

## Automated choice of the best renormalization scheme

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High-precision predictions in BSM models require calculations at the loop-level and thus a renormalization of (some of) the BSM parameter. Here many choices for the renormalization scheme (RS) are possible. A given RS can be well suited to yield “stable” and “well behaved” higher-order corrections in one part of the BSM parameter space, but can fail completely in other parts. The latter may not even be noticed numerically if an isolated parameter point is investigated. Here we review a new method for choosing a “well behaved” RS. We demonstrate the feasibility of our new method in the chargino/neutralino sector of the Minimal Supersymmetric Standard Model (MSSM), but stress the general applicability of our method to all types of BSM models.

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## 1. Introduction

A reliable investigation of a model beyond the Standard Model (BSM) requires the inclusion of higher-order corrections to, e.g., the production cross sections of BSM particles at the HL-LHC. This in turn requires the renormalization of the BSM model. The renormalization of BSM models is much less explored than the renormalization of the SM. Examples for “full one-loop renormalizations” can be found for the Minimal Supersymmetric Standard Model (MSSM) [1, 2], and the Next-to-MSSM (NMSSM) [3]. These analyses showed that many different choices of renormalization schemes (RS) are possible. This can concern the choice of the set of to-be-renormalized parameters out of a larger set of BSM parameters, but can also concern the type of renormalization condition that is chosen for a specific parameter.

BSM models naturally possess several new BSM parameters. The number of new parameters can vary from  $\mathcal{O}(1)$  to  $\mathcal{O}(10)$ , or even higher. Often multi-dimensional parameter scans are employed, or methods such as Markov-Chain Monte-Carlo (MCMC) analyses to find the phenomenological best-appealing parameters in the multi-dimensional BSM parameter space. The above mentioned BSM analyses also demonstrated that a given RS can be well suited to yield “stable” and “well behaved” higher-order corrections (more details will be given below) in one part of the BSM parameter space, but can fail completely in other parts. The latter may not even be noticed numerically if only isolated parameter points are investigated, which is natural in a scan, or MCMC analyses. Consequently, the exploration of BSM models requires a choice of a good RS *before* the calculation is performed.

An RS “fails” if one of the counterterms (or a linear combination of counterterms) does not (or only marginally) depend on the parameter itself, but is rather determined via other parameters of the model. This failure can manifest itself in *(i)* “unnaturally” large higher-order corrections, *(ii)* large (numerical) differences between  $\overline{\text{DR}}$  and OS masses, *(iii)* (numerical) differences between  $\overline{\text{DR}}$  and OS parameters. In this work we review a new method how such a situation can be avoided, i.e. how a “good” RS can be chosen. This method is based on the properties of the transformation matrix that connects the various counter terms with the underlying parameter. This allows a point-by-point test of all “available” or “possible” RS, and the “best” one can be chosen to perform the calculation. Our idea is designed to work in all cases of RS choices (in BSM models).

The numerical examples will be performed within the MSSM, concretely in the sector of charginos and neutralinos, the supersymmetric (SUSY) partners of the SM gauge bosons and the 2HDM-like Higgs sector. While this constitutes a very specific example, we would like to stress the general applicability of our method to all types of BSM models and types of RS choices.

## 2. Renormalization: theoretical considerations and concrete implementations

### 2.1 The general idea

As discussed above, the idea of how to choose a stable and well behaved RS is generally applicable. However, here we will outline it focusing a more concrete problem: in our theory we have  $m$  underlying Lagrangian parameters and  $n > m$  particles or particle masses that can be renormalized OS. Each choice of  $m$  particles renormalized OS defines an  $\text{RS}_l$ , of which we have  $N$  in total. How can one choose the “best”  $\text{RS}_l$ ? Our starting point will be the following: The masses

of the BSM particles under investigation have not (yet) been measured. Then we start with  $\overline{\text{DR}}$  parameters. The general idea for the automated choice of the  $\text{RS}_L$  in the  $\overline{\text{DR}}$  case can be outlined for two possible levels of refinement. The first one is called “semi-OS scheme”, and the second one “full-OS scheme” (where in our numerical examples we will focus on the latter). The two cases are defined as follows.

### Semi-OS scheme:

1. We start with  $m$   $\overline{\text{DR}}$  parameters,  $P_i^{\overline{\text{DR}}}$ , from the Lagrangian and  $N$   $\text{RS}_l$ .
2. For each  $\text{RS}_l$ , i.e. each different choice of  $m$  particles renormalized OS, we evaluate the corresponding OS parameters

$$P_{i,l}^{\text{OS}} = P_i^{\overline{\text{DR}}} - \delta P_{i,l|\text{fin}}^{\text{OS}} \quad (2.1)$$

with the transformation matrix  $\mathbf{A}_l^{\overline{\text{DR}}}$  (more details will be given below).

3. It will be argued that a “bad” scheme  $\text{RS}_l$  has a small or even vanishing  $|\det \mathbf{A}_l^{\overline{\text{DR}}}|$ .
4. Comparing the various  $|\det \mathbf{A}_l^{\overline{\text{DR}}}|$  yields  $\text{RS}_L$ .
5. Inserting  $P_{i,l}^{\text{OS}}$  into the Lagrangian yields  $n$  particle masses out of which  $m$  are by definition given as their OS values. The remaining OS masses have to be determined calculating  $n - m$  finite shifts.
6. The counterterms for the  $P_{i,l}^{\text{OS}}$  are already known from Eq. (2.1) as  $\delta P_{i,l}^{\text{OS}}$  and can be inserted as counterterms in a loop calculation.

This procedure yields all ingredients for an OS scheme. However, the OS counterterms  $\delta P_{i,l}^{\text{OS}}$  and thus also the OS parameters themselves,  $P_{i,l}^{\text{OS}}$ , are calculated in terms of  $\overline{\text{DR}}$  parameters, i.e. one has  $\delta P_{i,l}^{\text{OS}}(P_i^{\overline{\text{DR}}})$  and  $P_{i,l}^{\text{OS}}(P_i^{\overline{\text{DR}}})$ . This is unsatisfactory for a “true” OS scheme, i.e. one would like to have  $\delta P_{i,l}^{\text{OS}}(P_{i,l}^{\text{OS}})$ . Furthermore, when a  $\text{RS}_l$  “starts to turn bad” as a function of a  $\overline{\text{DR}}$  parameter, large differences between the  $P_{i,l}^{\text{OS}}$  and  $P_i^{\overline{\text{DR}}}$  occur, shedding doubt on the above outlined procedure. These problems can be circumvented by extending the above scheme to an evaluation of the counterterms in terms of OS parameters. The general idea starts as above, but deviates from step 4 on.

### Full-OS scheme:

The first two steps are as in the semi-OS scheme. We then continue with

3. Inserting  $P_{i,l}^{\text{OS}}$  into the Lagrangian yields  $n$  particle masses out of which  $m$  are by definition given as their  $\text{os}_l$  values. The remaining  $\text{os}_l$  masses have to be determined calculating  $n - m$  finite shifts.
4.  $\text{RS}_l$  is applied again on the  $\text{OS}_l$  Lagrangian.
5. This yields now OS counterterms in terms of  $\text{os}_l$  parameters,

$$\delta P_{i,l}^{\text{OS}}(P_{i,l}^{\text{OS}}) \quad (2.2)$$

with the transformation matrix  $\mathbf{A}_l^{\text{OS}}$  (more details will be given below).

6. It will be argued that a “bad” scheme  $RS_l$  has a small/vanishing  $|\det \mathbf{A}_l^{\overline{\text{DR}}}|$  and/or  $|\det \mathbf{A}_l^{\text{OS}}|$ .
7. Comparing the various  $\min \{|\det \mathbf{A}_l^{\overline{\text{DR}}}|, |\det \mathbf{A}_l^{\text{OS}}|\}$  yields  $RS_L$ .
8. The counterterms for the  $P_{i,L}^{\text{OS}}$  are already known from Eq. (2.2) as  $\delta P_{i,L}^{\text{OS}}$  and can be inserted as counterterms in a loop calculation.

Steps 3-5 could be iterated until convergence is reached. We will not do this.

## 2.2 Application to the chargino/neutralino sector of the MSSM

The concrete implementation concerns the calculation of physics processes with (external) charginos and/or neutralinos,  $\tilde{\chi}_c^\pm (c = 1, 2)$  and  $\tilde{\chi}_n^0 (n = 1, 2, 3, 4)$  at the loop level. This requires the choice of a (numerically well behaved) RS. The possible scheme choices are ( $n'' > n' > n$ )

$$\text{CCN}_n, \quad \text{CNN}_{cn'n'}, \quad \text{NNN}_{nn'n''} \quad c = 1, 2; n, n', n'' = 1, 2, 3, 4. \quad (2.3)$$

Here  $\text{CCN}_n$  denotes a scheme where the two charginos and the neutralino  $n$ ,  $\tilde{\chi}_n^0$ , are renormalized OS.  $\text{CNN}_{cn'n'}$  denotes a scheme where chargino  $c$ ,  $\tilde{\chi}_c^\pm$ , as well as neutralinos  $n, n'$ ,  $\tilde{\chi}_n^0, \tilde{\chi}_{n'}^0$ , are renormalized OS. Finally  $\text{NNN}_{nn'n''}$  denotes a scheme with three neutralinos renormalized OS. For sake of simplicity, in the following we neglect the  $\text{NNN}_{nn'n''}$  schemes.

To fix our notation we briefly describe the chargino/neutralino sector of the MSSM. The bilinear term in the Lagrangian is given by,

$$\begin{aligned} \mathcal{L}_{\tilde{\chi}^-, \tilde{\chi}^0}^{\text{bil.}} = & \overline{\tilde{\chi}_i^-} \not{p} \omega_- \tilde{\chi}_i^- + \overline{\tilde{\chi}_i^-} \not{p} \omega_+ \tilde{\chi}_i^- - \overline{\tilde{\chi}_i^-} [\mathbf{V}^* \mathbf{X}^\top \mathbf{U}^\dagger]_{ij} \omega_- \tilde{\chi}_j^- - \overline{\tilde{\chi}_i^-} [\mathbf{U} \mathbf{X}^* \mathbf{V}^\top]_{ij} \omega_+ \tilde{\chi}_j^- \\ & + \frac{1}{2} \left( \overline{\tilde{\chi}_k^0} \not{p} \omega_- \tilde{\chi}_k^0, + \overline{\tilde{\chi}_k^0} \not{p} \omega_+ \tilde{\chi}_k^0 - \overline{\tilde{\chi}_k^0} [\mathbf{N}^* \mathbf{Y} \mathbf{N}^\dagger]_{kl} \omega_- \tilde{\chi}_l^0 - \overline{\tilde{\chi}_k^0} [\mathbf{N} \mathbf{Y}^* \mathbf{N}^\top]_{kl} \omega_+ \tilde{\chi}_l^0 \right), \end{aligned} \quad (2.4)$$

already expressed in terms of the chargino and neutralino mass eigenstates  $\tilde{\chi}_i^-$  and  $\tilde{\chi}_k^0$ , respectively, and  $i, j = 1, 2$  and  $k, l = 1, 2, 3, 4$ . The mass eigenstates can be determined via unitary transformations where the corresponding matrices diagonalize the chargino and neutralino mass matrix,  $\mathbf{X}$  and  $\mathbf{Y}$ , respectively.

In the chargino case, two  $2 \times 2$  matrices  $\mathbf{U}$  and  $\mathbf{V}$  are necessary for the diagonalization of the chargino mass matrix  $\mathbf{X}$ ,

$$\mathbf{M}_{\tilde{\chi}^-} = \mathbf{V}^* \mathbf{X}^\top \mathbf{U}^\dagger = \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix} \quad \text{with} \quad \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} \sin \beta M_W \\ \sqrt{2} \cos \beta M_W & \mu \end{pmatrix}, \quad (2.5)$$

where  $\mathbf{M}_{\tilde{\chi}^-}$  is the diagonal mass matrix with the chargino masses  $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}$  as entries, which are determined as the (real and positive) singular values of  $\mathbf{X}$ . The singular value decomposition of  $\mathbf{X}$  also yields results for  $\mathbf{U}$  and  $\mathbf{V}$ .

In the neutralino case, as the neutralino mass matrix  $\mathbf{Y}$  is symmetric, one  $4 \times 4$  matrix is sufficient for the diagonalization

$$\mathbf{M}_{\tilde{\chi}^0} = \mathbf{N}^* \mathbf{Y} \mathbf{N}^\dagger = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}) \quad (2.6)$$

with

$$\mathbf{Y} = \begin{pmatrix} M_1 & 0 & -M_Z s_w \cos \beta & M_Z s_w \sin \beta \\ 0 & M_2 & M_Z c_w \cos \beta & -M_Z c_w \sin \beta \\ -M_Z s_w \cos \beta & M_Z c_w \cos \beta & 0 & -\mu \\ M_Z s_w \sin \beta & -M_Z c_w \sin \beta & -\mu & 0 \end{pmatrix}. \quad (2.7)$$

$M_Z$  and  $M_W$  are the masses of the  $Z$  and  $W$  boson,  $c_w = M_W/M_Z$  and  $s_w = \sqrt{1 - c_w^2}$ . The unitary  $4 \times 4$  matrix  $\mathbf{N}$  and the physical neutralino (tree-level) masses  $m_{\tilde{\chi}_k^0}$  ( $k = 1, 2, 3, 4$ ) result from a numerical Takagi factorization of  $\mathbf{Y}$ .

Concerning the renormalization of this sector, the following replacements of the parameters and the fields are performed according to the multiplicative renormalization procedure, which is formally identical for the two set-ups:

$$M_1 \rightarrow M_1 + \delta M_1, \quad M_2 \rightarrow M_2 + \delta M_2, \quad \mu \rightarrow \mu + \delta \mu, \quad (2.8)$$

$$\omega_{-/+} \tilde{\chi}_i^\pm \rightarrow \left[ \mathbb{1} + \frac{1}{2} \delta \mathbf{Z}_{\tilde{\chi}^\pm}^{L/R} \right]_{ij} \omega_{-/+} \tilde{\chi}_j^\pm \quad (i, j = 1, 2), \quad (2.9)$$

$$\omega_{-/+} \tilde{\chi}_k^0 \rightarrow \left[ \mathbb{1} + \frac{1}{2} \delta \mathbf{Z}_{\tilde{\chi}^0}^{*/} \right]_{kl} \omega_{-/+} \tilde{\chi}_l^0 \quad (k, l = 1, 2, 3, 4). \quad (2.10)$$

It should be noted that the parameter counterterms are complex counterterms which each need two renormalization conditions to be fixed. The transformation matrices are not renormalized, so that, using the notation of replacing a matrix by its renormalized matrix and a counterterm matrix

$$\mathbf{X} \rightarrow \mathbf{X} + \delta \mathbf{X}, \quad \mathbf{Y} \rightarrow \mathbf{Y} + \delta \mathbf{Y} \quad (2.11)$$

with

$$\delta \mathbf{X} = \begin{pmatrix} \delta M_2 & \sqrt{2} \delta(M_W \sin \beta) \\ \sqrt{2} \delta(M_W \cos \beta) & \delta \mu \end{pmatrix}, \quad (2.12)$$

$$\delta \mathbf{Y} = \begin{pmatrix} \delta M_1 & 0 & -\delta(M_Z s_w \cos \beta) & \delta(M_Z s_w \sin \beta) \\ 0 & \delta M_2 & \delta(M_Z c_w \cos \beta) & -\delta(M_Z c_w \sin \beta) \\ -\delta(M_Z s_w \cos \beta) & \delta(M_Z c_w \cos \beta) & 0 & -\delta \mu \\ \delta(M_Z s_w \sin \beta) & -\delta(M_Z c_w \sin \beta) & -\delta \mu & 0 \end{pmatrix}, \quad (2.13)$$

the replacements of the matrices  $\mathbf{M}_{\tilde{\chi}^-}$  and  $\mathbf{M}_{\tilde{\chi}^0}$  can be expressed as

$$\mathbf{M}_{\tilde{\chi}^-} \rightarrow \mathbf{M}_{\tilde{\chi}^-} + \delta \mathbf{M}_{\tilde{\chi}^-} = \mathbf{M}_{\tilde{\chi}^-} + \mathbf{V}^* \delta \mathbf{X}^\top \mathbf{U}^\dagger \quad (2.14)$$

$$\mathbf{M}_{\tilde{\chi}^0} \rightarrow \mathbf{M}_{\tilde{\chi}^0} + \delta \mathbf{M}_{\tilde{\chi}^0} = \mathbf{M}_{\tilde{\chi}^0} + \mathbf{N}^* \delta \mathbf{Y} \mathbf{N}^\dagger. \quad (2.15)$$

More details on the renormalization can be found in Ref. [4].

### 2.3 Concrete renormalization in the semi-OS scheme

We start with  $\overline{\text{DR}}$  mass matrices for charginos and neutralinos, collectively denoted as  $\mathbf{X}^{\overline{\text{DR}}}(P_i^{\overline{\text{DR}}})$ , depending on the three input parameters,

$$P_i^{\overline{\text{DR}}} = M_1^{\overline{\text{DR}}}, M_2^{\overline{\text{DR}}}, \mu^{\overline{\text{DR}}} = \{p_i^{\overline{\text{DR}}}\}. \quad (2.16)$$

The mass matrices can be diagonalized,

$$\mathbf{X}^{\overline{\text{DR}}} \rightarrow \mathbf{M}^{\overline{\text{DR}}} := (\mathbf{N}^{\overline{\text{DR}}})^\dagger \mathbf{X}^{\overline{\text{DR}}} \mathbf{N}^{\overline{\text{DR}}}, \quad (2.17)$$

containing on the diagonal two charginos and four neutralino masses,  $m_j$ .

The  $\mathbf{X}^{\overline{\text{DR}}}$  can be renormalized,

$$\mathbf{X}^{\overline{\text{DR}}} \rightarrow \mathbf{X}^{\overline{\text{DR}}} + \delta \mathbf{X}^{\overline{\text{DR}}}(\delta P_i^{\overline{\text{DR}}}) \quad (2.18)$$

$$\mathbf{M}^{\overline{\text{DR}}} \rightarrow \mathbf{M}^{\overline{\text{DR}}} + \delta \mathbf{M}^{\overline{\text{DR}}}(\delta P_i^{\overline{\text{DR}}}) = \mathbf{M}^{\overline{\text{DR}}} + (\mathbf{N}^{\overline{\text{DR}}})^\dagger \delta \mathbf{X}^{\overline{\text{DR}}}(\delta P_i^{\overline{\text{DR}}}) \mathbf{N}^{\overline{\text{DR}}}. \quad (2.19)$$

So far, the  $\delta P_i^{\overline{\text{DR}}}$  are unknown. The self-energies of the charginos and neutralinos can be written down as  $\Sigma_j(P_i^{\overline{\text{DR}}}, \mathbf{X}^{\overline{\text{DR}}})$ . Now the RS is chosen: CCN<sub>c</sub> or CNN<sub>cm'</sub>. For each of these  $N = 28$  schemes we perform the following. The scheme is denoted as RS<sub>l</sub> ( $l = 1 \dots 28$ ). Three renormalized self-energies are chosen to be zero,

$$\hat{\Sigma}_{k,l}(P_i^{\overline{\text{DR}}}, \mathbf{X}^{\overline{\text{DR}}}) = 0 \quad (k = 1, 2, 3), \quad (2.20)$$

corresponding to three os masses,  $m_k^{\text{os}}$ . The three renormalized self-energies yield three conditions on  $\delta \mathbf{M}_k^{\overline{\text{DR}}}$ ,

$$\delta \mathbf{M}_{k,l}^{\overline{\text{DR}}} = f_{k,l}^{\overline{\text{DR}}}(m_{k',l}^{\overline{\text{DR}}}, \Sigma_{k'',l}) + F_{k,l}^{\overline{\text{DR}}}(\delta \tan \beta, \delta M_Z^2, \dots) \quad (2.21)$$

$$\downarrow \mathbf{A}_l^{\overline{\text{DR}}} \quad (2.22)$$

$$\delta P_{i,l}^{\text{os}} = g_{i,l}^{\overline{\text{DR}}}(m_{k',l}^{\overline{\text{DR}}}, \Sigma_{k'',l}) + G_{i,l}^{\overline{\text{DR}}}(\delta \tan \beta, \delta M_Z^2, \dots), \quad (2.23)$$

yielding the os values

$$P_i^{\overline{\text{DR}}} \rightarrow P_i^{\overline{\text{DR}}} - \delta P_{i,l}^{\text{os}} = P_{i,l}^{\text{os}}. \quad (2.24)$$

It is worth noticing that in the r.h.s. of Eq. (2.21)  $f_{k,l}$  is linear in  $\delta P_{i,l}^{\text{os}}$ , while  $F_{k,l}$  only depends on the counterterm of the remaining model parameters. These relations define  $\mathbf{A}_l^{\overline{\text{DR}}}$ , the transformation matrix from the set of mass counterterms to parameter counterterms,

$$\delta P_{i,l}^{\text{os}} = (\mathbf{A}_l^{\overline{\text{DR}}})_{ik}^{-1} (\delta \mathbf{M}_{k,l}^{\overline{\text{DR}}} - F_{k,l}(\delta \tan \beta, \delta M_Z^2, \dots)). \quad (2.25)$$

os masses  $m_{k,l}^{\text{os}}$  are derived from

$$\mathbf{X}_l^{\text{os}}(P_{i,l}^{\text{os}}) \rightarrow \mathbf{M}_l^{\text{os}} := (\mathbf{N}_l^{\text{os}})^\dagger \mathbf{X}_l^{\text{os}}(P_{i,l}^{\text{os}}) \mathbf{N}_l^{\text{os}}. \quad (2.26)$$

The three masses that are not obtained as os masses so far can be evaluated by adding finite shifts to them, see Ref. [4].

As discussed above, an RS “fails” if one of the counterterms (or a linear combination of counterterms) does not (or only marginally) depend on the parameter itself, but is rather determined via other parameters of the model. This is exactly given in our ansatz if the matrix  $\mathbf{A}_l^{\overline{\text{DR}}}$  does not provide a numerically “well behaved” transition

$$\delta \mathbf{M}_{k,l}^{\overline{\text{DR}}} \xrightarrow{\mathbf{A}_l^{\overline{\text{DR}}}} \delta P_{i,l}^{\text{os}}, \quad (2.27)$$

see Eqs. (2.22), suppressing terms involving other counterterms ( $\delta \tan \beta$ ,  $\delta M_Z^2$ , ...). Following the argument of the “well behaved” transition,  $RS_l$  fails if  $\mathbf{A}_l^{\overline{DR}}$  becomes (approximately) singular, or the normalized determinant,

$$\mathbf{D}_l^{\overline{DR}} := \frac{|\det \mathbf{A}_l^{\overline{DR}}|}{\|\mathbf{A}_l^{\overline{DR}}\|} \ll 1, \quad (2.28)$$

Conversely, the “best” scheme  $RS_L$  can be chosen via the condition of the maximum normalized determinant,

$$RS_L^{\text{OS}} \Leftrightarrow \mathbf{D}_L^{\overline{DR}} = \max_l \{\mathbf{D}_l^{\overline{DR}}\}. \quad (2.29)$$

## 2.4 Concrete implementation in the full OS renormalization

For each  $RS_l$  as evaluated in Sect. 2.3 we now have os mass matrices for charginos and neutralinos, collectively denoted as  $\mathbf{X}^{\text{OS}}(P_{i,l}^{\text{OS}})$  following Eq. (2.26). We also have os parameters  $P_{i,l}^{\text{OS}}(P_i^{\overline{DR}})$  following Eq. (2.24) and  $\delta P_{i,l}^{\text{OS}}(P_i^{\overline{DR}})$  following Eq. (2.23). This is unsatisfactory for a “true” OS scheme, i.e. one would like to have  $\delta P_{i,l}^{\text{OS}}(P_{i,l}^{\text{OS}})$ . Furthermore, when a  $RS_l$  “starts to turn bad” as a function of a  $\overline{DR}$  parameter, large differences between the  $P_{i,l}^{\text{OS}}$  and  $P_i^{\overline{DR}}$  occur, shedding doubt on the above outlined procedure. These problems can be circumvented by extending the above scheme to an evaluation of the counterterms in terms of OS parameters.

We start with the os parameters obtained in Sect. 2.3,  $P_{i,l}^{\text{OS}}$ . The mass matrices depend on these three input parameters. Now the renormalization process in  $RS_l$  is applied again, starting from the above os values. Following the same steps as in Sect. 2.3, defining the matrix  $\mathbf{A}_l^{\text{OS}}$ . As in the case of the semi-OS scheme, a bad  $RS_l$  is indicated if in our ansatz if the matrix  $\mathbf{A}_l^{\text{OS}}$  does not provide a numerically “well behaved” transition

$$\delta \mathbf{M}_{k,l}^{\text{OS}} \xrightarrow{\mathbf{A}_l^{\text{OS}}} \delta P_{i,l}^{\text{OS}}, \quad (2.30)$$

and suppressing terms involving other counterterms ( $\delta \tan \beta$ ,  $\delta M_Z^2$ , ...). Following the argument of the “well behaved” transition,  $RS_l$  fails if  $\mathbf{A}_l^{\overline{DR}}$  or  $\mathbf{A}_l^{\text{OS}}$  become (approximately) singular, or the normalized determinant,

$$\mathbf{D}_l^{\overline{DR}} := \frac{|\det \mathbf{A}_l^{\overline{DR}}|}{\|\mathbf{A}_l^{\overline{DR}}\|} \ll 1 \quad \text{or} \quad \mathbf{D}_l^{\text{OS}} := \frac{|\det \mathbf{A}_l^{\text{OS}}|}{\|\mathbf{A}_l^{\text{OS}}\|} \ll 1, \quad (2.31)$$

equivalent to  $\mathbf{D}_l^{\text{OS}} := \min \{\mathbf{D}_l^{\overline{DR}}, \mathbf{D}_l^{\text{OS}}\} \ll 1$ . Conversely, the “best” scheme  $RS_L$  can be chosen via the condition of the maximum normalized determinant,

$$RS_L^{\text{OS}} \Leftrightarrow \mathbf{D}_L^{\text{OS}} = \max_l \{\mathbf{D}_l^{\text{OS}}\}. \quad (2.32)$$

Now all ingredients for physics calculations are at hand. (i) The physical parameters  $P_{i,L}^{\text{OS}}$  are given via the OS analogon to Eq. (2.24). (ii) The counterterms for the  $P_{i,L}^{\text{OS}}$  are known from the OS analogon to Eq. (2.23) as  $\delta P_{i,L}^{\text{OS}}$  and can be inserted as counterterms in a loop calculation. (iii) Inserting  $P_{i,L}^{\text{OS}}$  into the Lagrangian yields six particle masses out of which three are by definition given as their OS values. The remaining OS masses have to be determined calculating three finite shifts, see Ref. [4].

### 3. Numerical example

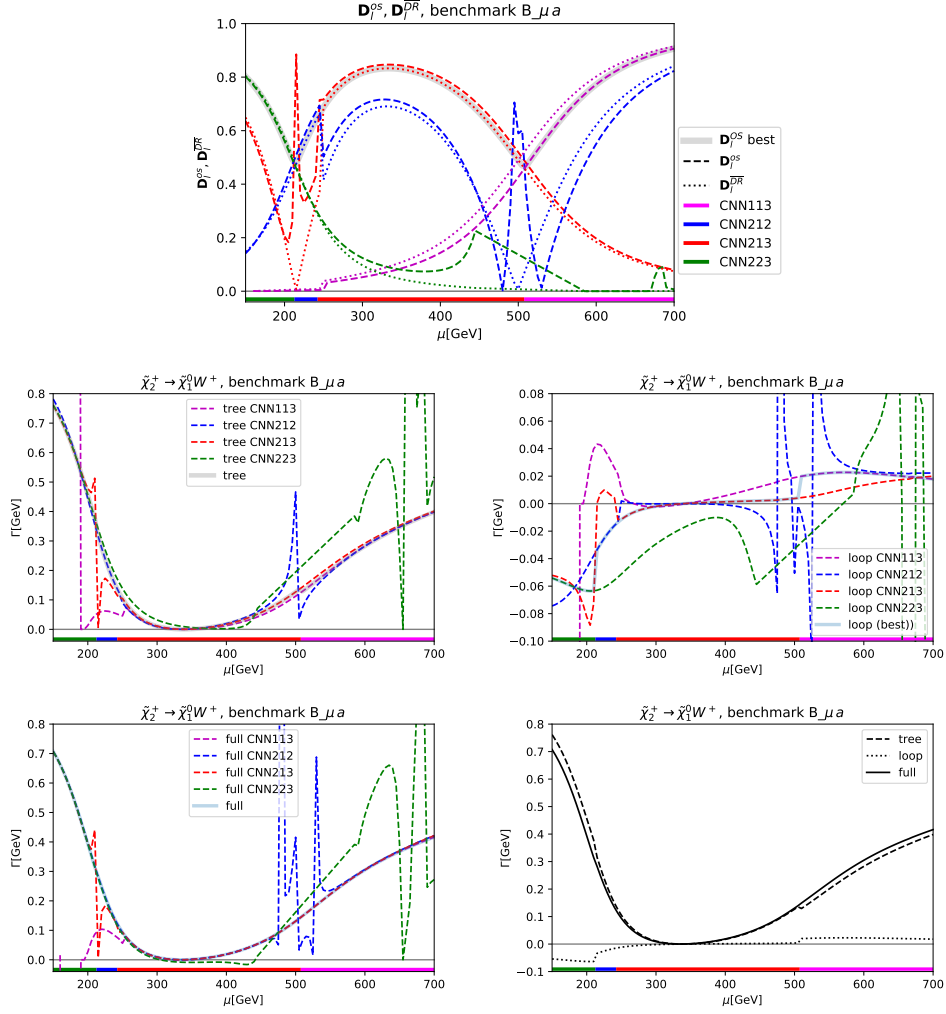
As numerical example of the application of our procedure we show in Fig. 1 the results for the decay width for  $\Gamma(\tilde{\chi}_2^+ \rightarrow \tilde{\chi}_1^0 W^+)$  as a function of  $\mu$  for  $M_1 = 200$  GeV,  $M_2 = 500$  GeV and  $\tan\beta = 10$ . The results were obtained using the `FeynArts/FormCalc/LoopTools` set-up [5–7] with the MSSM model file as defined in Ref. [2]. The upper plot shows the normalized determinants  $\mathbf{D}_I^{\overline{\text{DR}}}$  (dotted) and  $\mathbf{D}_I^{\text{OS}}$  (dashed), see Eq. (2.31) in four colors for the four “best RS”. The results of the “selected best RS” are overlaid with a gray band. The horizontal colored bar indicates this best RS for the corresponding value of  $\mu$ , following the same color coding as the curves:  $\text{CNN}_{223}$  for  $\mu \lesssim 210$  GeV,  $\text{CNN}_{212}$  for  $215 \text{ GeV} \lesssim \mu \lesssim 240$  GeV,  $\text{CNN}_{213}$  for  $245 \text{ GeV} \lesssim \mu \lesssim 505$  GeV,  $\text{CNN}_{113}$  for  $510 \text{ GeV} \lesssim \mu$ . In this example the selected best scheme has determinants larger than  $\sim 0.5$ , indicating that the counter terms can be determined reliably. The middle left figure shows the tree results for the same four selected RS as colored dashed lines, and the results of the “selected best RS” are again overlaid with a gray band. One can observe that where a scheme is chosen, the tree level width behaves “well” and smooth. It reaches zero at  $\mu \sim 330$  GeV because the involved tree-level coupling has an (accidental) zero crossing. On the other hand, outside the selected interval the tree-level result behave highly irregular, induced by the shifts in the mass matrices to obtain OS masses. The middle right plot shows the “loop plus real photon emission” results with the same color coding as in the middle left plot. As for the tree-level result one sees that where a scheme is chosen the loop corrections behave smooth and the overall size stays at the level of  $\sim 10\%$  or less compared to the tree-level result. As above, outside the chosen interval the loop corrections take irregular values, which sometimes even diverge, owing to a vanishing determinant. The lower left plot, using again the same color coding, shows the sum of tree and higher-order corrections, i.e. of the two previous plots. The same pattern of numerical behavior can be observed. The chosen scheme yields a reliable higher-order corrected result, whereas other schemes result in highly irregular and clearly unreliable results. This is summarized in the lower right plot, where show the selected tree-level result as dashed line, the loop result as dotted, and the full result as solid line. The overall behavior is completely well-behaved and smooth. A remarkable feature can be observed at  $\mu \sim 500$  GeV. Here the selected tree-level result has a kink, because of a change in the shift in the OS values of the involved chargino/neutralino masses, caused by the change from switching from  $\text{CNN}_{213}$  to  $\text{CNN}_{113}$ . However, the loop corrections contain also a corresponding kink, leading to a completely smooth full one-loop result.

This shown example demonstrates the power of the new algorithm used to select *beforehand* the best RS out of many. It also demonstrates that without such a scheme choice completely unreliable results can be obtained.

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**Figure 1:**  $\Gamma(\tilde{\chi}_2^+ \rightarrow \tilde{\chi}_1^0 W^+)$  as a function of  $\mu$  for  $M_1 = 200$  GeV,  $M_2 = 500$  GeV and  $\tan\beta = 10$  (see text).

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Program, user's guide and model files are available via: <http://www.feynarts.de>.
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Program and user's guide are available via: <http://www.feynarts.de/formcalc/>.