PROCEEDINGS OF SCIENCE

Quantum correlations in neutrino mixing and oscillations

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The phenomenon of neutrino mixing and oscillations has a deep quantum nature, which can be conveniently described in a quantum information language. This allows for an efficient investigation of the quantum correlations involved in this phenomenon and their possible use as quantum resources. Furthemore, complete complementarity relations provide a full characterization of such correlations.

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In the simplest case, neutrino mixing between two generations is defined by $\left|\nu^f\right\rangle = U(\theta)\left|\nu^m\right\rangle$, where $\left|\nu^f\right\rangle = (\left|\nu_e\right\rangle, \left|\nu_\mu\right\rangle)$ are the states with definite flavor and $\left|\nu^m\right\rangle = (\left|\nu_1\right\rangle, \left|\nu_2\right\rangle)$ those with definite mass. $U(\theta)$ is a unitary matrix. Time evolution of flavor states is given by $\left|\nu^f(t)\right\rangle = \tilde{U}(t)\left|\nu^f\right\rangle$, with $\tilde{U}(t) = U(\theta)U_0(t)U(\theta)^{-1}$, where $\left|\nu^f\right\rangle$ is the flavor state at $t = 0$, $U_0(t) = diag(e^{-iE_1 t}, e^{-iE_2 t})$ and $\tilde{U}(t=0) = \mathbf{1}$.

In a quantum information language, it is possible to identify the mass eigenstates as qubits [1]:

$$\left|\nu_1\right\rangle = |1\rangle_1 \otimes |0\rangle_2 = |10\rangle_{12}\,, \qquad \left|\nu_2\right\rangle = |0\rangle_1 \otimes |1\rangle_2 = |01\rangle_{12}\,. \tag{1}$$

The flavor neutrino states are then entangled states of the mass eigenstates. Such a *static entanglement* can be calculated [1] and it depends only on the mixing angle. It is worth to be remarked that the tensor product structure appearing in Eq.(1) is mandatory. Indeed, neutrino mass eigenstates are the single-particle excitations of the corresponding (quantum) fields which have different masses and thus are associated to different (unitarily inequivalent) Hilbert spaces (see for example Ref.[2]).

Since entanglement is a concept which depends on the chosen observables, one can also choose flavor eigenstates as qubits [1]:

$$\left|\nu_e\right\rangle = |1\rangle_e \otimes |0\rangle_\mu = |10\rangle_{e\mu}\,, \qquad \left|\nu_\mu\right\rangle = |0\rangle_e \otimes |1\rangle_\mu = |01\rangle_{e\mu} \tag{2}$$

In this case, entanglement arise as a consequence of time evolution (*dynamical entanglement*) and is naturally related to flavor oscillations. At time t, the probability associated with the transition $\nu_\alpha \rightarrow \nu_\beta$ is $P_{\nu_\alpha\rightarrow\nu_\beta} = |\left\langle \nu_\beta \middle| \nu_\alpha(t)\right\rangle|^2 = |\tilde{U}_{\alpha\beta}(t)|^2$. The entanglement contained in a state $\left|\nu_\alpha(t)\right\rangle$ is quantified by the linear entropy:

$$S_{L\alpha}^{(e,\mu)} = 4|\tilde{U}_{\alpha e}(t)|^2|\tilde{U}_{\alpha\mu}(t)|^2. \tag{3}$$

Recently, entanglement associated to neutrino oscillations has attracted much attention, in view of the possible applications in quantum information protocols [3]. From such studies, a rich variety of quantum correlations has emerged, calling for a unified approach.

While entanglement encompasses any possible form of correlation in pure bipartite states, for mixed states the situation is more delicate. In such a case, several non-classical correlations have been identified. In order of decreasing strength, these can be classified as: non-local advantage of quantum coherence (NAQC), Bell non-locality, steering, entanglement and quantum discord. Mixed states in neutrino oscillations arise naturally in the wave-packet approach, which provides a more realistic description of this phenomenon [4] with respect to the plane-wave approximation.

A unified approach for quantum correlations in neutrino mixing and oscillations has been recently provided [5] by exploiting the complete complementarity relations (CCR). For a bipartite pure state described by a density matrix ρ_{AB} one has [6]:

$$C_{re}(\rho_A) + P_{vn}(\rho_A) + S_{vn}(\rho_A) = \log_2 d_A, \tag{4}$$

where ρ_A is the reduced density matrix of subsystem A, d_A is the dimension of the subsystem A, $C_{re}(\rho_A)$ is the relative entropy of coherence, a measure of visibility, $P_{vn}(\rho_A)$ is a measure of predictability in terms of the von Neumann entropy and $S_{vn}(\rho_A)$ is the von Neumann entropy, a

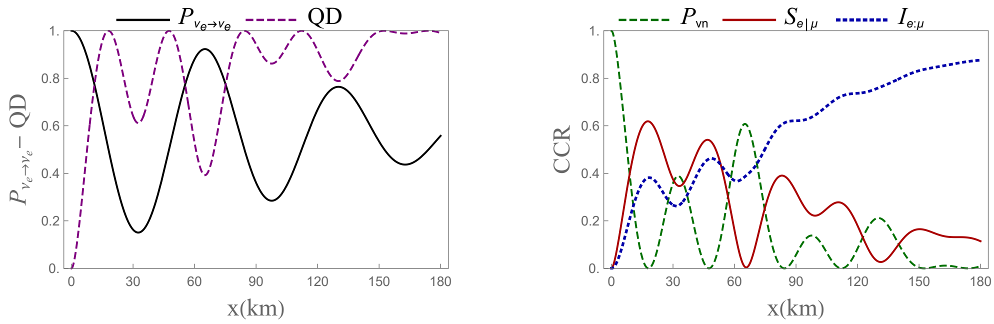

Figure 1: Left panel: survival probability and quantum discord. Right panel: predictability, conditional entropy and mutual information. Parameters are those of KamLand experiment.

measure of the entanglement between A and B. For a global bipartite mixed state, CCR are modified with the introduction of two non-local terms [6]:

$$C_{re}(\rho_A) + P_{vn}(\rho_A) + I_{A:B}(\rho_{AB}) + S_{A|B}(\rho_{AB}) = \log_2 d_A, \tag{5}$$

where $I_{A:B}(\rho_{AB})$ is the mutual information between A and B and $S_{A|B}(\rho_{AB})$ is a quantity that measures the ignorance about the whole system that one has by looking only at subsystem A.

In the wave packet approach to neutrino oscillations, the density matrix $\rho^{\alpha}_{e\mu}(x)$ represents a mixed state[1]. We find [5] that in Eq.(5) the local coherence term vanishes and the sum of the two non-local terms coincides with the quantum discord. In Fig.1, the quantities of Eq.(5) are plotted for the case of an initial electron neutrino with parameters as in the KamLand experiment. CCR guarantee that all correlations among electronic and muonic subsystems are taken into account. We observe the persistence of non-locality also at large distances from the source, in connection with the non-vanishing value of quantum discord. Finally, it is interesting to stress that both the quantum discord and the mutual information display a behaviour which is not simply reducible to the oscillation probability.

References

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[1] One starts with a pure state $\rho^{e}_{e\mu}(x,t) = |\nu_e(x,t)><\nu_e(x,t)|$ and obtains a mixed state after time integration.