reach 1% error criterion (see text for details).

Table 1 shows the percentage of Earth's surface for which the given rigidity step size is sufficient enough. Evaluated effective cut-off rigidities with the biggest rigidity step size of 10^{-1} GV are precise enough (1% criterion) on 57% of Earth's surface. It is possible to precisely calculate effective cut-off rigidity values on more than 80% (56.78% + 25.61%) of the Earth's surface with the rigidity step sizes up to 10^{-2} GV, which is the most commonly used step size in the current calculations in the COR system. With the rigidity step sizes up to 10^{-3}, we can calculate effective cut-off rigidity values on more than 90% (56.78% + 25.61% + 11.61%) of the Earth's surface. With rigidity step size 10^{-4} GV we could evaluate effective cut-off rigidity precisely enough (1% criterion) on more than 99% of Earth's surface. Smaller rigidity step sizes, 10^{-5} GV - 10^{-6} GV are needed only on an area smaller than 0.7% of Earth's surface.

5. Conclusion

We introduced the method to estimate the error of cut-off rigidity evaluation as the function of the rigidity step size. We set a 1% error criterium in the value of effective cut-off rigidity and estimate the maximal rigidity step needed to reach this criterium for different geographical locations. While for low latitude positions step 0.1 GV is sufficient, for middle latitude and polar regions smaller step is needed, with values reaching $\sim 10^{-4}$ GV. With the most commonly used rigidity step size in current simulations (10^{-2} GV) it is possible to precisely calculate effective cut-off rigidity values on more than 80% of the Earth's surface.

6. Acknowledgment

This work was partially supported by the TUKE Space Forum ESA PECS project, Slovak Academy of Sciences grant MVTS JEM-EUSO as well as by VEGA grant agency project 2/0077/20.

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