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Theoretical models on the structure of the neutrino mixing matrix

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We briefly review symmetry approaches to the flavour problem in the lepton sector, focusing on the structure of the neutrino mixing matrix. We first discuss traditional (linearly realised) non-Abelian discrete flavour symmetries and then modular flavour symmetries. The importance of future neutrino oscillation experiments for testing and narrowing down the broad classes of flavour models based on these symmetries is emphasised.

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1. Introduction

The origin of flavour is one of the biggest puzzles in particle physics. In the search for possible explanations of the observed fermion mass and mixing patterns, many ideas have been put forward. Despite that there is arguably no baseline theory of flavour. Of all the ideas, flavour symmetries are perhaps the best tool we have at our disposal. When it comes to the lepton sector, flavour symmetries able to describe/predict large mixing angles are needed. Below we briefly discuss such symmetries and give examples of models based on them.

2. Non-Abelian discrete flavour symmetries

At high energies, the theory is assumed to be invariant under the following transformations:

$$\varphi(x) \to \rho(g)\,\varphi(x), \quad g \in G_f\,, \tag{1}$$

where G_f is a non-Abelian finite group describing flavour symmetry, ρ is a unitary representation of G_f, and φ denotes a multiplet of fermion fields. Very often it is assumed that three lepton families are unified at high energies, and $\varphi = (L_e, L_\mu, L_\tau)^T$ furnishes a 3-dimensional irreducible representation of G_f. Here L_α, $\alpha = e, \mu, \tau$, stand for the electroweak lepton doublets. Such a symmetry has to be broken at low energies, since electron, muon and tau lepton have different masses. While being completely broken at the Lagrangian level, G_f may leave an imprint on the structures of the charged lepton and neutrino mass matrices if the latter remain invariant under its Abelian subgroups G_e and G_ν, respectively:

$$\rho(g_e)^\dagger M_e M_e^\dagger \rho(g_e) = M_e M_e^\dagger\,, \quad g_e \in G_e \qquad \text{and} \qquad \rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu\,, \quad g_\nu \in G_\nu\,. \tag{2}$$

Since $\rho(g_e)$ and $M_e M_e^\dagger$ commute, they can be diagonalised by the same unitary matrix U_e. The same applies to $\rho(g_\nu)$ and $M_\nu^\dagger M_\nu$, which can be diagonalised by U_ν. Thus, fixing the *residual symmetries* G_e and G_ν, one can derive the form of the PMNS neutrino mixing matrix $U = U_e^\dagger U_\nu$. Such an approach has been extensively studied over the past decades, see [1–5] for reviews.

The minimal (in terms of the number of elements) non-Abelian finite groups, which possess a 3-dimensional irreducible representation, are permutation groups A_4, S_4, A_5 (and their double covers). If the residual symmetries G_e and G_ν are both larger than Z_2, these groups lead to highly symmetric mixing patterns. The examples being tri-bimaximal (TBM) mixing [6] from A_4 [7, 8] or S_4 [9] and golden ratio (GR) mixing [10] from A_5 [11]. Both patterns are characterised by maximal atmospheric mixing, $\sin^2\theta_{23} = 1/2$, and zero reactor mixing angle, $\sin^2\theta_{13} = 0$. The solar mixing parameter $\sin^2\theta_{12} = 1/3$ ($(\sqrt{5}r)^{-1} \approx 0.276$) for TBM (GR) mixing, with $r = (1+\sqrt{5})/2$ being the golden ratio. While both solar and atmospheric mixing angles are in agreement with the current global data [12–14] at approximately 2σ, the zero reactor angle is not experimentally viable. Still, these highly symmetric mixing patterns can be viewed as a leading-order approximation.

One of the ways to reconcile TBM mixing with the data is to assume that $G_\nu = Z_2$ (while keeping $G_e > Z_2$). This fixes U_ν up to a (complex) rotation in one of the three planes. The rotation in the 1-3 (2-3) plane leads to trimaximal mixing 2, TM2 [15] (TM1 [16]), for which the second (first) column of the TBM mixing matrix is preserved. The *a priori* free angle parameter of the

rotation matrix is fixed by the experimentally determined value of θ_{13}. In addition, certain relations between $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$, as well as between the cosine of the Dirac CP-violating phase δ, θ_{13} and θ_{23} are predicted. These relations are often referred to as *neutrino mixing sum rules*. Similarly, one can assume $G_e = Z_2$ and $G_\nu > Z_2$, or both residual symmetries to be (different) Z_2. A systematic classification of all possible cases and derivation of the corresponding sum rules have been performed in [17] and their phenomenological viability further assessed in [18].

Another possibility consists in breaking G_e completely and assuming a certain ansatz for the matrix U_e, which will correct $U_\nu = U_{\rm TBM}$ fixed by $G_\nu = Z_2 \times Z_2$. Oftentimes, the U_e contribution to the PMNS matrix is referred to as *charged lepton corrections*. For example, if U_e is given by a complex rotation matrix in the 1-2 plane, $\sin^2\theta_{23}$ is related to $\sin^2\theta_{13}$, whereas $\cos\delta$ to θ_{13} and θ_{12}. If instead U_e is given by a product of two complex rotation matrices (in the 2-3 plane followed by the 1-2 plane), one gets a sum rule by which $\cos\delta$ is expressed in terms of the three mixing angles [19, 20]. A systematic study of other forms of U_e has been performed in [21].

The most salient feature of the approach considered is that the predictions of the mixing sum rules can be tested at current and future neutrino oscillation experiments [18, 20, 22–25]. In this regard, future projects such as DUNE, T2HK, ESSnuSB and JUNO will play a crucial role in narrowing down the broad class of currently viable lepton flavour models, see *e.g.* [26–28].

However, the approach faces several challenges. At the moment, there exist many viable models based on different discrete groups. In concrete models, multiple flavon fields with elaborated potentials are usually needed to realise a desired symmetry breaking pattern. Higher-dimensional operators with multiple flavon insertions may spoil successful leading-order predictions. The bottom-up approach is mainly focused on the neutrino mixing parameters, which are "decoupled" from neutrino masses.

3. Modular flavour symmetries

In [29], modular invariance was proposed to play the role of flavour symmetry. Below we briefly summarise key features of this proposal. One considers a supersymmetric $\mathcal{N} = 1$ theory. The theory depends on a set of chiral supermultiplets φ comprising the dimensionless modulus τ ($\mathrm{Im}\,\tau > 0$) and other superfields φ_I (I numbers sectors of the theory, *e.g.* electroweak lepton doublets and singlets). In the case of rigid supersymmetry and switching off gauge interactions, the Lagrangian is fully specified by the Kähler potential $K(\varphi, \bar\varphi)$ and the superpotential $W(\varphi)$:

$$\mathcal{L} = \int d^2\theta d^2\bar\theta\, K(\varphi,\bar\varphi) + \int d^2\theta\, W(\varphi) + \int d^2\bar\theta\, \overline{W}(\bar\varphi)\,. \tag{3}$$

It is assumed to be invariant under transformations γ of the modular group $\overline{\Gamma} = PSL(2,Z)$:[1]

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}\,, \qquad \varphi_I \to (c\tau + d)^{-k_I}\rho_I(\tilde\gamma)\,\varphi_I\,, \tag{4}$$

where a, b, c, d are integers obeying $ad - bc = 1$. Such transformations are generated by the two elements of $\overline{\Gamma}$:

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \text{and} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{5}$$

[1]The construction can be extended to the homogeneous modular group $\Gamma = SL(2,Z)$, see *e.g.* [30].

The matrix $\rho_I(\tilde{\gamma})$ is a unitary representation of the group $\Gamma_N = \overline{\Gamma}/\overline{\Gamma}(N)$, obtained as a quotient between the group $\overline{\Gamma}$ and a principal congruence subgroup $\overline{\Gamma}(N)$, the positive integer N being the level of the representation. The level N is kept fixed in the construction, and $\tilde{\gamma}$ represents the equivalence class of γ in Γ_N. Here we assume the weights k_I to be integer. While the modular group is infinite, the quotient group is finite. Remarkably, for $N \leq 5$, the finite modular groups Γ_N are isomorphic to permutation groups: $\Gamma_2 \cong S_3$, $\Gamma_3 \cong A_4$, $\Gamma_4 \cong S_4$ and $\Gamma_5 \cong A_5$, see *e.g.* [31].

Expanding the superpotential in powers of the matter multiplets φ_I,

$$W(\varphi) = \sum_{I_1,\ldots,I_n} Y_{I_1\ldots I_n}(\tau)\, \varphi_{I_1} \ldots \varphi_{I_n}\,, \tag{6}$$

and requiring invariance of W under the transformations in Eq. (4), we find that the τ-dependent Yukawa couplings $Y_{I_1\ldots I_n}(\tau)$ should transform as

$$Y_{I_1\ldots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho_Y(\tilde{\gamma})\, Y_{I_1\ldots I_n}(\tau)\,, \tag{7}$$

where ρ_Y is a representation of Γ_N, and k_Y and ρ_Y are such that

$$k_Y = k_{I_1} + \ldots + k_{I_n} \qquad \text{and} \qquad \rho_Y \otimes \rho_{I_1} \otimes \cdots \otimes \rho_{I_n} \supset \mathbf{1}\,. \tag{8}$$

Thus, $Y_{I_1\ldots I_n}(\tau)$ are modular forms of weight k_Y and level N furnishing the representation ρ_Y of the finite modular group Γ_N. Such forms span a linear space of finite dimension, which depends on N and k_Y. For example, for $N = 3$ and $k_Y = 2$, this dimension is three, implying that there are three modular forms arranging themselves into a triplet of $\Gamma_3 \cong A_4$ [29]. Higher weight modular forms can be obtained from the weight-2 forms. For $N = 4$ ($N = 5$), there are 5 (11) weight-2 modular forms, which arrange themselves into a double and a triplet of $\Gamma_4 \cong S_4$ [32] (two triplets and a quintet of $\Gamma_5 \cong A_5$ [33]).

Modular invariance can be consistently combined with CP symmetry [34]. The minimal (with no flavons) viable modular and CP invariant model at level $N = 4$, in which neutrino masses are generated via the seesaw mechanism, contains 7 real parameters (for 12 observables in the lepton sector). Being perfectly compatible with the global oscillation data, it leads to normal ordering of neutrino masses, $m_1 = 0.012$ eV and $\delta = \pm 1.64\pi$ (the Majorana phases are also predicted) [34].

Compared to the conventional discrete symmetry approach, modular invariance has a number of advantages. Usually complicated scalar sectors of traditional models are replaced by the moduli space. Yukawa couplings are the known functions of the modulus τ. The familiar discrete flavour symmetries A_4, S_4, A_5 arise as quotients of the modular group. Both mass and mixing parameters are simultaneously constrained. In addition, charged fermion mass hierarchies can be generated by a small deviation of the modulus from the special points $\tau = i$, $e^{\frac{2\pi i}{3}}$, and $i\infty$, which preserve S, ST and T elements, respectively [35–37]. However, there are several challenges associated with the considered bottom-up approach. Unlike the superpotential, the Kähler potential is not constrained by the modular symmetry alone, which brings in extra free parameters and results in a reduced predictability of the set-up [29, 38]. In most of models constructed so far, τ is regarded as a spurion parameterising modular symmetry breaking, and the dynamical mechanism of its stabilisation is not known.[2] The level N, weights k_I and representations ρ_I are inputs, which are not fixed by the theory itself. Interestingly, under certain mild assumptions, modular-invariant models exhibit a universal (independent of N, k_I and the form of the Kähler potential) behaviour around $\tau = i$ [40].

[2] See [39] for a recent analysis of supergravity-motivated potentials for the modulus.

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