PROCEEDINGS OF SCIENCE

# Low-energy charmonium, bottomonium and tetraquark production cross sections from a statistical model

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Many proposed and on-going experiments require the preliminary knowledge of low-energy production cross-sections of different quarkonium and/or exotic states in hadronic e.g. in proton-antiproton collisions, to be able to make estimates to the expected yields, momentum distributions etc. These are necessary ingredients to simulate the detector systems, and to plan the experiments. Here, we propose a statistical based model to estimate the low-energy cross sections of some charmonium, bottomonium, and the X(3872) possible tetraquark state in proton-proton, pion-proton, and proton-antiproton collisions at a few GeV center-of-mass energies. The X(3872) cross-sections are calculated, using the assumption that it is a diquark-antidiquark bound state in the triplet-antitriplet representation, which gave a good match with the available high energy data in proton-proton collisions at 7 TeV. The estimated low-energy cross sections can be used as inputs e.g. in transport simulations of heavy-ion collisions, which can be used as event generators for detector studies, which is an important task during the construction of the detector systems. In each case the calculated cross sections are compared to the available measured data, giving a good match between the two.

*FAIR next generation scientists - 7th Edition Workshop (FAIRness2022)*
*23-27 May 2022*
*Paralia (Pieria, Greece)*

*Speaker



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PoS(FAIRness2022)004

## 1. Introduction

Hadronic cross sections are important ingredients of the non-equilibrium studies of low-energy heavy-ion collisions, which aim to better understand the structure of the strong interaction, and the strongly interacting matter, and it is one of the most important areas of research today. With relativistic transport codes it is possible to study such non-equilibrium processes with a wide range of energies, from a few hundreds of MeV's up to a few TeV center-of-mass energies per nucleon. In the lower energy regime up to a few GeV/nucleon center-of-mass energies, the necessary degrees of freedoms are the hadrons, therefore the most important inputs of the transport codes are the elementary hadron+hadron cross sections. In this energy regime a very hot and dense nuclear matter could be created, which could be used to extract information about the equation of state of the strongly interacting matter, or to study the vacuum structure of the strong interaction through the different condensates of quarks and gluons, which could be measured through observing the mass and width changes of the different vector mesons during the collision processes [1]. To be able to estimate processes, which are not well measured at such low energies, we have developed a statistical method, which is able to give reliable estimates to several exclusive, and inclusive cross sections with or without heavy quarks, and is also capable to be used to describe tetraquark production with the inclusion of diquarks. To the full description of the model, we refer the reader to [2–5], where we also have made many estimations to different processes.

## 2. Model description

The model starts from the assumption, that the cross section of a two body reaction can be factorized into two terms, which describes the initial dynamics, and the final state hadronization of the processes as:

$$\sigma(\sqrt{s}) = \left( \int \prod_{i=1}^{2} d^3 p_i R(\sqrt{s}, p_1, p_2) \right) \times \left( \int_{i=1}^{k} d^3 q_i w(\sqrt{s}, q_1, ..., q_k) \right), \tag{1}$$

where $p_1$, and $p_2$ are the momenta of the colliding particles, $\sqrt{s}$ is the center-of-mass energy of the collision, and $q_1, \ldots, q_k$ are the momenta of the outgoing particles. The $R(\sqrt{s}, p_1, p_2)$ function describes the initial, dynamical evolution of the collision, and is assumed to be described by the inelastic cross section of the two-body reaction as $\sigma_I^{A+B\to X}(\sqrt{s}) = \int \prod_{i=1}^{2} d^3 p_i R(\sqrt{s}, p_1, p_2)$, where $\sigma_I^{A+B\to X}(\sqrt{s})$ is the inelastic cross section of the reaction $A + B \to X$, and $X$ is the combination of any number and type of hadrons, which is allowed by the conservation laws. The integral in the second bracket in Eq. (1) then describes the hadronization probability to a k-body final state. Here, we assume that during the collision a short lived fireball is created with an invariant mass of $\sqrt{s}$, which could decay into smaller fireballs, with smaller invariant masses, which of course satisfies the energy conservation. The smaller fireballs could also decay towards to even smaller fireballs, and at the end of the decay chain, the formed fireballs will hadronize into specific hadronic final states. The fireball decay chain is described by a simple probabilistic approach, where each fireball is allowed to decay into smaller fireballs uniformly, if each has at least enough energy to have hadronized into two neutral pions. The probability of the actual type of hadronic final states can be specified by calculating the phase space integrals (where for non-stable resonances an extra

Breit-Wigner factor is also introduced), spin multiplicities, and symmetry factors, the density of states, and the so-called quark combinatorial factors. The most general form of the hadronization probability of $k$-fireballs can be expressed as:

$$W_{k,i_1..i_k}(E) = P_k^{fb}(E)\frac{1}{Z_k}\frac{1}{N_{i_1,..i_k}!}\int_{x_{min}}^{x_{max}}\prod_{a=1}^{k}\left[dx_a \times \frac{T_{i_a}(x_a)}{\sum_j T_j(x_a)}\delta\Big(\sum_{a=1}^{k}x_a - E\Big)\right], \tag{2}$$

where $P_k^{fb}(E)$ is the probability of the $k$-fireball scheme, $N_{i_1,..i_k}$ is the number of fireballs with the same hadrons in their final states, while $T(x) = C_Q(x)P_n^H(x)$ is a function containing the phase space integrals, Breit-Wigner -, and quark combinatorial factors as:

$$P_n^H(x) = P_n^d\frac{\Phi_n(x, m_1, .., m_n)}{\rho(x)(2\pi)^{3(n-1)}N_I!}\prod_{l=1}^{n}(2s_l + 1), \tag{3}$$

where $x$ is the invariant mass of the fireball, $P_n^d$ is the probability that one get $n$ hadrons from one fireball, $\Phi_n(E, m_1, .., m_n)$ is the n-body phase space integral, consisting the Breit-Wigner factors for resonant particles, $\rho(E)$ is the density of states obtained from the statistical Bootstrap, $N_I$ is a symmetry factor, and $s_l$ is the total spin of the $l'th$ particle. The remaining factor in Eq. (2) $Z_k$ is an overall normalization function, where the summation goes for all of the possible processes, which have the correct quantum numbers. One of the most important ingredient of the model is the $C_Q(x)$ quark combinatorial factor, which counts down all the possible quark/antiquark combinations, which could give a specific hadronic final state. To do this, the total number of quarks and antiquarks has to be estimated for which, simple phase space consideration has been used. To distinct between the different type of quarks the so-called quark creational probabilities ($P_i$) have been introduced, which serve as free parameters of the models, and have been fitted by comparing the model calculations to experimentally measured cross sections. To calculate exotics like tetraquarks, we also have introduced diquarks to the model. To do this only the quark combinatorial factors, specifically the quark creational probabilities had to be extended by assuming that the probability to create a diquark is $P_{ij} = P_iP_j$. Knowing the quark creational probabilities for the necessary quarks, the number distribution for quarks and diquarks can be expressed by a multinomial distribution as:

$$F(x, n_i, n_{ij}) = \frac{N(x)!}{\prod_i n_i! \prod_{ij} n_{ij}!}\prod_i P_i^{n_i}\prod_{ij} P_{ij}^{n_{ij}}, \tag{4}$$

where $N(x)$ is the total number of quarks and antiquarks, $n_i$ is the number of a specific type of quark (i=u,d,s,c,b), and $P_i$ is the quark creational probability of the quark with flavor $i$, with the constraint that the sum of the number of specific type of quarks has to give back the total number of quarks. The expected number of quarks corresponds to the maximum of the distribution function if $n_i \geq 1$ and is given by $\langle n_i(x)\rangle = P_iN(x)$. For the diquarks $n_{ij}$ means that one take $n_{ij}$ number of $i$ quarks and $j$ quarks separately, so the maximum number of diquarks is $N/2$ and the expected number is $\langle n_{ij}\rangle = P_iP_jN/2$.

## 3. Charmonium, Bottomonium, and Tetraquark cross sections

The statistical model described in the previous section can be readily used to describe inclusive charmonium and bottomonium production cross sections in proton, antiproton, and pion induced



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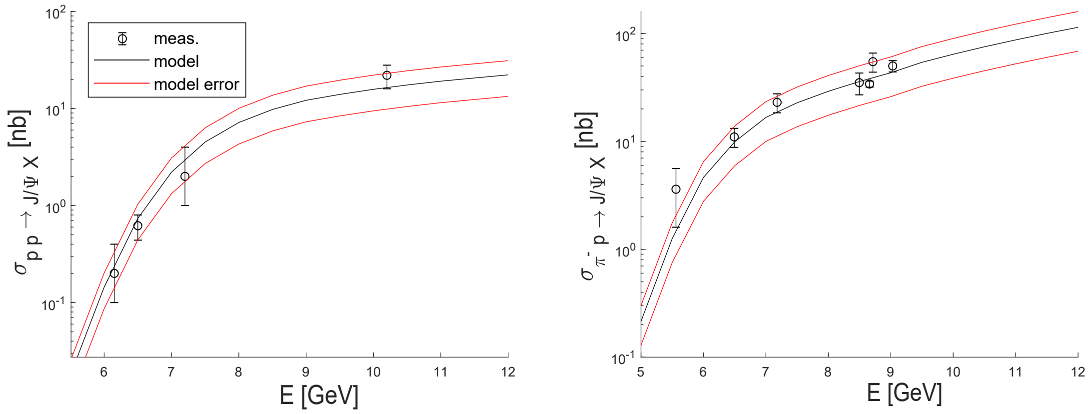


**Figure 1:** Model calculation for the $pp \to J/\Psi X$, and $\pi^- p \to J/\Psi X$ inclusive cross sections.

reactions, of which we show some examples in this section. In Fig. 1 the inclusive $J/\Psi$ production cross sections are shown in $pp$, and in $\pi^- p$ reactions, where the model errors, and the existing measurements are also shown. In Fig. 2 the results for bottomonium production is shown, where on the left side the inclusive $\pi^- p \to \Upsilon X$, while on the right side the $pp \to \Upsilon X$ process can be seen. In all cases a very good match has been obtained, considering the measurement and model errors. For the tetraquark production, we have calculated the inclusive production cross sections of the $X(3872)$ possible tetraquark state in $pp$, $\pi^- p$, and $\overline{p}p$ reactions as well. As the actual structure of this particle is still unknown it is interesting to compare the calculations to other models, where e.g. the $X(3872)$ is assumed to have a loosely bound molecule, or a closely bound diquark-antidiquark structure. Here, we assume that the particle is a diquark-antidiquark bound state, so it can be easily include into the statistical model. The diquarks could be in the triplet-antitriplet, and sextet-antisextet color representation, for which we had made calculations with a varying triplet-antitriplet probability ($P_3 \in [0, 1]$) and compared the results with a measured $\Psi(2S)/X(3872)$ cross section ratio at $\sqrt{s} = 7$ TeV. This can be followed on the left panel in Fig. 3, where the two dashed lines shows the upper and lower limits, coming from the uncertainties of the measurements. It can be seen, that the results are in the determined uncertainty range for almost all of the $P_3$ probabilities, so to make further calculations, we have assumed that the diquarks are in the triplet-antitriplet representation, with $P_3 = 1$. On the right side in Fig. 3 the low energy inclusive $X(3872)$ production cross section estimations can be seen in different reactions, giving the same low energy hierarchy as what was expected from the charmonium, and bottomonium results.

## 4. Conclusions

Low energy hadronic cross sections are important inputs of heavy ion transport simulations, where we could examine the strongly interacting matter in a controlled environment. These simulations are necessary to better understand the underlying physical processes, which arise in heavy ion colliding experiments. The statistical method described here is able to give reliable estimates to many low energy (few GeV center-of-mass energies) hadronic reactions, and could even be used as an event generator for heavy ion transport codes.

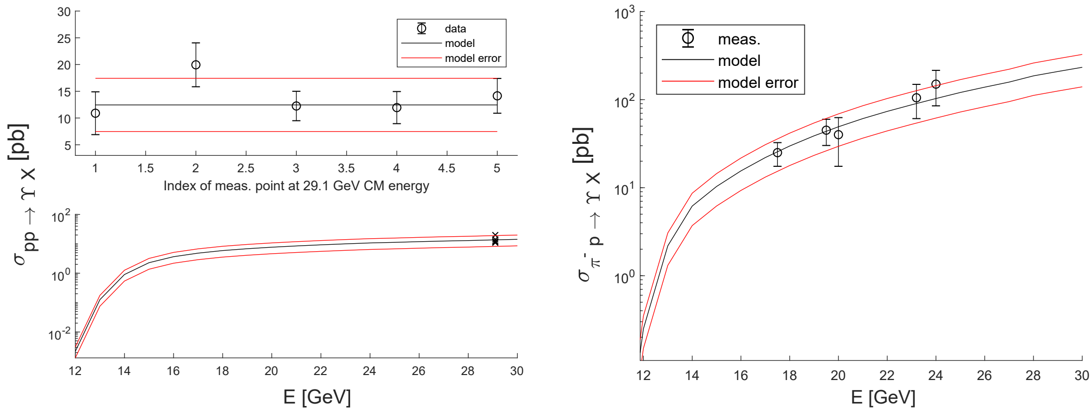


**Figure 2:** Bottomonium production cross sections in $pp$ and $\pi^- p$ collisions.

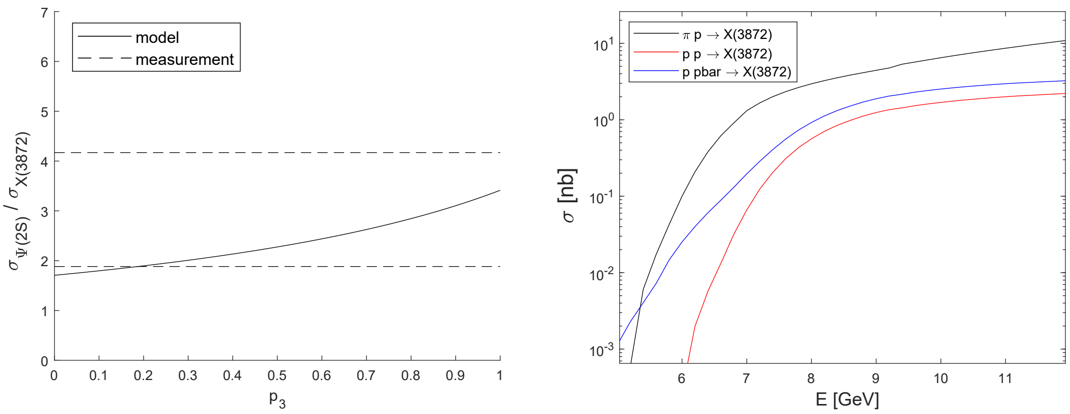


**Figure 3:** X(3872) production cross sections in $pp$, $\pi^- p$, and $\overline{p}p$ collisions.

## References

[1] Gy. Wolf, G. Balassa, P. Kovács, M. Zétényi, Su Houng Lee, *Mass shift of charmonium states in pbarA collision*, *Phys. Lett. B* **780** (2018) 25.

[2] G. Balassa, P. Kovács, Gy. Wolf, *A statistical method to estimate low-energy hadronic cross sections*, *Eur. Phys. J. A* **54** (2018) 25.

[3] G. Balassa, Gy. Wolf, *A statistical model to calculate inclusive hadronic cross sections*, *Eur. Phys. J. A* **56** (2020) 174.

[4] G. Balassa, Gy. Wolf, *Charmed and bottomed hadronic cross sections from a statistical model*, *Eur. Phys. J. A* **56** (2020) 237.

[5] G. Balassa, Gy. Wolf, *Production cross sections of tetraquark states in elementary hadronic collisions*, *Eur. Phys. J. A* **57** (2021) 246.