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Electroweak renormalization based on gauge-invariant vacuum expectation values

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We briefly review a recently proposed scheme for a gauge-invariant treatment of tadpole corrections in spontaneously broken gauge theories called *Gauge-Invariant Vacuum expectation value Scheme (GIVS)*. The tadpole scheme matters in higher-order predictions of observables if not all free parameters are fixed by renormalization conditions based on S-matrix elements, such as in $\overline{\mathrm{MS}}$ renormalization. In contrast to previously used tadpole schemes, the GIVS unifies the properties of gauge invariance and perturbative stability. The application of the GIVS to the Standard Model, for instance, leads to very moderate electroweak corrections in the conversion of on-shell-renormalized to $\overline{\mathrm{MS}}$-renormalized masses. Moreover, in models with extended Higgs sectors, the GIVS is less prone to perturbative instabilities in the $\overline{\mathrm{MS}}$ renormalization of Higgs mixing angles than observed for the traditional gauge-independent tadpole treatment. We illustrate this by considering the next-to-leading-order (electroweak and QCD) corrections to the decay processes $\mathrm{h/H} \to \mathrm{WW/ZZ} \to 4$ fermions of the CP-even neutral Higgs bosons h and H in a singlet Higgs extension of the Standard Model and in the Two-Higgs-Doublet Model.

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1. Introduction

Electroweak (EW) corrections in the Standard Model (SM) and its extensions are an important ingredient in precision calculations for present and future collider phenomenology. Renormalization, which is an important part of this task, in particular includes the definition of vacuum expectation value (vev) parameters, such as v in the SM, in higher orders—an issue that is connected to the treatment of tadpole loop contributions. In case, all model parameters are fixed by renormalization conditions based on S-matrix elements, i.e. by so-called *on-shell (OS)* conditions, higher-order predictions of observables do not depend on the tadpole scheme. However, this is different, for instance, in $\overline{\text{MS}}$ renormalization schemes, which are frequently employed for mass parameters and mixing angles in extended Higgs sectors.

Explicit tadpole contributions are most conveniently removed via tadpole counterterms which can be generated in two different ways in the Lagrangian: in the course of parameter renormalization [1, 2], or alternatively via Higgs field redefinitions [3]. The former, called *Parameter Renormalized Tadpole Scheme (PRTS)* in the following, typically leads to small corrections originating from tadpoles, but in general suffers from gauge dependences if $\overline{\text{MS}}$ renormalization conditions are used for mass parameters. The latter, usually called *Fleischer–Jegerlehner Tadpole Scheme (FJTS)*[1], is free from gauge dependences, but is prone to very large corrections in $\overline{\text{MS}}$ schemes, jeopardizing perturbative stability of $\overline{\text{MS}}$ predictions. More details on tadpole renormalization can, e.g., be found in Refs. [5–10].

In Refs. [5, 6] we have proposed a new scheme for tadpole renormalization, dubbed *Gauge-Invariant Vacuum expectation value Scheme (GIVS)*, which is a hybrid scheme of the two mentioned types, with the benefits of being gauge independent and perturbatively stable.[2] The GIVS is based on the gauge-invariance property of Higgs fields, and the corresponding parameters like v, in non-linear representations of Higgs multiplets. In this article we briefly summarize the salient features of the GIVS and its first applications within the SM [5], a Singlet Higgs Extension of the SM (SESM) [6], and to the Two-Higgs-Doublet Model (THDM) [6].

2. Non-linear representations of Higgs sectors

One of the basic ideas underlying the GIVS is the use of a field basis in which the Higgs fields developing non-vanishing vevs are gauge invariant. This implies that explicit tadpole contributions induced by loop diagrams are gauge independent. In such a field basis the would-be Goldstone-boson part of the scalar Lagrangian is necessarily parametrized in a non-linear fashion. In the following, we briefly sketch the structure of appropriate non-linear representations for the SM, the SESM, and the THDM. For the full details, we refer to Refs. [5, 6].

Standard Model

Non-linear field representations are most conveniently formulated via matrix fields. Denoting the usual, two-component SM Higgs doublet Φ and its charge conjugate Φ^{c}, we form the 2×2 matrix

[1]The FJTS [3] is equivalent to the β_t scheme of Ref. [4].

[2]Based on completely different considerations about symmetry breaking in R_ξ gauges, a similar idea for a hybrid scheme has been mentioned in Sect. D of Ref. [10].

$\mathbf{\Phi} = (\Phi^c, \Phi)$ and write it in some "polar representation"

$$\mathbf{\Phi} = \tfrac{1}{\sqrt{2}}(v+h)U(\boldsymbol{\zeta}), \qquad U(\boldsymbol{\zeta}) \equiv \exp\left(\frac{2\mathrm{i}\boldsymbol{\zeta}}{v}\right), \qquad \boldsymbol{\zeta} \equiv \tfrac{1}{2}\zeta_j\sigma_j, \tag{1}$$

in which h corresponds to the physical Higgs field, which is gauge invariant, ζ_j ($j = 1,2,3$) are real would-be Goldstone-boson fields and σ_j denotes the Pauli matrices. Since the matrix $U(\boldsymbol{\zeta})$ is unitary, the gauge-invariant square $\mathrm{tr}\big[\mathbf{\Phi}^\dagger\mathbf{\Phi}\big] = (v+h)^2$ does not involve would-be Goldestone-boson fields ζ_j. In the SM Higgs Lagrangian

$$\mathcal{L}_{\mathrm{H,SM}} = \tfrac{1}{2}\mathrm{tr}\left[(D_\mu\mathbf{\Phi})^\dagger(D^\mu\mathbf{\Phi})\right] - V_{\mathrm{SM}} \tag{2}$$

the first term is the kinetic part, which encodes the gauge interactions of the scalar fields in the covariant derivative D_μ. Owing to the non-linear representation of $\mathbf{\Phi}$, this part is non-polynomial in ζ_j and induces interaction vertices of arbitrarily many would-be Goldstone-boson fields, but this is only a minor complication, and the usual perturbative Feynman diagram calculus works as usual. Note that the SM Higgs potential

$$V_{\mathrm{SM}} = -\tfrac{1}{2}\mu_2^2\mathrm{tr}\big[\mathbf{\Phi}^\dagger\mathbf{\Phi}\big] + \tfrac{1}{16}\lambda_2\big(\mathrm{tr}\big[\mathbf{\Phi}^\dagger\mathbf{\Phi}\big]\big)^2 = -\tfrac{1}{2}\mu_2^2(v+h)^2 + \tfrac{1}{16}\lambda_2(v+h)^4, \tag{3}$$

with the free parameters μ_2^2 and λ_2, is free from would-be Goldstone-boson fields. With the of help the Nielsen identities [11], it is easy to show that the one-point vertex function Γ^h_{nl} of the gauge-invariant Higgs field h is gauge independent; a simple one-loop calculation confirms this.

Singlet extension of the SM (SESM)

The SESM extends the SM by a real singlet scalar field σ, leading to a second CP-even Higgs boson H. The SESM Higgs Lagrangian is given by

$$\mathcal{L}_{\mathrm{H,SESM}} = \tfrac{1}{2}(\partial\sigma)^2 + \tfrac{1}{2}\mathrm{tr}\left[(D_\mu\mathbf{\Phi})^\dagger(D^\mu\mathbf{\Phi})\right] - V_{\mathrm{SESM}}, \tag{4}$$

with a non-polynomial kinetic part similar as in the SM and the gauge-invariant Higgs potential

$$V_{\mathrm{SESM}} = -\tfrac{1}{2}\mu_2^2\mathrm{tr}\big[\mathbf{\Phi}^\dagger\mathbf{\Phi}\big] - \mu_1^2\sigma^2 + \tfrac{1}{16}\lambda_2\big(\mathrm{tr}\big[\mathbf{\Phi}^\dagger\mathbf{\Phi}\big]\big)^2 + \lambda_1\sigma^4 + \tfrac{1}{2}\lambda_{12}\mathrm{tr}\big[\mathbf{\Phi}^\dagger\mathbf{\Phi}\big]\,\sigma^2, \tag{5}$$

which again does not involve would-be Goldstone-boson fields ζ_j. The Higgs fields h_1 and h_2 corresponding to particle excitations in σ and $\mathbf{\Phi}$, respectively, are identified by

$$\sigma = v_1 + h_1, \qquad \mathbf{\Phi} = \tfrac{1}{\sqrt{2}}(v_2 + h_2)U(\boldsymbol{\zeta}), \tag{6}$$

with vev parameters v_1 and v_2, and $U(\boldsymbol{\zeta})$ denoting the same unitary would-be Goldstone-boson matrix as in the SM, where $v = v_2$. The fields (h_1, h_2) are rotated into a field basis of h and H corresponding to mass eigenstates,

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha)\begin{pmatrix} H \\ h \end{pmatrix}, \qquad R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \tag{7}$$

where α is a real-valued mixing angle which is determined by the parameters of the Higgs potential. In analogy to the SM case, the one-point functions Γ^h_{nl} and Γ^H_{nl} of the fields h and H are gauge independent in the non-linear representation.

Two-Higgs-Doublet Model (THDM)

The THDM comprises two complex Higgs doublets $\mathbf{\Phi}_n$ ($n = 1, 2$), which together contain eight real d.o.f.s. Denoting the vev parameters of $\mathbf{\Phi}_n$ by v_n, we parametrize the 2×2 matrix fields according to

$$\mathbf{\Phi}_n = \frac{1}{\sqrt{2}} U(\boldsymbol{\zeta}) \left[(v_n + h_n)\mathbb{1} + \mathrm{i}c_n\sigma_j\rho_j\right], \tag{8}$$

with the unitary would-be Goldstone-boson matrix $U(\boldsymbol{\zeta})$ as in the SM with

$$v \equiv \sqrt{v_1^2 + v_2^2}, \qquad c_1 = -\frac{v_2}{v} \equiv -\sin\beta, \qquad c_2 = \frac{v_1}{v} \equiv \cos\beta. \tag{9}$$

The gauge-invariant fields h_1 and h_2 are CP even (under appropriate assumptions on the Higgs potential) and are rotated into the fields h and H as in (7) corresponding to neutral CP-even mass eigenstates h and H. The three remaining fields ρ_j describe a neutral CP-odd Higgs boson A_0 via the gauge-invariant field ρ_3 and two charged Higgs bosons $H^\pm$ via the fields $\rho^\pm = (\rho_2 \pm \mathrm{i}\rho_1)/\sqrt{2}$. The proper normalization of these fields fixes the constants c_n as above. The two Higgs mixing angles α and β are frequently taken as basic input parameters of the THDM. The Higgs Lagrangian of the THDM is given by

$$\mathcal{L}_{\mathrm{H,THDM}} = \frac{1}{2}\mathrm{tr}\left[(D_\mu\mathbf{\Phi}_1)^\dagger(D^\mu\mathbf{\Phi}_1)\right] + \frac{1}{2}\mathrm{tr}\left[(D_\mu\mathbf{\Phi}_2)^\dagger(D^\mu\mathbf{\Phi}_2)\right] - V_{\mathrm{THDM}}, \tag{10}$$

in which the kinetic term is again non-polynomial in the would-be Goldstone-boson fields ζ_j but polynomial in the fields h_n and ρ_j corresponding to physical Higgs bosons. In the non-linear Higgs representation the gauge-invariant Higgs potential can be written as

$$\begin{aligned} V_{\mathrm{THDM}} = {} & \frac{1}{2}m_{11}^2\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_1\right] + \frac{1}{2}m_{22}^2\mathrm{tr}\left[\mathbf{\Phi}_2^\dagger\mathbf{\Phi}_2\right] - m_{12}^2\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_2\right] \\ & + \frac{1}{8}\lambda_1\left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_1\right]\right)^2 + \frac{1}{8}\lambda_2\left(\mathrm{tr}\left[\mathbf{\Phi}_2^\dagger\mathbf{\Phi}_2\right]\right)^2 + \frac{1}{4}\lambda_3\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_1\right]\mathrm{tr}\left[\mathbf{\Phi}_2^\dagger\mathbf{\Phi}_2\right] \\ & + \lambda_4\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_2\Omega_+\right]\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_2\Omega_-\right] + \frac{1}{2}\lambda_5\left[\left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_2\Omega_+\right]\right)^2 + \left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_2\Omega_-\right]\right)^2\right] \end{aligned} \tag{11}$$

with the two-dimensional projection operators $\Omega_\pm = \frac{1}{2}(1 \pm \sigma_3)$, which select the original Higgs doublet Φ_n or its charge conjugate Φ_n^{c} from the matrix field $\mathbf{\Phi}_n$. Obviously, the unitary Goldstone-boson matrix $U(\boldsymbol{\zeta})$ again drops out in V_{THDM}. To avoid flavour-changing neutral currents at tree level, we assume the $\mathbb{Z}_2$ symmetry $\Phi_1 \to -\Phi_1$ and $\Phi_2 \to \Phi_2$ that is only softly broken by the m_{12}^2 term in V_{THDM}. Moreover, all couplings in V_{THDM} are assumed to be real in order to conserve CP. Owing to the gauge invariance of the fields h and H the one-point functions Γ_{nl}^h and Γ_{nl}^H are again gauge independent in the non-linear representation.

3. The GIVS in the SM

Before formulating the GIVS as introduced in Ref. [5] for the SM, we briefly sketch the FJTS and PRTS for treating tadpoles in the course of renormalizing the theory. The full renormalization procedure of the SM can be found in Ref. [9]. Renormalization starts with the transformation that replaces the original *bare* quantities in terms of *renormalized* ones and renormalization constants. We mark bare parameters by subscripts “0” and bare fields by subscripts “B”.

In the following, we denote the physical Higgs-boson field of the usually adopted linear Higgs representation by $v + \eta(x)$. One-particle-irreducible Green function $\Gamma^{\cdots}$, so-called *vertex functions*, involve tadpole contributions if the splitting $v_0+\eta_\mathrm{B}(x)$ does not provide an expansion of the effective Higgs potential about its true minimum (see for instance App. C of Ref. [12]). Technically, it is desirable to eliminate such tadpole contributions by appropriate parameter and field definitions. Choosing v_0 such that $v_0^2 = 4\mu_{2,0}^2/\lambda_{2,0}$ at least to leading order (LO) avoids tadpole contributions at tree level. In higher orders, the explicit (unrenormalized) tadpole function Γ^η can be cancelled upon generating a tadpole counterterm $\delta t\,\eta$ in the counterterm Lagrangian $\delta\mathcal{L}$. This is achieved by a tadpole renormalization condition for the renormalized one-point function Γ_R^η (in momentum space) of the physical Higgs field,

$$\Gamma_\mathrm{R}^\eta = \Gamma^\eta + \delta t \overset{!}{=} 0 \quad \Rightarrow \quad \delta t = -\Gamma^\eta. \tag{12}$$

Note that Γ^η is a gauge-dependent quantity in contrast to its counterpart Γ_nl^h in the non-linear representation. The tadpole counterterm $\delta t\,\eta$ is generated in the Lagrangian by appropriately choosing v_0 and, if needed, by a further redefinition of the bare Higgs field η_B. Inserting the field decomposition $v_0 + \eta_\mathrm{B}(x)$ into the bare SM Lagrangian $\mathcal{L}$, produces a term $t_0\,\eta$ in $\mathcal{L}$ with

$$t_0 = \tfrac{1}{4} v_0 (4\mu_{2,0}^2 - \lambda_{2,0} v_0^2) \tag{13}$$

at the one-loop level, where t_0 can be viewed as *bare tadpole constant*. The tadpole schemes described below impose different conditions on t_0, partially accompanied by appropriate field redefinitions of η_B, in order to generate the desired tadpole counterterm $\delta t h$ in the counterterm Lagrangian $\delta\mathcal{L}$.

In the *FJTS* the bare tadpole constant is consistently set to zero, $t_0 = 0$, so that no tadpole counterterm is introduced via parameter redefinitions. Instead, the tadpole counterterm is introduced by an additional field redefinition

$$\eta_B(x) \;\longrightarrow\; \eta_B(x) + \Delta v^\mathrm{FJTS}, \qquad \Delta v^\mathrm{FJTS} = -\frac{\delta t^\mathrm{FJTS}}{M_\mathrm{H}^2} = \frac{\Gamma^\eta}{M_\mathrm{H}^2}, \tag{14}$$

in the bare Lagrangian. The field shift (14) distributes tadpole renormalization constants to many counterterms in $\delta\mathcal{L}$ (see, e.g., App. A of Ref. [9]). Since the field shift (14) is a mere reparametrization of the functional integral over the Higgs field, it does not alter predictions for observables. Omitting the field shift would mean that explicit tadpole diagrams had to be included, but the result would still be the same as in the FJTS. In the FJTS, tadpole contributions correct for the fact that the effective Higgs potential is not expanded about the location of its minimum, but about the minimum of the potential in lowest order, which in the course of renormalization receives further corrections. For this reason, renormalization constants to mass parameters receive tadpole corrections in the FJTS, which are rather large by experience. In OS renormalization schemes these corrections cancel in predictions, but in other renormalization schemes such as $\overline{\mathrm{MS}}$ schemes this cancellation is only partial, and large corrections typically remain. On the positive side, the FJTS respects gauge invariance, i.e. the gauge independence of the parametrization of an observable in terms of v_0 and the other bare parameters carries over to the renormalized version of these parameters if the corresponding renormalization constants do not introduce gauge dependences, which is for instance the case in OS and $\overline{\mathrm{MS}}$ schemes in the FJTS.

The idea behind the *PRTS* is to achieve an expansion of the Higgs field about the true minimum of the renormalized effective Higgs potential (as obtained from the effective action after renormalization) by appropriate relations among the parameters of the theory. To this end, the bare parameter $v_0 = v + \delta v$ is renormalized in such a way that the renormalized parameter v is fixed by the renormalized W-boson mass M_W and and the SU(2) gauge coupling $g_2 = e/s_\mathrm{w}$ according to $v = 2M_\mathrm{W}/g_2$, where $s_\mathrm{w} = \sin\theta_\mathrm{w}$ is the sinus of the weak mixing angle θ_w and e the electric unit charge. The corresponding renormalization constant δv is directly fixed by the renormalization conditions on e, M_W, and $s_\mathrm{w}^2 = 1 - M_\mathrm{W}^2/M_\mathrm{Z}^2$. In order to guarantee the compensation of all tadpole contributions after renormalization, the bare tadpole constant $t_0 = t^\mathrm{PRTS} + \delta t^\mathrm{PRTS}$ is split into a renormalized value t^PRTS and a corresponding renormalization constant δt^PRTS and we demand $t^\mathrm{PRTS} = 0$, so that

$$\delta t^\mathrm{PRTS} = v_0(\mu_{2,0}^2 - \tfrac{1}{4}\lambda_{2,0}v_0^2) = v(\mu_{2,0}^2 - \tfrac{1}{4}\lambda_{2,0}v^2 - \tfrac{1}{2}\lambda_{2,0}v\delta v), \tag{15}$$

where the second equality holds in one-loop approximation. Since the renormalized parameter v, which is directly fixed by measurements, and the original bare parameters $\mu_{2,0}^2$ and $\lambda_{2,0}$ are gauge independent, the gauge dependence of δt^PRTS goes over to δv, where it shows up as gauge dependence in the mass renormalization constant δM_W^2. Trading the two bare parameters $\mu_{2,0}^2$ and $\lambda_{2,0}$ of the Higgs sector for v_0 and $M_{\mathrm{H},0}$, the PRTS tadpole counterterms can be generated by the replacements

$$\lambda_{2,0} \;\to\; \lambda_{2,0} + \frac{2\delta t^\mathrm{PRTS}}{v^3}, \qquad \mu_{2,0}^2 \;\to\; \mu_{2,0}^2 + \frac{3\delta t^\mathrm{PRTS}}{2v} \tag{16}$$

in the bare Lagrangian with $t_0 = 0$, i.e. several vertex counterterms receive contributions from δt^PRTS (see, e.g., App. A of Ref. [9]). If $\overline{\mathrm{MS}}$-renormalized mass parameters are used as input, the gauge dependence of δt^PRTS enters the parametrization of observables in the step where $\mu_{2,0}^2$ and $\lambda_{2,0}$ are traded for v_0 and $M_{\mathrm{H},0}$. Note that this flaw of introducing gauge dependences does not invalidate the applicability of the PRTS. On the positive side, the PRTS has the practical advantage over the FJTS that contributions to mass renormalization constants are much smaller, which, in particular, implies that conversions of renormalized mass parameters between OS and $\overline{\mathrm{MS}}$ renormalization schemes are typically much smaller in the PRTS as compared to the FJTS.

The *GIVS* aims to unify the benefits of the FJTS and the PRTS: the gauge-invariance property of the former and the perturbative stability of the latter. To avoid potentially large corrections induced by tadpole loops as inherent in the FJTS, the vev of the Higgs field is tied to the true minimum of the effective Higgs potential. Gauge dependences are avoided by switching to the non-linear Higgs representation (1) where the condition $v_0 = v + \delta v$ applies to the gauge-invariant component $v_0 + h_\mathrm{B}(x)$. In detail, in the non-linear Higgs representation the GIVS is identical to the PRTS described above, i.e. the tadpole renormalization constant is fixed according to

$$\delta t_\mathrm{nl}^\mathrm{GIVS} = \delta t_\mathrm{nl}^\mathrm{PRTS} = -\Gamma_\mathrm{nl}^h, \tag{17}$$

which is gauge independent as pointed out in the previous section. However, actual next-to-leading-order (NLO) calculations are typically carried out in the linear representation, and simply taking $\delta t_\mathrm{nl}^\mathrm{PRTS}$ as tadpole renormalization constant there does not fully cancel explicit tadpole loops. In order to fix this, we calculate δt^GIVS in the linear representations from $\delta t_\mathrm{nl}^\mathrm{GIVS}$ plus an extra term to restore $\delta t = -\Gamma^\eta$ as demanded in (12):

$$\delta t^\mathrm{GIVS} = \delta t_1^\mathrm{GIVS} + \delta t_2^\mathrm{GIVS}, \qquad \delta t_1^\mathrm{GIVS} = -\Gamma_\mathrm{nl}^h, \qquad \delta t_2^\mathrm{GIVS} = \Gamma_\mathrm{nl}^h - \Gamma^\eta. \tag{18}$$

	M^{OS} [GeV]	$\Delta M_{\mathrm{EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}}$ [GeV]				M^{OS} [GeV]	$\Delta M_{\mathrm{EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}}$ [GeV]		
		FJTS	PRTS	GIVS			FJTS	PRTS	GIVS
W boson	80.379	−2.22	0.82	0.74	top quark	172.4	10.75	0.99	0.54
Z boson	91.1876	−0.77	1.25	1.14	bottom quark	4.93	−1.79	0.10	0.13
Higgs boson	125.1	6.34	3.16	2.80	τ lepton	1.77686	−0.93	−0.028	−0.015

Table 1: On-shell masses M^{OS} of the heaviest SM particles and differences $\Delta M_{\mathrm{EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}}$ between the $\overline{\mathrm{MS}}$ mass $\overline{M}(\mu)$ and M^{OS} induced by NLO EW corrections using the FJTS, PRTS, or GIVS at the renormalization scale $\mu = M^{\mathrm{OS}}$ (taken from Ref. [5]).

The gauge-independent part $\delta t_1^{\mathrm{GIVS}}$ occurs in PRTS-like tadpole contributions to counterterm which are generated as in (16) with $\delta t_1^{\mathrm{GIVS}}$ playing the role of δt^{PRTS}. This part absorbs potentially large corrections to the location of the minimum in the effective Higgs potential into renormalized input parameters. The gauge-dependent part $\delta t_2^{\mathrm{GIVS}}$ occurs in FJTS-like tadpole contributions to counterterm which are generated by the field shift

$$\eta_B(x) \;\longrightarrow\; \eta_B(x) + \Delta v^{\mathrm{GIVS}}, \qquad \Delta v^{\mathrm{GIVS}} = -\frac{\delta t_2^{\mathrm{GIVS}}}{M_{\mathrm{H}}^2} \tag{19}$$

analogous to (14). In summary, knowing the tadpole counterterms of the FJTS and the PRTS, as e.g., given in the SM Feynman rules of App. A of Ref. [9], the generation of the one-loop GIVS tadpole counterterms is easily accomplished by the substitutions

$$\delta t^{\mathrm{PRTS}} \;\longrightarrow\; \delta t_1^{\mathrm{GIVS}}, \qquad \delta t^{\mathrm{FJTS}} \;\longrightarrow\; \delta t_2^{\mathrm{GIVS}}. \tag{20}$$

In Ref. [5] we have illustrated the perturbative stability of the GIVS by considering the conversion of OS-renormalized masses of SM particles to $\overline{\mathrm{MS}}$ masses with the various tadpole schemes. Table 1 shows the corresponding mass shifts induced by NLO EW corrections for the heaviest particles in the SM. The masses entering in $\Delta M_{\mathrm{EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}}$ are chosen according to the OS mass values given in Tab. 1, and for the PRTS the 't Hooft–Feynman gauge is chosen.

The values obtained in the PRTS and the GIVS are of comparable size while in general the FJTS leads to larger differences between the OS and the $\overline{\mathrm{MS}}$ masses. As emphasized in the literature [13–15] for the top quark before, the FJTS shift $\Delta m_{\mathrm{t,EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}} = 10.75\,\mathrm{GeV}$ in the conversion of fermion masses is much larger than the typical size of EW corrections of the percent level. For the lighter fermions b and τ, the relative corrections $\Delta M_{\mathrm{EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}}/M^{\mathrm{OS}}$ are even larger than for the top quark in the FJTS, reaching up to $\sim 50\%$, while the shifts in the PRTS and GIVS remain all moderate. Despite these large corrections, the FJTS often is favoured in the literature in this context, since it leads to a gauge-independent result in contrast to the PRTS. Note, however, that these large EW one-loop corrections entail an enhancement of the theoretical uncertainties due to missing higher-order corrections. The GIVS, on the other hand, provides gauge-independent mass shifts that are moderate and, thus, leads to smaller EW theory uncertainties, when those uncertainties are estimated by the propagation of the known corrections to higher order as typically done.

4. The GIVS in extended Higgs sectors

The generalization of the GIVS from the SM to extended Higgs sectors is fully straightforward. The tadpole renormalization constant $\delta t_{h_n} = -\Gamma^{h_n}$ of any Higgs field $v_n + h_n(x)$ that can acquire a vev v_n is generated in the GIVS from two parts as in (20). The gauge-independent PRTS-like part $\delta t^{\mathrm{GIVS}}_{h_n,1} = -\Gamma^{h_n}_{\mathrm{nl}}$ is calculated in the non-linear Higgs representation, and the gauge-dependent FJTS-like part $\delta t^{\mathrm{GIVS}}_{h_n,2} = \Gamma^{h_n}_{\mathrm{nl}} - \Gamma^{h_n}$ accounts for the remaining contributions. For the SESM and THDM, the application of the GIVS and its impact on the $\overline{\mathrm{MS}}$ renormalization of the Higgs mixing angles are described in Ref. [6] in detail. For the $\overline{\mathrm{MS}}$ renormalization of the THDM mixing angle β we find that the PRTS in any R_ξ gauge coincides with the GIVS, a fact that puts existing PRTS results on a gauge-independent basis when reinterpreted as GIVS results. We conjecture that this feature of the $\overline{\mathrm{MS}}$ renormalization of β also carries over to supersymmetric theories.

The renormalization of Higgs mixing angles as well as the weaknesses and strengths of various schemes was discussed for the SESM and THDM already in Refs. [7, 8, 12, 16, 17]. In particular, Ref. [12] analyzes existing and newly suggested renormalization schemes wrt. the following criteria: (i) gauge independence, (ii) symmetry wrt. mixing degrees of freedom, (iii) perturbative stability, and (iv) smoothness for degenerate masses or extreme mixing angles. While the suggested schemes based on field-theoretical symmetries or on OS conditions widely meet these requirements, $\overline{\mathrm{MS}}$ renormalization with FJTS tadpole treatment turned out to be particularly prone to perturbative instabilities in specific parameter regions (large or degenerate Higgs masses, extreme mixing angles). These features were demonstrated in a comprehensive discussion of NLO predictions for various Higgs-boson production and decay processes in Ref. [12], extending the earlier discussions of Refs. [7, 8, 16, 17].

The results of Ref. [6] for the Higgs-boson decays $\mathrm{h/H} \to \mathrm{WW/ZZ} \to 4f$ in the SESM and THDM demonstrate that $\overline{\mathrm{MS}}$ renormalization with GIVS tadpole treatment mitigates perturbative instabilities significantly and produces gauge-independent results very close to the gauge-dependent PRTS. Table 2 shows some results for the decays $\mathrm{h} \to 4f$ in two THDM scenarios, which illustrate that the scale uncertainty of the $\overline{\mathrm{MS}}$ FJTS results is not always reduced in the transition from LO to NLO. It should also be mentioned that all $\overline{\mathrm{MS}}$ variants run into problems with perturbative stability in extreme parameter regions; for instance, the $\overline{\mathrm{MS}}$ schemes do not give reliable results for the $\mathrm{H} \to 4f$ decays of the heavy Higgs boson H in the THDM.

Ren. scheme	tadpoles	A1 LO	A1 NLO	A2 LO	A2 NLO
OS12 (α,β)		$0.89832(3)$	$0.96194(7)^{-0.1\%}_{+0.1\%}$	$0.87110(3)$	$0.92947(7)^{-0.2\%}_{+0.1\%}$
$\overline{\mathrm{MS}}$ (α,β)	FJTS	$0.89996(3)^{+0.7\%}_{-7.4\%}$	$0.96283(7)^{+0.8\%}_{-0.2\%}$	$0.88508(3)^{+2.2\%}_{-10.0\%}$	$0.93604(7)^{+3.1\%}_{-11.0\%}$
$\overline{\mathrm{MS}}$ (α,β)	PRTS	$0.89035(3)^{-2.8\%}_{+0.9\%}$	$0.96103(7)^{+1.2\%}_{+0.4\%}$	$0.86130(3)^{-6.1\%}_{+2.3\%}$	$0.92784(7)^{+1.3\%}_{+1.3\%}$
$\overline{\mathrm{MS}}$ (α,β)	GIVS	$0.89082(3)^{-2.7\%}_{+0.9\%}$	$0.96106(7)^{+1.2\%}_{+0.5\%}$	$0.86249(3)^{-5.8\%}_{+2.3\%}$	$0.92808(7)^{+1.3\%}_{+1.3\%}$
$\overline{\mathrm{MS}}$ (λ_3,β)	FJTS	$0.89246(3)^{-15.1\%}_{+1.6\%}$	$0.96108(7)^{+17.3\%}_{+1.9\%}$	$0.85590(3)^{-29.8\%}_{+5.5\%}$	$0.92723(7)^{+18.3\%}_{+2.8\%}$
$\overline{\mathrm{MS}}$ (λ_3,β)	PRTS/GIVS	$0.89156(3)^{-8.4\%}_{+1.7\%}$	$0.96111(7)^{+3.8\%}_{+2.1\%}$	$0.85841(3)^{-12.7\%}_{+5.0\%}$	$0.92729(7)^{+4.6\%}_{+2.6\%}$

Table 2: LO and NLO decay widths $\Gamma^{\mathrm{h}\to 4f}$[MeV] of the light CP-even Higgs boson h for THDM scenarios A1 and A2 in different renormalization schemes, with the on-shell scheme OS12 as input scheme (and full conversion of the input parameters into the other schemes). The scale variation (given in percent) corresponds to the scales $\mu = \mu_0/2$ and $\mu = 2\mu_0$, where μ_0 is the average Higgs mass (for all details, see Ref. [6]).

5. Conclusions

Recently we have proposed a new scheme for a gauge-invariant treatment of tadpole corrections in spontaneously broken gauge theories called *Gauge-Invariant Vacuum expectation value Scheme (GIVS)*. The GIVS is a hybrid scheme of the two most commonly used tadpole schemes, in which tadpole counterterms are introduced either via parameter or via field renormalization transformations. In the construction of tadpole counterterms, the GIVS makes use of non-linear representations of Higgs sectors, in which fields that can develop non-vanishing vacuum expectation values are gauge invariant, but actual loop calculations are performed in the GIVS as usual. In contrast to previously used tadpole schemes, the GIVS unifies the properties of gauge independence and perturbative stability. We have illustrated these virtues by discussing the conversion of on-shell-renormalized to $\overline{\text{MS}}$-renormalized masses in the SM and the $\overline{\text{MS}}$ renormalization of Higgs mixing angles in two frequently considered models with extended Higgs sectors—a singlet Higgs extension of the SM and the Two-Higgs-Doublet Model. We expect that the generalization of the GIVS beyond the one-loop level works without problems.

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